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Conversion of substrate to product by an organism (with negligible biomass production)

\[ \begin{align*}
X & \rightarrow \text{Product} \\
S & \rightarrow P
\end{align*} \]

Monod growth kinetics
\[
\mu_m := \frac{1}{\text{(h}^{-1})} \quad K_s := 0.01 \text{ (g/L)} \quad \mu(s) := \frac{\mu_m s}{K_s + s}
\]

Yield coefficient
\[
Y_s := 0.5 \text{ (g cell/g substrate)}
\]

Product formation
\[
\alpha := 0.4 \text{ (mg product/g cell)} \quad \beta := 0.5 \text{ (h}^{-1}\text{mg product/g cell)}
\]

Operating condition

Dilution rate \(D := 0.8 \mu_m\) \(D = 0.8 \text{ (h}^{-1})\)
Feed substrate concentration \(s_0 := 1 \text{ (g/L)}\)

Find steady-states (via analytical solutions)
\[
\begin{align*}
\frac{dx}{dt} & = 0 = (\mu(s) - D)x \quad \mu(s) = D \quad s(D) = \frac{K_s}{D} \\
\frac{ds}{dt} & = 0 = D(s_0 - s) - \frac{1}{Y_s} \mu(s)x \quad x(D) = \frac{D}{s(s_0 - s)} \\
\frac{dp}{dt} & = 0 = -Dp + \alpha \mu(s)x + \beta x \quad p(D) = \left(\alpha + \frac{\beta}{D}\right)x(D)
\end{align*}
\]

Productivity at the given \(D\)
\(Dp(D) = 0.394\) \(\text{ (mg product/h)}\)

Plot steady-states \(D := 0.01, 0.02 \ldots 0.99\)