Pretransistorization of Granular Chain via Simulink

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***Abstract*—After a review of the coupled Newton's equations for a granular alignment, the equations are put into block diagrams of Simulink with attention to formulating them suitably for realization by MOS transistor circuits. Simulink simulations are given for 22 grain systems with cubic potential energy. We obtain the expected granular solitary waves in our simulations.**

I. INTRODUCTION

The types of grains under consideration comprise a one-dimensional chain of symmetric elastic grains such that an input pulse travels through compression along the chain. By experiment [1], [2], through simulations [3], and by series approximations [4], [5], the pulses are known to be able to form into solitary waves and since action potentials are solitary waves, these are similar to the signals used by biological neurons [6, p. 42] and of considerable interest for mimicking neural information processing. Therefore, these grains can be seen as an alternate means of forming the pulses used in silicon based pulse coded neural networks [7]. Alternatively, their equations can be put into a form which allows for an equivalent transistor structure having the key properties of the elastic spherical grains. Consequently, toward that goal, in this paper we present a Simulink model of these grains in a form that allows for future conversion into VLSI circuits.

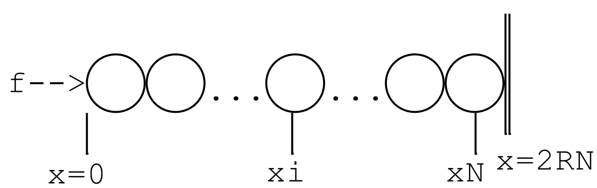

Figure 1. Chain of grains of radii R placed between rigid walls.

II. DESCRIBING EQUATIONS

Figure 1 gives a one-dimensional representation of the granular spheres which we here assume all have the same radius R. We consider N grains with x_i being the coordinate of the center of the i^{th} grain. For i=1 an external impulse-like force is assumed applied while the final grain, at x_N, a rigid wall is assumed. We use the Hamiltonian, H(p,q) representation where p = momentum N-vector and q = position N-vector and H is the sum of the kinetic and the potential, V(.,.), energies. Thus

$$H(p,q) = \sum_{l=1}^{N}\left(\frac{1}{2m}p_i + V(q_{i-1},q_i)\right),$$

$$p_i = m\frac{\partial q_i}{\partial t}, \qquad (1a, b, c)$$

$$V(q_{i-1},q_i) = k[(x_{i-1}+R)-(x_i-R)]^{r+1}.$$

Here x_i is the position of the center of the i^{th} grain measured from an origin $x_0+R = 0$. The potential energy depends on the overlap of two adjacent grains and is zero if there is no overlap, so following [3] the symbol [x] = (x+|x|)/2 is used in (1) to designate x if x > 0 and zero if x < 0. The power r+1 is due to Hertz [8] and known to be 5/2 for elastic spheres though we simulate with other r as well [9]; especially r = 2 is convenient for transistorization. The mass of a grain is m and k comprises various constants including Young's modulus. Although (1) can be realized by transistors, we obtain similar results for other values, especially for r = 2 which is easier to realize with transistors, so we set up the equations with r as a parameter.

Circuit realizations are most easily obtained through the state variable equations which in this case are the Hamilton differential equations.

$$\frac{\partial q_i}{\partial t} = \frac{\partial H}{\partial p_i} = \frac{1}{m}p_i,$$

$$\frac{\partial p_i}{\partial t} = -\frac{\partial H}{\partial q_i} = m\frac{\partial^2 q_i}{\partial t^2}. \qquad (2a,b)$$

By normalizations, and introducing possible loss (by the parameter k_{loss}) we recast these into the following form which are the actual ones we put into Simulink in the following paragraphs.

$$\frac{dx_i}{dt} = z_i,$$

$$\frac{dz_i}{dt} = \frac{d(\frac{dx_i}{dt})}{dt} = (\frac{k}{m})\{(\mathrm{sign}(x_{i-1} - x_i)(x_{i-1} - x_i))^r - ((\mathrm{sign}(x_i - x_{i+1})(x_{i+1} - x_i))^r\} - k_{loss} z_i. \quad (3a,b)$$

Equations (3) are for i = 2, …, N while at i = 1 an additive input term, f(t), is to be added and the x_0 term omitted while at i = N a fixed boundary is imposed by fixing $x_{N+1} = (2N+1)R$. However, since the differences of position hold, the x_i can be interpreted as the incremental displacement of the center of the i^{th} sphere. For solitary waves of velocity c we have $x_i(t) = u(x_i\text{-}ct)$. Following normalizations of Chatterjee [3], this gives the second order differential equation for the solitary wave

$$\ddot{u} = [u(t+1) - u(t)]^r - [u(t) - u(t-1)]^r. \quad (4)$$

From these Chatterjee shows MatLab simulations indicating the existence of solitary waves while Sen & Manciu [4] give a series solution approximation. In detail, [4], with α = xi-ct and n a parameter.

$$u(\alpha, n)) = \frac{A}{2}(1 - \tanh(F(\alpha(n))),$$

$$F(\alpha, n)) = \frac{1}{2}\sum_{q=0}^{\infty} C_{2q+1}(n)\alpha^{2q+1}. \quad (5a,b)$$

The C_{2q+1} have been evaluated for q = 0,…,5 and the results shown to be solitary type waves (see Fig. 3.1 of [7]). In short these grains are known to support solitary waves.

Consequently we know that we can obtain solitary waves from the state variable equations (3) so it is to them we turn for possible transistor realization. Toward that we obtain next a suitable block diagram realization.

III. Simulink Block Diagrams

Although we simulate for much larger N, for convenience of illustration Fig. 2 shows a Simulink block diagram for N=5 stages of grains with the subblocks for i=2,3,4 being given in detail by Fig. 3 while the input, i=1, and output, i=N=5, stages are given in Figs. 4 & 5. Figure 3 realizes equations (3) while the input and output stages, are simple modifications reflecting their different loading. In Fig. 2 an input pulse is applied on the left to the input stage.

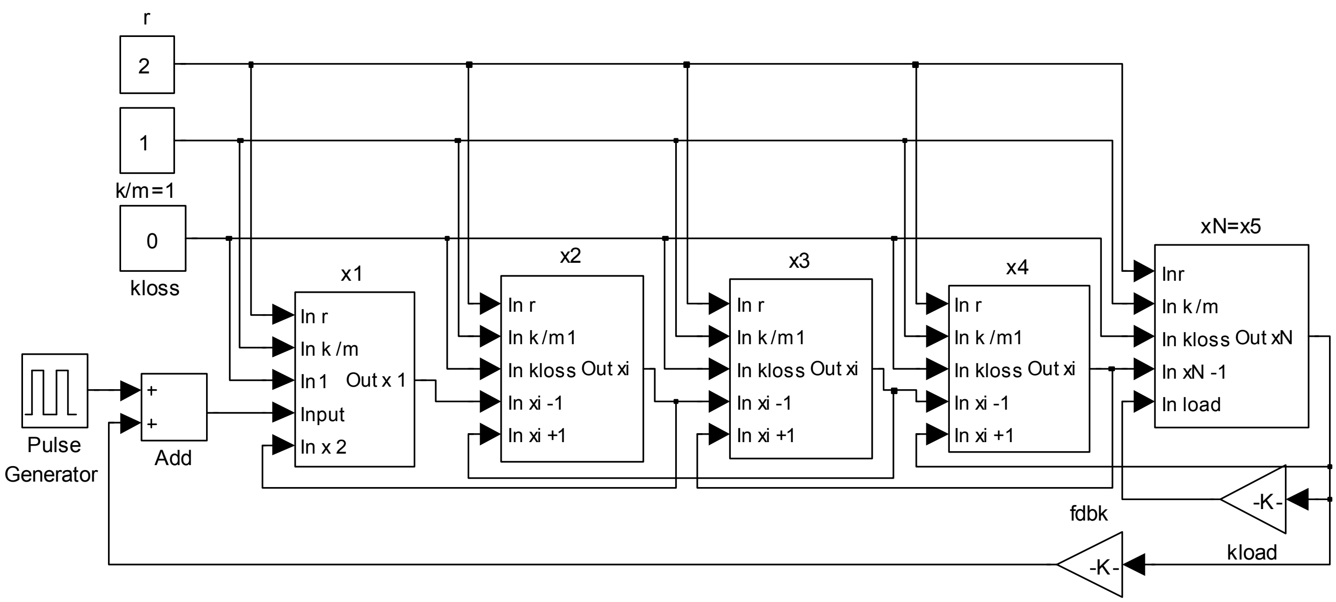

Figure 2. Simulink 5 grains Simulink block connection

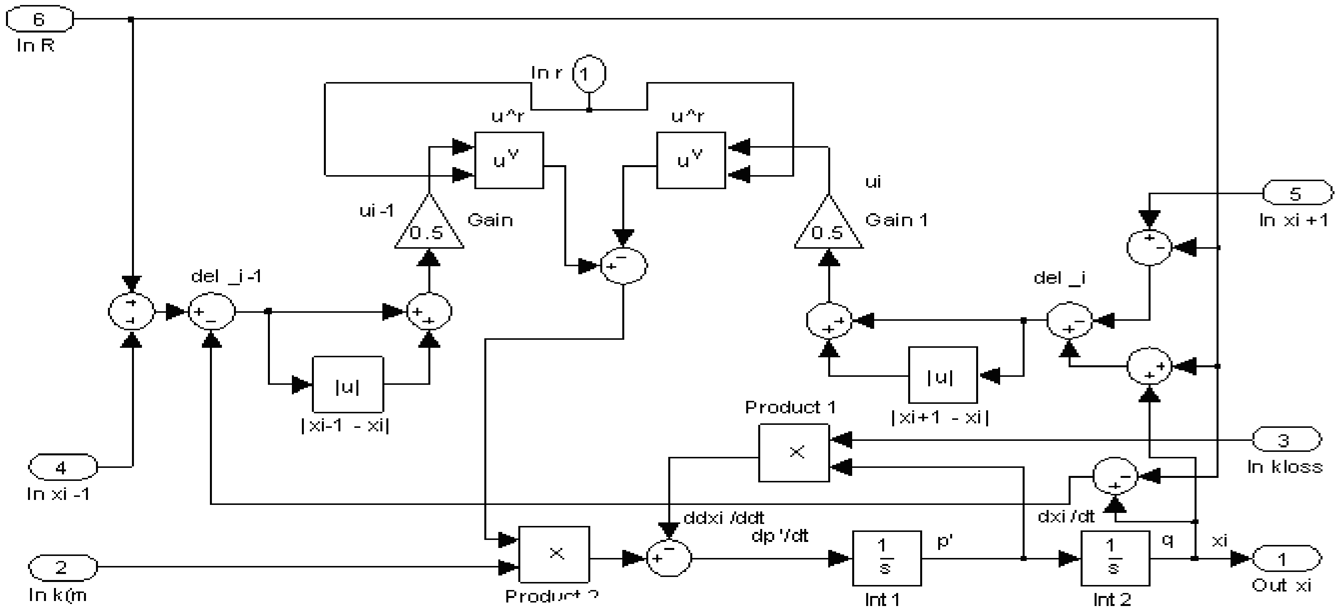

Figure 3. Simulink ith internal grain stage.

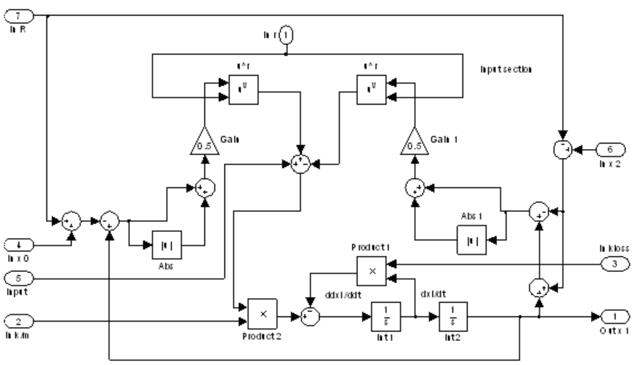

Figure 4. Detailed Simulink input grain, i=1, stage

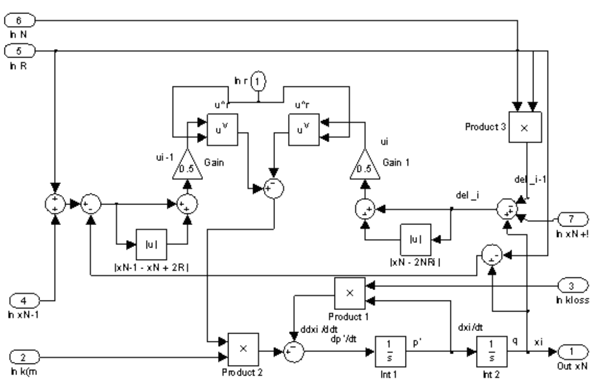

Figure 5. Detailed Simulink output grain stage, i=N

IV. Simulation Results

Figure 6(at the end of the paper) gives a plot of dx/dt at the fourth and the 21st stages for r=2 and m/k normalized to 1, showing solitary waves as well as their reflection from the N=22 end wall. For Fig. 6 a square input pulse of amplitude 10^{-6} and pulse width t=5x10. This results in a traveling wave of amplitude $3.5\text{x}10^{-9}$ with a delay of t=2,500 at the 4th grain, for r=2. A reflected wave can be seen at t=32,000 with the solitary pulse arriving at the 21st grain at t=15,000, all in normalized time.

V. Discussion

As a step toward realizing the grains equations in MOS transistor form we have put the grains differential equations into state variable form, (3) above. From these we are able to set up block diagrams which use only integrators, multipliers, square roots, and summers. From those put into Simulink we have shown that solitary signals can be generated. In the case of grains satisfying the Hertz potentials these blocks necessitate square roots in obtaining the 3/2 power. However, from the simulations we obtain similar results for powers of r=2 and 3/2, though with a different time scaling as for example the 4th stage peak occurs at t=200 for r = 3/2. For transistor realizations, in using r=2 we can avoid the need for circuits to give square roots. We have normalized m/k=1 and can consider x_i as the absolute displacement of the ith grain or the change in displacement; we have chosen the latter for the given block diagrams (which also uses R=1 for the right wall). In the Simulink system we have the capability of using any real r as well as the added possibility of including loss though it is not present in the basic grains equations. The above can be generalized to allow for different radii and for different materials of different individual grains within the system ([21]] gives material constants for various materials).

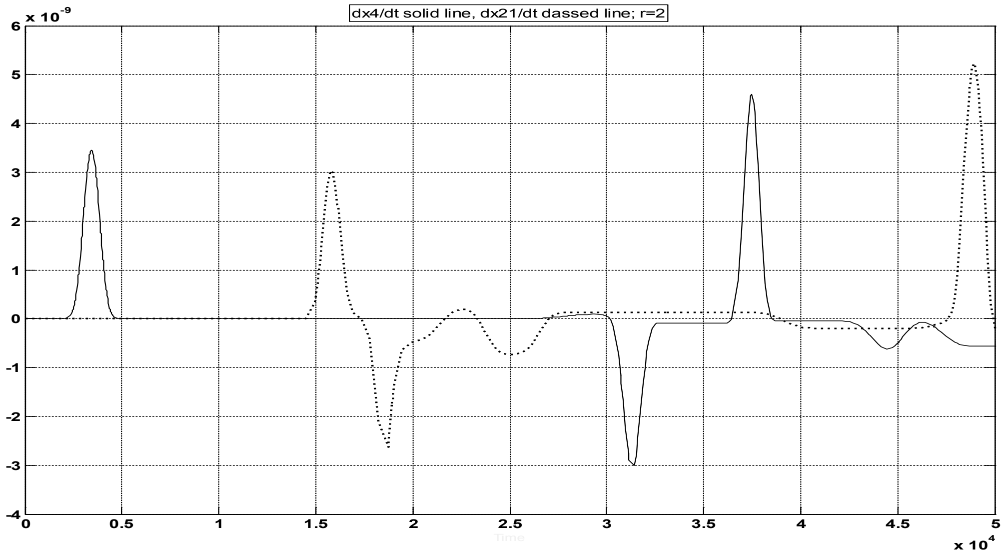

Figure 6. Simulation result:for r=2; solid line dx/dt at 4thgrain ; dashed line dx/dt at 21st grain both for N=22 grain

Since this is an area not too familiar to many readers, some useful additional references are included [12-21]. For example, there are other effects which can be included, one of which uses the "coefficient of restitution" [11], while generalization to higher dimensions is possible, such as for sand at the beach. Also the above equations are normalized and will need eventual denormalization when realized by physical transistors.

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