

Psychic Fields
Basics and Main Ideas

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Abstract:

The theory of psychic fields is introduced and results derived therein presented. The theory proceeds in an n-dimensional psychic to psychic fields, obtains within a Clifford algebra. By separating the psychic field quantities into excitation and intensity vectors and the n coordinates into time and psychic space psychic effects can be justified. These include a) precognition, through noncausal waves, b) love, hate, and passion, through the looping of fields and forces of psychic charge and currents, c) the male and female psyche, through signed charges, and d) interactions with a supreme being through boundary conditions and bias fields. A hitherto unconsidered holography is proposed as a conceivable means of experimentation.

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Prologue:

“What has made us so different from our forebears is that we have become conscious of the movement which is carrying us along.” [1, p.215]

I. Introduction

“Religion and science are the two conjugated faces or phases of one and the same complete act of knowledge – the only one which can embrace the past and future of evolution so as to contemplate, measure and fulfill them.” [1, p.285]

Interactions between individuals on the one hand and individuals and their creator on another are of most fundamental importance to humans in their specific achievements and the human race in its attainments as a whole. Since a primary way of characterizing interactions at a distance is through fields, it seems not unwarranted to develop the theory of fields for these fundamental interactions. This is what is initiated here.

As may well be imagined, and will be seen, there is a religious aspect to the interpretation of the physical laws which are to be presented mathematically. The physical laws generalize those of electromagnetism based upon Maxwell's equations [2], as used in communication electronics of the day, while the religious aspects follow the ideas of Pierre Teilhard de Cardin [1][3]. Some bare developments in these directions were initiated on a scientific basis by Stromberg [4], though in both areas we will find extensions and complete redevelopments of available outlooks.

In order not to cloud the main ideas the necessary mathematical theories will not be developed here, though the results will be presented and interpretations begun. For this, presentation of mathematical detail and derivations are being initiated elsewhere [5]. Nevertheless, in order that this work may be complete in itself the mathematical concepts are all outlined in the Appendix. To orient ourselves it is worth mentioning that this mathematics uses heavily Clifford algebras in their space – time framework, incorporating differential operators, of M. Riesz [6] with solutions to wave equations developed through convolution operators within the theory of distributions of L. Schwartz [7]. Indeed it was the clarity of the Clifford algebra representation in organizing these ideas which led to the following formulation of psychic laws. And, as with all physical laws, the justification in formulating them is not the replacement of emotions and religion by scientific quantities – quite the opposite.

Interpretations are given in Section III which it is hoped may be sufficiently self-contained to be understood on its own.

II. The Mathematical Framework

“I think it important to point out that two basic assumptions go hand in hand to support and govern every development of the theme. The first is the primacy accorded to the psychic and to thought in the stuff of the universe.” [1, p.30]

The fields under consideration will be called psychic fields and their interactions called psychic interactions. These interactions are often thought to occur in a “space” of dimension beyond that for the earth. Consequently, the psychic reals will be taken to be (real Euclidean vector space of dimension n with $n > 4$, this being fundamentally hypothesized. There are then coordinates x_1, x_2, \dots, x_n in terms of which psychic field quantities are expressed; \underline{x} will denote a (n -dimensional, vector) point in the psychic reals with these coordinates. Since time t and normal 3-dimensional space, here called real space (of coordinates x, y, z), occur in psychic phenomena, it can be assumed that there is a privileged coordinate system in which the first coordinate of the psychic realm is t , the second x , etc.; thus, it will be assumed that

$$x_1 = t, \text{ time} \quad (\text{II.1a})$$

$$x_2 = x, x_3 = y, x_4 = z \text{ (real space)} \quad (\text{II.1b})$$

Within this privileged coordinate system psychic “distances” are measured. For such measurements the means of measuring distance ds between two adjacent points \underline{x} and $\underline{x} + d\underline{x}$ must be defined; this will be taken according to the (fundamental metric) law

$$(ds)^2 = (dx_1)^2 - \sum_{i=2}^n (dx_i)^2 \quad (\text{II.2})$$

where $d\underline{x}$ is the vector of differences in coordinates of the two adjacent points. Within the psychic realm we then have a time subrealm (of dimension 1) and a space subrealm (of dimension $n-1$) called psychic space. Distances between two adjacent points in the psychic realm are measured in terms of those in the space and time subrealms according to (II.2). In actual fact (II.2) is to be taken as valid only at points of the psychic realm where no “psychic substance” is present; otherwise another fundamental metric (the general M of the Appendix) is taken to describe the psychic media. What is important is that there is some fundamental metric law according to which psychic distances can be measured in the n dimensions.

Given any quantities defined over the psychic realm we are in a position, via (II.2), to make measurements upon them. In particular if E represents psychic excitation field quantities we can measure changes on E in moving from one psychic realm point to another. Mathematically this means that the gradient, ∇E , of E can be calculated. Changes in the psychic field quantities would naturally come from psychic sources C . Thus, the psychic excitation field is postulated to satisfy the n -dimensional basic equation

$$\nabla E = C \quad (\text{II.3})$$

Technically, this basic equation is in a $2n$ -dimensional Clifford algebra formed on the n -dimensional psychic realm; E is a 2-vector in the Clifford algebra, C is a 1-vector, and ∇ is a 1-vector differential operator which takes first derivatives with respect to the coordinates while incorporating the metric of the psychic realm (see Appendix). The

basic equation is rather all encompassing, and, hence, useful for further developments and extensions of the theory, though difficult to use in deriving specific results. Thus, derived equations will be investigated in order to obtain concrete interpretations.

It is first possible to express the psychic excitation field quantities E as the gradient of a (Clifford algebra 1-vector) potential P ; thus, $E = \nabla P$. That is, the excitation field is itself due to the changes in another quantity, the potential P , in which case C is equal to second changes in the potential. Then the basic equation, (II.3), becomes the wave equation for the psychic potential P

$$\square P = C \quad (II.4)$$

where the wave operator $\square = \nabla \nabla$ takes second derivatives with respect to the coordinate variables again taking into account the metric of the psychic reals. This wave equation has travelling wave solutions for P , and hence E , which are of interest to us.

In order to obtain further physically interpretable equations from the basic equation it is important to introduce added quantities. In particular, introduction of the intensity field I , related directly to E through the metric M of the psychic realm (see Appendix, yields the decomposition

$$\nabla \cdot E = C \quad (II.5a)$$

$$\nabla \wedge I = 0 \quad (II.5b)$$

where \cdot and \wedge are divergence and curl operations, respectively. The first of (II.5) says that the psychic excitation field diverges from sources while the second says that the intensity field is free of curl, that is, its flow is smooth and free of eddies. We will see that psychic forces result from the intensity field.

By isolating time as a special coordinate we can separate the excitation and intensity fields further. This will give an amazing set of equations, (II.6), from which our main interpretations will result. Thus, we delineate the following field vectors, which will be interpreted in the next section (for these quantities the subscript e denotes the entity results from a partition of E while the subscript I denotes the same with respect to I).

F_e = psychic excitation field

S_e = soul excitation field

F_i = psychic field intensity

S_i = soul induction field.

We delineate as well two types of sources for these fields

C_p = psychic charge density

C_S = soul current density

The field equations which relate these arise from (II.3) (see Appendix) and are

$$\underline{\nabla} \cdot \mathbf{F}_e = C_p \quad (\text{II.6a})$$

$$\frac{\partial \mathbf{F}_e}{\partial t} + (-1)^n \underline{i}_s \underline{\nabla} \wedge \mathbf{S}_e = C_s \quad (\text{II.6b})$$

$$\underline{i}_s \frac{\partial \mathbf{S}_i}{\partial t} + \underline{\nabla} \wedge \mathbf{F}_i = 0 \quad (\text{II.6c})$$

$$\underline{i}_s \underline{\nabla} \cdot \mathbf{S}_i = 0 \quad (\text{II.6d})$$

Here \underline{i}_s labels the 2^{n-1} th basis vector in the 2^{n-1} dimensional Clifford subalgebra used to represent the fields in the psychic space subreals while $\underline{\nabla}$ operates in this same subalgebra and is essentially the last $n-1$ (psychic) space) components of ∇ ; ∂ denotes the partial derivative, that is changes due just to the quantity indicated, as time in this case.

These field equations are quite analogous to the classical Maxwell equations of electromagnetic theory from which interpretations become available. Consequently, using the considerable body of knowledge of electromagnetism, including radio wave propagation, we have readily available information on psychic phenomena. In fact since these equations can be seen to reduce to the electromagnetic theory equations when $n = 4$ [5], it may have been appropriate to use the same symbols used in electromagnetic field theory for our field quantities. But the notation above seems more suggestive of the physical meanings to be attached.

III. Physical Interpretations – Implications

“We have not sufficiently meditated upon the three-fold property possessed by every consciousness: (i) of centering everything partially upon itself; (ii) of being able to center itself upon itself constantly; and (iii) of being brought more by this very super-concentration into association with all the other centres surrounding it.” [1, p. 259]

The content of the theory of psychic fields is in the detailed field equations (II.6) and the wave equation (II.4). Thus, it is through interpretation of them and the quantities therein that psychic theory can most solidly proceed. Our attempt is just this in this most important section of the work.

To begin, the wave equation, (II.4), can be seen to have travelling wave solutions [8] (see Appendix) emanating from the psychic sources. The nature of these waves depends upon the psychic dimension; for n even there is a wave packet while for n odd the “region” behind the wave front is filled. Thus, different people working in different psychic dimensions would see possibly different results. Of further special interest in the psychic case is that besides causal solutions there are noncausal solutions to the wave

equation, which undoubtedly should not be ignored, as is usually done when $n = 4$. For the psychic waves there is a parameter ψ , $0 \leq \psi \leq 1$, (see Appendix), which indicates how much of the noncausal wave is present. Perhaps by changing ψ from zero to one we go from an awake state to a dream state. In any event precognition seems to be mathematically justified by $\psi \neq 0$, while telepathy could be considered as two psychic realm points lying at the same wave front value by having the same psychic realm distances a given times.

Next we move to the field equations where first (II.6a) says that the psychic charge, C_p , excites the psychic realm, setting up the psychic excitation field vector, F_e . This latter diverges from positive psychic charge and converges upon negative psychic charge; although the sign association is arbitrary, let us take $C_p > 0$ to represent the male psyche and $C_p < 0$ to represent the female. In this regard similar genders are attractive with the intensity of attraction inversely proportional to the $(n-3)$ rd power of the distance apart in the psychic space subrealm. Indeed, if an individual has a psychic space location, then it can be assumed that the individual's psychic charge resides at this location from which an excitation field is set up throughout all of psychic space. These field lines terminate upon opposite psychic charges and bend away from like ones; they move in time according to the psychic trajectories of those involved. By orienting psychic charge distributions the excitation field can be directed.

According to (II.6b), movement of this excitation field is equivalent to setting up a soul current vector, C_s , to initiate the soul excitation vector, S_e . As the soul excitation curls around one in psychic space it induces a soul current, and conversely, an individual's soul current sets up an enveloping soul excitation field which radiates through psychic space to be felt by others. As the soul excitation vector changes it also changes the soul induction field, S_i , which is related to it through the metric, M , of the psychic realm. By (II.6c) these changes in the soul induction set up the psychic field intensity, F_i , which curls around any time changes in the soul induction field; since the psychic excitation is also related to the psychic field intensity through the metric, changes in the psychic field intensity are also responsible for the soul current, by (II.6b). Finally, (II.6d) says that the soul induction field does not diverge from any type of psychic source but that its field lines close upon themselves; in doing so they may change their configuration and travel through psychic space looping other individuals' psychic currents. If the soul inductions of two individuals loop a psychic current in the same direction the mutual effect is additive and we may say that a beneficial relationship is established. If they loop oppositely, then an antagonistic relationship results, while if they fail to loop, the relationship is neutral. Various degrees of looping in either direction are possible allowing for dynamic relationships of all degrees and with any number of individuals. Indeed we may say that love is a close coupling of additive soul induction fields, passionate if the psychic charges are opposite and close, while hate is a close coupling of subtractive soul induction fields. Nearness in the psychic realm should yield a more intense relationship, and thus one would expect physical nearness in real space to lend itself to more intense relationships, while, perhaps, as relationships develop the dimension of the psychic realm in which they take place increases. Further, one field may swamp others, so that love overcomes hate (and vice versa), or so that one love is larger than others as far as one of the individuals involved is concerned. Since the psychic media may be anisotropic, nonreciprocal relationships are allowed.

Using energy – stress density vectors, E^i , one associated with each (n-1)-dimensional subrealm (and with E^i giving the psychic energy leaving the subrealm when integrated over the boundary) psychic forces f can be mathematically calculated in terms of the field quantities (see Appendix)

$$f = C_p F_i + (-1)^n \underline{1}_g C_S \wedge S_i \quad (\text{III.1})$$

This shows that the entities just mentioned (love, hate, etc.) may be quantified. Specifically, the psychic force felt by an individual is the product of the individual's psychic charge and the psychic field intensity in which it lies plus or minus a more dynamic term due to the linking of the individual's soul current by ether soul induction fields. It is through (III.1) that we see that the components of I measure intensity of force, while (II.5a) shows that the components of E represent the excitation of the psychic field by sources. The intensity and excitation fields are related through the fundamental metric of the psychic realm, and it is through nonlinearities in the metric, that is, in the psychic media, that nonlinear effects would be exhibited (possibly including mental diseases).

Further, an enveloping boundary with attendant excitations and bias fields can be incorporated to bound the active region of the psychic space and to account for the actions of a supreme being, as God. By changing the metric of the psychic realm from point to point it is also possible to represent changes in the psychic media to take into account local psychic bodies, as for example souls. And most intriguing, by means of the field equations (II.6) one should be able to control psychic interactions through design of psychic transmitters and receivers matched to them. For this, one needs a means of creating psychic charge and/or soul current. Since this creation must take place in a space of higher dimensions, it seems quite unlikely that humans living in real (3-dimensional, $n - 4$) space could consciously carry out this creation to any extensive degree. In this regard it should then be true that we are most creative in our dreams, when it seems that we should be able to enter the higher dimensional realm, and that we have not too great a control over when and with whom we fall in love or hate (It seems the poets are correct: "Unbidden, love will come to those it picks" [9, p.107]). Likewise, the theory says nothing about the types of particles (perhaps called psi particles) which develop the source charges and currents. Again these seem out of reach to "living" humans and present an almost sure purpose for a supreme being.

At this point the causal – noncausal waves most likely need further discussion. First, these travel with a velocity normalized to unity at any given point (but of actual speed c , see Appendix). There is a portion of the wave going forward and a portion going backward in time, the latter of which is predictive. Here there is no question of tunneling through a space – time barrier [10, p.2]; the question is that of having a nonzero proportionality constant ψ . The more intriguing question is: What excites the noncausal wave? For, if we knew that we could know the future at will. But since this excitation is in a higher dimension than the "physical" world, there must be a transmitting antenna matched to that higher dimensional world to excite the wave. Supposedly ψ is most often very small, meaning very little of the predictive wave present, or else for most humans the psychic receiver is not matched to the proper media to detect the wave. Further, there is nothing to prevent "inanimate" objects from having associated psychic sources, if the creator so designs, in which case psychic "pictures" can develop on an active basis. More

likely, though, the surroundings of an active body are seen through the reflection and transmission properties of the surroundings as it affects the media in which the waves propagate. It should also be noted that a theory of psychic holography, as discussed in the next section, is possible and developed through the field equations.

The theory presented is actually a special case of a more general one which would include mass fields, as in general relativity (for $n = 4$), and quantum particles as in quantum mechanics. Basically, though, these do not yield much more in terms of physical interpretations beyond what is given here. A more interesting generalization is to formulate E within the underlying Clifford algebra to be of dimension up to the maximum of 2^n possible (only E of dimension two are treated here). The normal theory of electromagnetism is contained within this theory by setting $n = 4$ (and taking the [unnormalized] velocity of wave propagation to be the speed of light) in which case one wonders why electromagnetic waves are not received as psychic waves. Several possibilities are available, including invalidity of this theory and mismatch of the psychic body to the reception of electromagnetic waves.

The question of the size of the dimension n must be raised. Perhaps it is, as implied by Teilhard de Chardin, that we “fill up” one (bounded) subrealm and then move into the next higher in climbing the evolutionary scale. But this leaves unexplained the differences between even and odd dimensions (which occur throughout the theory), what I means to fill up a subspace, and why dimension would need to jump from one integer value to another. Since dreams appear to take place in the psychic realm and since dreams appear to be switched off and on, the jumping of dimensions appears quite plausible (at least for going between $n = 4$ and $n > 4$).

IV. An Experiment

“The aim is to try to define experimentally this mysterious human by determining its position in relation to the other forms assumed around us by the stuff of the cosmos.” [3, p.13]

Because the psychic realm envelopes the 3-dimensional real space of every day life, the difficulties in checking any theory of psychic phenomena involve transcending this real space and hence appear to be immense. Nevertheless, it does seem that there is a possible experiment which could, and should, be carried out in order to really explore a portion of psychic space. This is a holographic one: By means of a 3-dimensional space hologram we should be able to enter the 4-dimensional space world (i.e. $n = 4$). A 3-dimensional hologram should relatively easily be constructed and conceivably excited into a holographic image in $n = 4$ psychic space. Focusing the image upon psychic bodies should allow penetration into individual psychic states.

Several basic and perhaps severe technical difficulties must, however, first be overcome. Foremost is that of obtaining a “coherent” source needed to excite the 3-dimensional hologram in 4-dimensions; that is, a very ideal psychic “medium” is needed. Along with this is the problem of reception. Here one needs a means of detecting a 3-dimensional holographic image which most likely must be in the range of a dreaming human serving as a receiver. For all of this to occur, a proper design of the hologram must also be made, this including material, frequency of excitation, and pattern layout. To be sure, the theory of n -dimensional holography needs still to be developed, this resting

upon the fact that n-dimensional holographic fields should be specified by their values on (n-1)-dimensional surfaces (this basically follows from the equations of Helmholtz and Green's theorem [11, Chapitre IX] but uses the more complicated Hankel functions, which reduce to the tractable exponentials as used in 3-dimensional holography; see [12, p.46 & Chapter 8][13, p.38] for this basis of holography). Certainly a working knowledge of n-dimensional spaces is needed [14].

Epilogue:

“We have seen and admitted that evolution is an ascent towards consciousness. Therefore, it should culminate forwards in some sort of supreme consciousness.” [1, p.258]

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Appendix: Outline of Mathematical Relationships

1. (II.2)

Equation (II.2) can be rewritten as

$$(ds)^2 = (d\mathbf{x})^{\sim} M(d\mathbf{x}) \quad \sim = \text{matrix transpose}$$

where the fundamental metric matrix M defines the measurement structure on the n-dimensional space. We are using at (II.2)

$$M = [m^{ij}] = \begin{bmatrix} +1 & & & \\ & -1 & & \\ & & \dots & \\ & & & -1 \end{bmatrix} \quad \text{and } G = M^{-1} = [g_{ij}]$$

M is used to define a Clifford algebra of dimension 2^n using basis vectors \underline{e}^i of the underlying n-dimensional space.

$$\underline{e}^0 = 1, \underline{e}^i = -\underline{e}^i, \underline{e}^{ij} = \underline{e}^i \underline{e}^j, \dots, \underline{e}^{12\dots n} = \underline{e}^1 \underline{e}^2 \dots \underline{e}^n$$

$$\underline{e}^i \underline{e}^i + \underline{e}^j \underline{e}^j = 2m^{ij} \underline{e}^0 \quad \text{which implies } (\underline{e}^1)^2 = \underline{e}^0, (\underline{e}^i)^2 = -\underline{e}^0 \text{ if } i \neq j$$

$$\underline{e}^i \underline{e}^j = -\underline{e}^j \underline{e}^i \text{ for the chosen } M$$

Any product of two elements in the Clifford algebra takes the form

$$ab = a \bullet b + a \wedge b$$

where

$$2a \bullet b = ab + ba \text{ and } 2a \wedge b = ab - ba$$

For the metric of (II.2)

$$\underline{e}^i \bullet \underline{e}^j = \begin{cases} 0 & \text{if } i \neq j \\ +1 & \text{if } i = j = 1 \\ -1 & \text{if } i = j \neq 1 \end{cases} \quad \underline{e}^i \wedge \underline{e}^j = \begin{cases} \underline{e}^i \underline{e}^j & \text{if } i \neq j \\ 0 & \text{if } i = j \end{cases}$$

Also let for $i, j, \dots \geq 2$

$$\underline{e}_s^i = \underline{e}^1 \underline{e}^i, \underline{e}_s^0 = \underline{e}^0, \underline{e}_s^{ij} = \underline{e}_s^i \underline{e}_s^j, \dots, \underline{e}_s^{23\dots n} = \underline{e}_s^2 \underline{e}_s^3 \dots \underline{e}_s^n$$

then the \underline{e}_s^{\dots} span the subspace of all even Clifford algebra vectors. This corresponds to the space subrealm when time is taken as the first coordinate. The coordinate vector \underline{x} can now be considered a 1-vector in the Clifford algebra, with the time subrealm of basis vector \underline{e}^1 and the space subrealm of bases $\underline{e}^2, \dots, \underline{e}^n$ or equivalently $\underline{e}_s^2, \dots, \underline{e}_s^n$. By changing M locally the local properties of the media are brought out.

2. (II.3)

$$C = \sum_{i=1}^n c_i \underline{e}^i = \text{Clifford 1 - vector of sources} = \sum_{i=1}^n c_i \underline{e}_i$$

$$E = \sum_{i=1}^n \sum_{j=1}^n E_{ij} \underline{e}^i \underline{e}^j = \text{Clifford 2 - vector of excitation field}$$

$$= \sum_{i=1}^n \sum_{j=1}^n E^{ij} \underline{e}_i \underline{e}_j, \quad E^{ij} = -E^{ji}$$

$$\nabla = \sum_{i=1}^n \partial_i \underline{e}^i = \text{Gradient operator}, \quad \partial_i = \frac{\partial}{\partial x^i} = \text{partial derivative}$$

$$\underline{e}_i = \sum_{j=1}^n g_{ij} \underline{e}^j = \text{Covariant basis vectors}; \quad \underline{e}^i = \text{Contravariant basis vectors}$$

$$x_i = \sum_{j=1}^n g_{ji} x^j \quad \text{and} \quad x^i = \sum_{j=1}^n m^{ji} x_j$$

3. (II.4)

$$\square = \nabla \nabla = \left(\sum_{i=1}^n \underline{e}^i \partial_i \right) \left(\sum_{j=1}^n \underline{e}^j \partial_j \right) = \sum_{i=1}^n \sum_{j=1}^n \underline{e}^i \underline{e}^j \partial_i \partial_j = \partial_i \partial_i - \sum_{i=2}^n \partial_i \partial_i$$

$$\square P = C \text{ is solved by letting } C = C_\delta = \sum_{i=1}^n \delta(t) \underline{e}_i \text{ with } \delta(\cdot) \text{ the unit impulse}$$

to get P_δ , $\square P_\delta = C_\delta$, and then convoluting:

$$P = P_\delta * C, \quad * = \text{convolution}$$

Setting r as the distance in the space subrealm, $r^2 = \sum_{i=2}^n x_i^2$, and $1(\cdot)$ as the unit step function, we have for the causal solution

$$P_{\delta_c} = \frac{1(t)}{(2\pi 2^k)} \left(\frac{1}{t} \frac{\partial}{\partial t} \right)^{k-1} \begin{cases} (t^2 - r^2)^{-1} 1(t-r) & n = 2k + 1, k = 1, 2, \dots \\ \frac{1}{2t} \delta(t-r) & n = 2k, k = 0, 1, 2, \dots \end{cases}$$

The completely noncausal solution is this with time reversed

$$P_{\delta_{nc}}(\underline{x}) = P_{\delta_c}(\hat{\underline{x}}) \quad \text{with } \hat{x}_1 = -x_1, \quad \hat{x}_i = x_i \quad \text{for } i \neq 1$$

The total impulse response is

$$P_\delta(\underline{x}) = \psi P_{\delta_{nc}}(\underline{x}) + (1 - \psi) P_{\delta_c}(\underline{x}) \quad \text{for } 0 \leq \psi \leq 1.$$

Waves due to point sources travel according to $t \pm r = \text{constant}$, that is with velocity one (in this “normalized” coordinate system).

4. (II.5)

$$\nabla E = \nabla \cdot E + \nabla \wedge E = C$$

Equating terms of equal dimension gives (II.5). $\nabla \cdot E$ and C are of dimension 1, $\nabla \wedge E$ has dimension 3; then use

$$I = \sum_{i=1}^n \sum_{j=1}^n \underline{e}^i I_{ij} \underline{e}^j = \text{Clifford 2 - vector intensity field}$$

$$I_{ij} = \frac{1}{\lambda \sqrt{|M|}} E_{ij} \quad \text{with} \quad E_{ij} = \sum_{k=1}^n \sum_{\ell=1}^n g_{ik} E^{k\ell} g_{\ell j},$$

$|M|$ = absolute value of determinant of M

$$\lambda = \text{unit dependence constant} \left(= \frac{1}{\mu c} = \epsilon c, \quad c = \text{speed of wave transmission} \right)$$

The basic formula of Riesz is

$$\nabla E = \sum_{i=1}^n \sum_{j=1}^n \partial_j E^{ij} \underline{e}_i + \frac{1}{i2} \sum_{k < r < s} (\partial_k E_{rs} + \partial_r E_{sk} + \partial_s E_{kr}) [\underline{e}^k, \underline{e}^r, \underline{e}^s]$$

where $[a, b, c] = 3(abc - cba)$

5. (II.6)

Let

$$E^* = \underline{e}^1 E \underline{e}^1$$

$$\begin{aligned} F_e &= \frac{1}{2} (E - E^*) = -F_e^* = \frac{1}{2} \sum_{i=1}^n \sum_{j=2}^n (\underline{e}^i \underline{e}^j - \underline{e}^1 \underline{e}^i \underline{e}^j \underline{e}^1) E_{ij} = \sum_{j=2}^n \underline{e}^1 \underline{e}^j E_{ij} \\ &= \sum_{j=2}^n \underline{e}_s^j E_{ih} \end{aligned}$$

$$\underline{i}_s = \underline{e}_s^{23\dots n} \quad \text{and} \quad \underline{i}^2 = \underline{e}^{12\dots n} = (-1)^{n-1} \underline{e}_{12\dots n}$$

Then

$$\underline{i}_s = (-2)^{[n/2]} (\underline{e}^1)^n \underline{i}, \quad \underline{i}^2 = (-1) \sum_{k=2}^n k, \quad \underline{i}^* = (-1)^{n-1} \underline{i}$$

where $[x]$ = largest integer not exceeding x . Now

$$C = \sum_{i=1}^n c_i \underline{e}^i = \underline{e}^1 (\underline{e}^1 C) \quad \text{where} \quad \underline{e}^1 C = \sum_{i=1}^n \underline{e}^1 \underline{e}^1 c_i = c_1 + \sum_{i=2}^n \underline{e}_s^i c_i = C_p + C_s$$

$$\underline{e}^1 \nabla = \sum \underline{e}^1 \underline{e}^1 \partial_i = \partial_1 + \sum_{i=2}^n \underline{e}_s^i \partial_i = \partial_1 + \underline{\nabla}$$

$$\underline{\nabla} \underline{i}_s = \sum_{i=2}^n \underline{e}_s^i \underline{e}_s^2 \dots \underline{e}_s^n \partial_i = (-1)^n \underline{e}_s^2 \dots \underline{e}_s^n \sum_{i=2}^n \underline{e}_s^i \partial_i = (-1)^n \underline{i}_s \underline{\nabla}$$

From these $\nabla E = C$ becomes, using $\nabla a = \nabla \bullet a + \nabla \wedge a$

$$\underline{e}^1 \nabla E = \underline{e}^1 C$$

$$(\partial_1 + \underline{\nabla})(F_e + \underline{i}_s S_e) = C_p + C_s$$

$$\begin{array}{cccccccc} \partial_1 F_e + \underline{i}_s \partial_1 S_e + \underline{\nabla} \bullet F_e + \underline{\nabla} \wedge F_e + (\underline{\nabla} \underline{i}_e) \bullet S_e + (\underline{\nabla} \underline{i}_s) \wedge S_e & = & C_p & + & C_s \\ \underline{e}_s^i \text{ dim} & = & 1 & & 2 & & 0 & & 2 & & 3 & & 1 & & 0 & & 1 \end{array}$$

Comparing \underline{e}_s^i dimensions gives (II.6) after dividing the terms of dimension two and three by $\lambda \sqrt{|M|}$. One can get similar types of results using

$$E_* = \underline{e}_1 E \underline{e}_1, \quad E^+ = \underline{e}^1 \overleftarrow{E} \underline{e}^1, \quad E_+ = \underline{e}_1 \overleftarrow{E} \underline{e}_1$$

where $\overleftarrow{}$ denotes reversal of all basis vector products. Analogues with electromagnetic field terms are given in [5].

6. (III.1)

Let

$$E^i = \frac{1}{\lambda \sqrt{|M|}} \frac{1}{2} \overleftarrow{E} \underline{e}^i E = \sum_{j=1}^n E_j^i \underline{e}^j = \text{Energy - stress density vector}$$

Then

$$\sum_{i=1}^n \partial_i E^i = \frac{1}{\lambda \sqrt{|M|}} \sum_{i=1}^n \partial_i \overleftarrow{E} \underline{e}^i E = \frac{1}{\lambda \sqrt{|M|}} (\overleftarrow{E} \nabla E + \overleftarrow{E} \overleftarrow{\nabla} E) = \frac{1}{\lambda \sqrt{|M|}} (-EC + CE)$$

where $\overleftarrow{\nabla}$ operates on the vector on its left. Then

$$\begin{aligned}
e^1 \sum_{i=1}^n \partial_i E^i &= \left(\sum_{i=1}^n \partial_i E_1^i \right) + \sum_{i=1}^n \sum_{j=2}^n \partial_i E_j^i \underline{e}_e^j = \left(\sum_{i=1}^n \partial_i E_1^i \right) + \sum_{j=2}^n \underline{e}_e^j \left(\sum_{i=2}^n \partial_i E_j^i \right) \\
&= \frac{1}{2\lambda\sqrt{|M|}} (-\underline{e}^1 E \underline{e}^1 \underline{e}^1 C + \underline{e}^1 C E) = \frac{1}{2\lambda\sqrt{|M|}} (-E^* (\underline{e}^1 C) + (\underline{e}^1 C) E) \\
&= \frac{1}{2\lambda\sqrt{|M|}} (-(F_e + \underline{i}_s S_e) \cdot (C_p + C_s) + (C_p + C_s)(F_e + \underline{i}_s S_e)) \\
&= \frac{1}{\lambda\sqrt{|M|}} (C_s \cdot F_e + C_p F_e + (-1)^n \underline{i}_s C_e \wedge S_e) \\
&= C_s \cdot F_i + C_p F_i + (-1)^n \underline{i}_s C_s \wedge S_i
\end{aligned}$$

On comparing dimensions in the Clifford algebra

$$\begin{aligned}
\sum_{i=1}^n \partial_i E_1^i &= C_s \cdot F_i \\
f &= \sum_{j=2}^n \underline{e}_s^j \left(\sum_{i=1}^n \partial_i E_j^i \right) = C_p F_i + (-1)^n \underline{i}_s C_s \wedge S_i
\end{aligned}$$