Active Scattering Synthesis

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***Abstract*—The scattering matrix one-port technique of an active synthesis of R. W. Wohlers is reviewed, an example given to illustrate, and an extension to n-ports given. This involves passive synthesis by the use of two bounded real rational (BR) scattering matrices with an additional negative impedance converter at each of the ports of one of them with the product realized through a 3n-port circulator.**

Keywords—Active n-port, BR scattering matrix, negative impedance converter, circulator.

I. Introduction

In1969 his book [1] on passive circuits, one finds a short treatment of active circuits in section 3.4, entitled "Active Lumped Networks." At the end of the section is a short page devoted to the synthesis of any real-rational reflection coefficient as the ratio of two real-rational bounded-real (BR) reflection coefficients; however no examples are given. Among the ideas is that a negative impedance converter, when loaded with a reflection coefficient circuit, inverts the reflection coefficient and then when multiplying with another gives their ratio; this product can then be physically realized via a passive 3-port circulator. Thus, given S(s) one writes it as $S(s)=S_1(s)/S_L(s)$. Then comes the key of the method in that both S_1 and S_L can be chosen BR, that is rational with real coefficient and bounded in magnitude in the closed right half s-plane. Such are known to be synthesized by passive one-ports [2][3][4]. Here we review the method of Wohlers, give an example, and show how to extend this theory to the realization of any real-rational n-port scattering matrix.

II. The Basic Background

As background, the necessary scattering concepts are reviewed In terms of the Laplace transform complex variable $s=\sigma+j\omega$, the scattering matrix, S(s), of an n-port circuit normalized to 1 Ohm resistor terminations at each port is defined through the n-vector incident voltage, v^i, and reflected, v^r, which are further defined in terms of the port n-vector voltages, v, and currents, i, via

$$2v^i = v = i. \quad 2v^r = v - i \qquad v^r = S(s)v^i \qquad (1a)$$

When the admittance matrix Y(s) exists it and S(s) are interrelated by [here 1_n is the nxn identity matrix]

$$S = (1_n - Y)(1_n+Y)^{-1} \text{ and } Y = (1_n-S)((1_n+S)^{-1} \quad (1b)$$

As we will treat only real circuits with a finite number of lumped components the entries of S(s) and Y(s) will be rational functions in s with real coefficients. And for passive circuits S(s) will be Bounded Real, abbreviated BR when rational, and Y(s) will be Positive Real, PR when rational. In the BR case this means that there are no poles in $\sigma \geq 0$ [that is, S is finite at infinity with strictly Hurwitz denominators] and on the $j\omega$ axis the Resistivity matrix

$$R(s) = 1_n - S(-s)^T S(s) \qquad (1c)$$

is a positive semi-definite matrix on the $j\omega$ axis [here superscript T denotes matrix transpose]. In the scalar, n=1, case treated by Wohlers, S is the reflection coefficient and the condition on $R(j\omega)$ is that the magnitude of S is no larger than 1, that is, $|S(j\omega| \leq 1$.

Of importance is the fact that the product $S=S_1S_2$ of two nxn BR matrices, S_1 and S_2 is also BR and that the product is physically realized by terminating 2n ports of a 3n-port circulator in two of the n-ports, S_1 & S_2, with the S_1S_2 product seen at the other n-ports. The circulator is passive, even lossless, and realized by 3n unit gyrators [2].

III. The Active Structure

For active synthesis it is noted that if a load admittance Y_L of a scattering matrix S_L is replaced by $-Y_L$ in the formula for S of (1b) then S_L is replaced by its inverse S_L^{-1}. But a negative Y is obtained by cascading a 2-port Negative Impedance Converter, NIC, with each port of the circuit for Y. Thus if $S_2=S_L^{-1}$ then the circulator input becomes $S=S_1S_L^{-1}$; if S_1 and S_L are BR then, by virtue of the active NICs, S need not be BR. In truth this allows any real-rational nxn matrix to be realized by passive circuits and n 2-port NICs where in fact the only active components needed are n resistors normalized to -1 Ohm, as is next discussed.

Figure 1 shows the structure for 1-ports as per Wohlers.

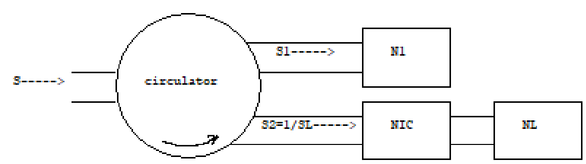

Figure 1: Wohlers' 1-port active 1-port configuration

In this the circulator has the 3-port scattering matrix

$$\text{Scirc} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad (2)$$

Figure 2 shows the negative impedance converter where the resistors are chosen to the scattering matrix resistance normalization in magnitude of R=1 Ohm with the middle one being negative.

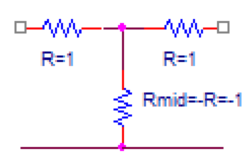

Figure 2: A negative impedance converter

For this NIC the admittance matrix is with G=1/R=1

$$\mathrm{Ynic} = \begin{pmatrix} 0 & G \\ G & 0 \end{pmatrix} \qquad (3)$$

When loaded by a 1-port admittance Y_L this gives an input admittance $Y_2 = -GY_LG = -Y_L$ when G=1. As seen above this gives $S_2(s)=1/S_L=(1+Y_L)/(1-Y_L)^{-1}$ allowing the circulator to multiply it by S_1 to give the desired $S_1S_L^{-1}$. Many means of physically realizing Ynic are available [5][6] but one can make it using two Operational Transconductance Amplifiers. OTAs, or the three resistors of Fig. 3 with one being the active negative resistor. The circulator also can be realized by 3 gyrators, of gyration conductance 1, having one gyrator for each nonzero entry in (2). So the question turns to making BR each of S_1 and S_L given a non-BR scattering matrix S. In the 1-port case where S=N/D with N(s) and D(s) are real numerator and denominator polynomials, one shifts D to the numerator of S_L and gives a common strictly Hurwitz denominator $D_L(s)$ of degree greater or equal to that of N(s) and with a constant multiplier to assure that the magnitudes of S_1 and S_L are smaller or equal to 1 on the jω axis. A simple example should make this clear.

Example 1: Given $S(s) = 3s^2/(s-1)$. This is not BR having a pole at infinity and a pole in the right-half plane at s=1. Dividing the numerator by a strictly Hurwitz degree two polynomial, say $(s+1)^2$ multiplied by the constant 3 allows the choice of $S_1(s)=3s^2/[3(s+1)^2]$ and $S_L(s)=(s-1)/[3(s+1)^2]$ both of which are BR. These are synthesized by conversion to their admittances, $Y_1(s)=(2s+1)/(2s^2+2s+1)$ and $Y_L(s)=(3s^2+5s+1)/(3s^2+7s+1)$ both of which are PR and synthesized by any realization method available, in general by the Bott-Duffin transformerless scheme [7]. Insertion into the blocks of Figure 1 gives a synthesis of the given reflection coefficient. The circuits for Y1 and Yl are shown in Figure 3.

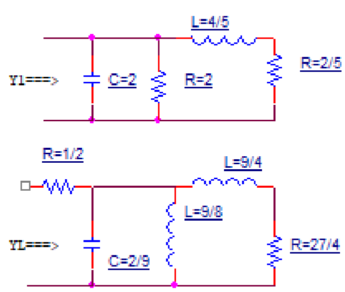

Figure 3: Circuits for example 1

We see that to realize this degree 2 active scattering matrix it takes 3 gyrators (for the circulator) 4 positive and one negative resistor (in the NIC), 2 capacitors and 3 inductors.

IV. Extension to Matrix S

The extension to matrices is almost immediate. One first replaces the 0's and 1's in Equations (2) and (3) by 0_n and 1_n, the nxn zero matrix and the nxn identity matrix, as used in (1), and also using S_L^{-1} for $1/S_L$ in Figure 1. The challenge is choosing a means to guarantee S_1 and S_L are BR matrices. The most obvious means is to first make all denominators of entries in the given S to be the least common denominator scalar polynomial $D_{lcd}(s)$ so that $S = N(s)/D_{lcd}(s)$. Then form a strictly Hurwitz polynomial h(s) of degree at least the highest of any polynomial in the nxn polynomial matrix N(s) while also including a positive scalar constant c. Then form $S_1= N(s)/[ch(s)]$ and a diagonal $S_L(s)=[D_{lcd}(s)/ch(s)]$. Since c^2 will appear in the denominator of the non-1_n term of the Resistivity matrix (1c), c can be chosen large enough to force the Resistivity matrices of both S_1 and S_L to guarantee that these two matrices are BR. The configuration of Figure 1 remains valid with all ports being n-ports. This gives the means to synthesize any real rational scattering matrix S(s).

Example 2: Given the non-BR 2-port scattering matrix

$$S(s) = \begin{pmatrix} \frac{s^2}{s-1} & 2s \\ \frac{-3}{s} & 4 \end{pmatrix} = \frac{1}{[s\cdot(s-1)]}\begin{bmatrix} s^3 & 2s^2\cdot(s-1) \\ -3\cdot(s-1) & 4\cdot s\cdot(s-1) \end{bmatrix}$$

Form

$$S_1(s) = \left[\frac{1}{c\cdot(s+1)^3}\right]\cdot\begin{bmatrix} s^3 & 2\cdot s^2\cdot(s-1) \\ -3\cdot(s-1) & 4\cdot s\cdot(s-1) \end{bmatrix}$$

$$S_L(s) = \left[\frac{s\cdot(s-1)}{c\cdot(s+1)^3}\right]\cdot\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Then there is a minimum constant c_{min} such that for any choice of $c \geq cmin \geq 4$, both of S_1 and S_L are BR and can be synthesized by passive 2-ports [2]. Using the 6-port circulator and two 2-port NICs in Figure 1 extended to 2-port inputs, this S is synthesized by an active circuit using two negative resistors (one for each port of S_L).

V. Conclusions

An example is given to illustrate Wohlers' synthesis of active 1-ports from their reflection coefficient. For this the theory is solid, depending upon the use of a negative impedance converter which is readily realized with present day transistor circuits using back to back OTAs of the same polarity. The theory is readily extended to the synthesis of active n-ports via the creation of BR scattering matrices to realize $S=S_1S_L^{-1}$. This also raises a number of interesting open problems concerning the best way to create S_1 and SL from S. For example, the use of co-prime factorizations [8][9] should prove of interest. As seen by Example 1, a large number of elements is used but all are practically realized with today's CMOS technology. However different choices of the strictly Hurwitz polynomial will lead to different circuits. The technique also uses a circulator which in turn can be realized by passive gyrators that are readily made with two OTAs, one being reversed in polarity from the other

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