Problem Statement. Consider a liquid metal ($Pr<<1$), with free stream conditions $u_\infty$ and $T_\infty$, in parallel flow over an isothermal flat plate at $T_s$. Assuming that $u=u_\infty$ throughout the thermal boundary layer, write the corresponding form of the boundary layer energy equation. Applying appropriate initial ($x=0$) and boundary conditions, solve this equation for the boundary layer temperature field, $T(x, y)$.

Use the result to obtain an expression for the local Nusselt number $Nu_x$. Hint: This problem is analogous to one-dimensional heat transfer in a semi-infinite medium with a sudden change in surface temperature.

Given $u=u_\infty$. Applying continuity equation $\frac{du}{dx} + \frac{dv}{dy} = 0$ leads to: $\frac{dv}{dy} = 0$

Since $v=0$ at $y=0$, $v=0$ for all $y$.

Energy equation in the boundary layer:

$$u \frac{dT}{dx} + v \frac{dT}{dy} = \alpha \frac{d^2 T}{dy^2}$$

Compare the above boundary layer energy equation to the transient thermal conduction equation (Eqn 5.29 of Bergman). Make the following substitution: $t\rightarrow x$, $x\rightarrow y$, $\alpha\rightarrow \alpha/u_\infty$. B.C. $T_{semi-infinite}(x,t=0)=T_i \rightarrow T(x=0,y)=T_\infty$, $T_{semi-infinite}(x=0,t)=T_s \rightarrow T(x,y=0)=T_s$ in the solution derived in Chapter 5.7 of Bergman for the semi-infinite solid.

Semi-infinite solid $\frac{T - T_s}{T_i - T_s} = \text{erf} \left( \frac{x}{\sqrt{4\alpha t}} \right)$

Semi-infinite solid $q''_s = k \frac{T_s - T_i}{\sqrt{\pi \alpha t}}$

local convective heat transfer coefficient $h_x = \frac{k q''_s}{T_s - T_\infty}$

local Nusselt number $Nu_x = \frac{h_x x}{k} = \frac{1}{\sqrt{\pi \alpha x}} \sqrt{\frac{u_\infty}{\alpha}} \sqrt{\frac{1}{\alpha \pi}} \sqrt{\frac{Re_x \cdot Pr}{0.564 \cdot \sqrt{Pe}}}$

where $Re_x = \frac{u_\infty x}{v}$, $Pr = \frac{\nu}{\alpha}$, $Pe = Re_x \cdot Pr$

Note that the average convective heat transfer coefficient and the average Nusselt number are 2X of the local values. Note that the average Nusselt number is not the local Nusselt number averaged over $x$. 

Given $u=u_\infty$, the correct form is:

$$h_{x,ave} = k \frac{1}{\sqrt{\pi \alpha}} \int_0^x \frac{1}{\sqrt{x'}} dx = k \frac{u_\infty}{\sqrt{\pi \alpha}} \sqrt{\frac{2}{3}} Nu_x$$

The wrong form is:

$$h_{x,ave} = k \frac{1}{\sqrt{\pi \alpha}} \int_0^x \sqrt{x'} dx = k \frac{u_\infty}{\sqrt{\pi \alpha}} 2 \cdot Nu_x$$