so that digital logic systems can be synthesized with such lines alone. Lines along which a signal propagates at a uniform velocity and without attenuation might seem logically similar to "lossless" linear transmission lines; however, there is an extremely important difference. Whereas signals propagating towards each other do not interact on a linear transmission line, two discharges propagating towards each other on a neuristor line are totally destroyed by the collision. (This, of course, is the basis for lighting back-fires—*i.e.*, destroying one fire with another.) This destructive collision property is basic to such a line, and offers a very powerful launching point for synthesizing digital logic.

Although the analysis and synthesis of logic networks based on such lines are equally as interesting, the main concern here is with the structure of the lines themselves. Two key properties unite this class of structures, *attenuationless* signal propagation and *recovery*. To achieve the first property, energy must be provided along the line, the propagating signal maintaining its condition by dissipating this energy (in a chemical fuse, temperature rise due to local combustion results in the ignition of the neighboring portion of line, and so on). The second property implies line monostability, each section of line recovering towards its initial resting condition after each propagation of a discharge.

In a structure of this type one does not have the usual input-output relations. Here, the form of the signal that propagates along a line is determined solely by the properties of the line itself, the input signal merely determining "when" a signal will propagate. (Sufficiently far from the trigger point, the burning zone along a fuse is completely independent of the actual trigger signal. Ignited with a very hot source the discharge will build-down towards its characteristic form. Ignited with a cool source, but sufficiently strong to cause ignition, the discharge will build towards its characteristic form.) A main problem then is the analysis and synthesis of structures that exhibit the desired mode of propagation, the specific input signal being insignificant except insofar as it determines the form of the transient response close to the trigger point.

An understanding of the transient behavior is also important, however, in predicting the performance of neuristor networks. It was indicated that there are ways in which neuristor lines may be interconnected so that digital logic systems can be synthesized with such lines alone. But whenever a propagating signal passes such an interconnection or junction region, it experiences a perturbation or transient. For example, in one of the primary junction types (called a T junction, T for triggering), a number of lines are brought together in such a manner that a discharge arriving at the junction on any one line initiates a discharge on each of the other connected lines. This would be the case, for example, where a set of chemical fuse lines are tied together in a knot so that a burning zone reaching the knot (junction) on any one line would ignite each of the other lines. Because of the increased trigger requirements at the junction, however, each of the new pulses will initially be somewhat weaker than normal, but will build up towards their characteristic value as they propagate along their respective line.

Although discussion here has been limited to the basic mode of propagation, a number of very interesting properties of such lines have already been uncovered: pulse locking, pulse trapping, rear-end collisions, multimode lines, fatigue, and so on. These properties will be discussed in future reports.

Studies of this type are not only interesting and challenging, but may be of long-range practical importance. One already sees the use of distributed passive components in "integrated" structures. And with the seemingly endless variety of solid-state active elements, it appears unquestionable that distributed active components will also appear in such structures, in which case techniques for the analysis of distributed, nonlinear structures will be necessary.

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Vratsanos' Theorem and Twoport Reciprocity*

In 1957 Vratsanos[1,2] published a detailed proof of the theorem[3] that in a network of linear impedances the following relation holds:

$$i^2 = I^2 \frac{\partial R}{\partial r}, \qquad (1)$$

where, following Vratsanos' notation, i and r are current and impedance in an arbitrary branch; I and R current and impedance at the input to the network.

The proof uses the mesh equations, and simpler proofs were soon offered by Ansell[4], Deards[5] and others[6–8]. An objection is that Vratsanos did not discuss the region of validity. It is not immediately evident that (1) still holds when the network contains other linear elements, as for instance ideal transformers, as long as the twoport which is obtained by removing of r is reciprocal.[6,9]

The following short derivation was given by the author in the Appendix to an earlier paper[10] in which use of (1) was made.

Denoting voltages by V and v, the following differential can be written as determinant:

$$I^2 \, d\left(\frac{V}{I}\right) = \begin{vmatrix} dV & dI \\ V & I \end{vmatrix}. \qquad (2)$$

V and I can be expressed by v and i and the A-matrix of the twoport. From Matrix theory it is known that the determinant of the product of two square matrices is the product of their determinants. For reciprocal twoports the determinant of the A-matrix is unity, so that

$$\begin{vmatrix} dV & dI \\ V & I \end{vmatrix} = \begin{vmatrix} dv & di \\ v & i \end{vmatrix} = i^2 \, d\left(\frac{v}{i}\right). \qquad (3)$$

Dividing (2) by (3) and going to the limit gives immediately (1), showing the connection between what is called Vratsanos' theorem and the two port reciprocity.

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[2] J. Vratsanos, "Calculation of current distribution in a linear network," IRE Trans. on Circuit Theory, vol. CT-4, p. 294; September, 1957. (Abstract.)

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On Causality, Passivity and Single-Valuedness*

Recently the importance of precisely defining the concepts used in electrical engineering has been recognized. As a consequence, several papers have appeared on this subject.[1–3] Of these the outstanding

* Received by the PGCT, September 22, 1961.

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[2] H. König and J. Meixner, "Lineare systeme und lineare transformationen," *Math. Nachrichten*, vol. 19, pp. 265–322; 1958.

[3] D. Youla, L. Castriota and H. Carlin, "Bounded real scattering matrices and the foundations of linear passive network theory," IRE Trans. on Circuit Theory, vol. CT-6, pp. 102–124; March, 1959.

paper of Youla, Castriota, and Carlin[3] is the most basic and rigorous. In this latter paper some important results are obtained by stating a set of postulates which in turn rest upon a set of definitions. Although some of these definitions are open for discussion, several of the results need clarification. It is the purpose of this note to treat some results concerning passivity, causality and single-valuedness. With the kind permission of Professor Youla we will give two interpretations, mine and what I believe to be his.

For conciseness we will adhere to the definitions and the notations of the original paper.[3] However, for utility several definitions will be repeated here. We recall that, in general, the domain $D(\Phi)$ of an n-port Φ is some subset of the set of all n-vectors whose components are individually measurable functions of the real variable t for $-\infty < t < \infty$.[4] This set of n-vectors is denoted by H_n and in the original paper $D(\Phi) = H_n$ can occur; H_1 is simply called H. Φ is causal when, for any two elements $v_1(t)$ and $v_2(t)$ in $D(\Phi)$ and any real τ, $v_1(t) = v_2(t)$, almost everywhere in $t \leq \tau$ implies $i_1(t) = i_2(t)$ almost everywhere in $t \leq \tau$, where $i_1(t)$ is any value of Φv_1 and $i_2(t)$ is any value of $\Phi v_2(t)$.[5] Φ is called single-valued if to each element v in $D(\Phi)$ there is associated exactly one value $i = \Phi v$.[6] Clearly, if Φ is not single-valued it can't be causal. Φ_a is the augmented network obtained by putting one ohm resistors in series with Φ. Φ is passive if for any $\tau > -\infty$ and any $v(t) \varepsilon D(\Phi)$,

$$Re \int_{-\infty}^{\tau} v^*(t) i(t)\, dt \geq 0 \qquad (1)$$

where $i = \phi v$ is any one of the values assigned to $v \varepsilon D(\Phi)$ by Φ; the integral is to be taken in the Lebesgue sense.[7]

This last definition is subject to two valid interpretations which will lead to two different conclusions. Youla's interpretation is as follows: the integral of any non-negative measurable function $f(x)$ taken over any measurable set Δ exists; it may be infinite. If $f(x)$ takes on positive and negative values, then $\int_\Delta f(x)\, d\mu$ exists if $\int_\Delta f_+(x)\, d\mu$ and $\int_\Delta f_-(x)\, d\mu$ are not both infinite and

$$\int_\Delta f(x)\, d\mu = \int_\Delta f_+(x)\, d\mu - \int_\Delta f_-(x)\, d\mu, \qquad (2)$$

where $f_+(x) = f(x)$ if $f(x) \geq 0$ and zero otherwise and $f_-(x) = -f(x)$ if $f(x) < 0$ and zero otherwise. If both integrals in (2) are finite, then $f(x)$ is said to be summable.[8] This definition can be found in well-known mathematical texts.[9] Passivity then says that the integral of (1) exists in the above sense and if it does take on an infinite value this value must be $+\infty$.[10] My interpretation is somewhat different. Following McShane, the Lebesgue integral is said to exist only if (2) is finite, *i.e.*, f summable.[11] This, in fact, appears to be the original meaning ascribed by Lebesgue.[12] Then I interpret passivity as: Φ is passive if (1) holds for every $v \varepsilon D(\Phi)$ for which the integral exists in the sense just mentioned.

The first definition given above for the integral has the advantage of integrating more functions than the second. However, it has the disadvantage that $\int_\Delta (f_1 + f_2)\, d\mu$ need not equal $\int_\Delta f_1\, d\mu + \int_\Delta f_2\, d\mu$ when the integrals in this sum are both infinite.[13] I would prefer to avoid this nonadditivity, as apparently would McShane and Lebesgue.

Now consider the network Φ of Fig. 1 where Φ_a is also drawn for convenience. Here

$$Y(z) = \frac{1}{\sqrt{2\pi}} \frac{z + i}{3z + 2i}$$

$$Y_a(z) = \frac{1}{\sqrt{2\pi}} \frac{z + i}{4z + 3i}.$$

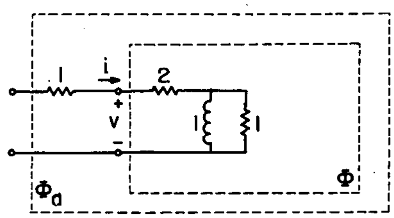

Fig. 1.

These can be found by using the Laplace transform variable p, multiplying by $1/\sqrt{2\pi}$ and replacing p by $-iz$. Considering Φ itself, we see that $i = Ae^{-2t/3}$ and $v = Be^{-t}$ accompany each other when A and B are arbitrary and $-\infty < t < \infty$. If we choose $D(\Phi) = H$ we see that $v \varepsilon D(\Phi)$ since v is measurable, being continuous. Because A and B are arbitrary, Φ is not single-valued and hence cannot be causal. Exactly similar reasoning applies to Φ_a. Now, under my interpretation of passivity, Φ is passive and it also satisfies the remaining postulates P_1 through P_4 of the original paper.[14] With the chosen $D(\Phi)$ and the interpretation given to passivity we have obtained the following two results:

1) The existence of an admittance matrix $Y(z)$ for a Φ satisfying the given postulates (P_1 through P_4) is *not* a sufficient condition for Φ to be single-valued and causal.[15]
2) When Φ is linear and passive, Φ_a need *not* be causal.[16]

With this, one wonders where the proof of Lemma 6 goes astray.[17] This occurs at the point where $0 \leq Re \int_{-\infty}^{\tau} v^* i dt$. As is seen by the Φ of Fig. 2, $v = -i = Ae^{-t}$ can occur, but in this case $\int_{-\infty}^{\tau} v^* i dt$ will not exist; however, $vi < 0$. Because of this, the second interpretation of the Lebesgue integral requires a smaller $D(\Phi)$ before Lemma 6 can be obtained. Consequently, to obtain the results of the paper under this interpretation, $D(\Phi)$ should be limited by a requirement such as $i(-\infty) = v(-\infty) = 0$. An alternative would be to insert causality as a separate postulate. For active networks this latter seems to be the preferable choice.

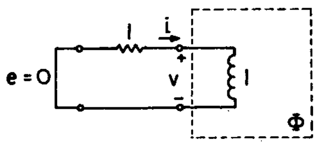

Fig. 2.

Youla gives a different and equally valid interpretation of the above examples. To him, when $D(\Phi) = H$ is chosen for the Φ of Fig. 1, Φ is not passive and hence the remaining theorems of the paper are not violated.[10] In order to see this, choose $i = e^{-2t/3}$, $v = -e^{-t}$; then the integral of (1) is $-\infty$, which is an allowed value of an integral under his interpretation. However, by choosing a smaller $D(\Phi)$, the postulates are satisfied and the theorems all hold. As he points out, this interpretation allows Φ to be passive or active depending upon the choice of $D(\Phi)$.[10] The main idea of the original paper is then that there is some $D(\Phi)$ such that Φ is passive and for which Φ_a has a domain dense in Hilbert space. If this latter is the case, everything in the paper remains true. In particular, with the first interpretation of the integral, Lemma 6 is correct as it stands; however, Theorem 1 seems to need more investigation.[8] Restricting (1) to summable functions seems to alleviate all troubles while putting no significant loss on the generality.[8]

In summary we have learned the following. An example with responses at $t = -\infty$ shows that two interpretations to a present theory are available. In one the passivity of Φ depends upon the choice of $D(\Phi)$, while in the other causality and single-valuedness depend upon this choice. Both are resolved and the results of the paper[3] are valid when $D(\Phi)$ is suitably restricted. ($D(\Phi)$ must still be large enough to allow P_4.) Desoer has suggested that the concept of the state, in particular that beginning with the zero state, would also solve the problem.[18]

In lectures at Stanford the author has followed a somewhat different approach to the entire theory and it is hoped to have this material available shortly. This essentially generalizes the material of McMillan[19]

[4] *Ibid*, pp. 109 and 102.
[5] *Ibid*, p. 111, definition 15.
[6] *Ibid*, p. 106, definition 7.
[7] *Ibid*, p. 110, definition 13.
[8] D. Youla, private communication, October 12, 1961.
[9] S. Saks, "Theory of the Integral," G. E. Stechert & Co., New York, N. Y., p. 20; 1937.
[10] D. Youla, private communication, September 26, 1961.
[11] E. McShane, "Integration," Princeton University Press, Princeton, N. J., p. 75, 1944, is very explicit on this point.
[12] H. Lebesgue, "Leçons sur l'integration," Gauthier-Villars, Paris, France, pp. 111–116, especially p. 115, footnote; 1904.
[13] Saks, *op. cit.* This is implied in theorem (11.9) on p. 24 which is limited to finite integrals. On p. 6 the funny convention $(+\infty) + (-\infty) = 0$ is established.
[14] Youla, *et al.*, *op. cit.*, p. 113.
[15] *Ibid*, p. 123, corollary 12(a).
[16] *Ibid*, p. 104 and p. 111, lemma 6.
[17] *Ibid*, p. 111.
[18] C. Desoer, private conversation. See, however, L. Zadeh, "An extended definition of linearity," Proc. IRE, vol. 49, pp. 1452–1453; September, 1961.
[19] B. McMillan, "Introduction to formal realizability theory—I," *Bell Sys. Tech. J.*, vol. 31, pp. 217–279; March, 1952.

and allows the use of distributions.

I would like to thank Professor Youla for his friendly and informing correspondence and Professor Desoer for his interesting discussions on the material.

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Transistor Precision One-Shot Multivibrator*

The problem of generating pulses of given amplitude and width is of well-known interest. The most popular and most versatile transistor pulse generator is the blocking oscillator. Its main disadvantage is its long recovery time. Very short recovery time is possible in transistor pulse generators with an RC timing network.

This paper is concerned with the analysis of the well-known emitter-coupled one-shot multivibrator with the timing network in the base circuit. Transistors have a low input impedance, store in the base region a charge that is large and, moreover, strongly different from one transistor to another. The input impedance at cutoff is strongly temperature dependent. Therefore, the basic transistor circuit has poor characteristics. These instabilities may be reduced by the use of the RLC-type timing circuits[1,2] with which, however, the advantage of a short recovery time is lost.

The recovery time is the time during which the energy stored in the whole circuit at the end of the output pulse is reset to the rest value. In order to reset the energy stored in an inductance by application of a voltage pulse in a short time, the amplitude of this pulse must be correspondently high. But this voltage pulse is limited by the parasitic capacitance of inductors and of other components (barrier capacitance of junctions) and by the rather low breakdown voltages of transistors. The resetting of electrostatic energy has not these difficulties because the inductance in series with the capacitors is really negligible and the transistors may carry all the current that is necessary.

The purpose of this note is, then, to discuss the results of the analysis in order to find what circuit modifications are necessary for obtaining a good performance with RC-type circuits. The characteristic phenomena of transistors and, in particular, those affecting the threshold level, the pulse width stability and precision, and the reproducibility are examined.

The circuit of Fig. 1 is analyzed. In the analysis the transistor, ideal large-signal equivalent circuit of Fig. 2 is used. Transistors are supposed never to saturate; the input admittance when transistors are cut-off is neglected. In order to allow the circuit to have one stable position, it is necessary for

$$v_{B_1} > v_{B_2} \tag{1}$$

where v_{B_1} and v_{B_2} (base voltages of TR_1 and TR_2 at rest) are temperature dependent.

The time duration τ_0 of the output pulse, when transient times are negligible, is given with good approximation by

$$\tau_0 = C(R_1 + R_2) \ln \frac{R_2}{R_1 + R_2} \cdot \frac{R_1 i_1 + R_2 I_{CO_2} + \frac{Q_0}{C} + (E_3 - E_2)}{R_2 I_{CO_2} - E_3 + v'_{B_1}}, \tag{2}$$

where v_{B_1}' (base voltage of TR_1 during the pulse) is given by

$$v'_{B_1} = \frac{E_1 R_4 - I_{CO_1} R_3 \cdot R_4}{\left(R_3 + R_4 - \frac{R_3 \cdot R_4}{\beta_1 R_5}\right)}, \tag{3}$$

and i_1 is given approximately by

$$i_1 = \frac{v'_{B_1}}{R_5}. \tag{4}$$

(Q_0 is the charge stored in the capacitor C at the end of the first transient.)

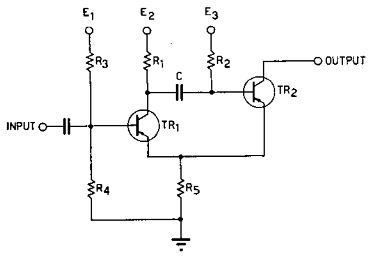

Fig. 1.

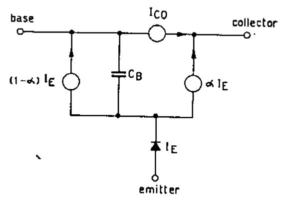

Fig. 2—C_B; Base-emitter capacitance is supposed infinite.

The time duration of the output pulse may be affected by the following:

a) Spread of common-base radian cutoff frequency of TR_2. During the first transient the diffusion charge Q_2 and the charge stored in junction capacitance Q_p must leak off through the capacitor C, so that at the end of the first transient

$$Q_0 = \frac{v_{B_2} - E_2}{C} - Q_2 - Q_p, \tag{5}$$

where Q_2 is given by

$$Q_2 = 1.22 \frac{i_{E_2}}{\omega_{\alpha_2}}.$$

If $\omega_{\alpha m}$ and $\omega_{\alpha M}$ are the minimum and the maximum radian cutoff frequency of the common-base current gain, the uncertainty in the value of the diffusion charge Q_2 is

$$\Delta Q_2 = 1.22 i_{E_2} \frac{\omega_{\alpha M} - \omega_{\alpha m}}{\omega_{\alpha M} \cdot \omega_{\alpha m}}. \tag{6}$$

The percentile change in the output pulse duration is nearly the same as the percentile change in the swing V_C of the voltage across the capacitor C during the pulse:

$$\frac{\Delta V_C}{V_C} = \frac{\Delta Q_2}{C} \cdot \frac{1}{V_C}. \tag{7}$$

For instance, for a transistor 2N404 $\omega_{\alpha m} = 25.10^6$ rad/sec and $\omega_{\alpha M} = 100.10^6$ rad/sec and with $i_{E_2} = 10$ ma and $C = 100$ pf, the uncertainty in V_C is 3.5 v, so that for $V_C = 10$ v, which is a median value for transistor multivibrators, the uncertainty in the output pulse duration is $\pm$ 17 per cent.

b) Variations of I_{CO_1}. The value of I_{CO} at a given temperature and the influence of the base-collector voltage over the variation of I_{CO} due to temperature variations are different from one transistor to another. The variation in the I_{CO_1} flowing in the input resistor R_3, R_4 causes a variation $\Delta v_{B_1}'$. The time duration of the output pulse changes first owing to the Δi_1 corresponding to $\Delta v_{B_1}'$ and given by (4), second to the corresponding variations in the "snap back" voltage v_{B_1}'. These two effects act in the sense of increasing the duration of the pulse with temperature.

c) Variations of I_{CO_2}. At rest these cause a Δv_{B_2} and then a ΔQ_0 from (5). During the pulse, I_{CO_2} flows through the timing network. These two phenomena affect the duration of the output pulse in an opposite way but the influence of the latter is larger than that of the former. The over-all effect is a decrease of the time duration of the pulse with increasing temperature.

d) Variations in the current gain β from one transistor to another. v_{B_2} is affected by β_2 and then the trigger level, Q_0 and τ_0 are dependent upon β_2. β_1 affects τ_0 in the same way as I_{CO_1} does.

e) Of course, in design also the well-known factors which affect the operation of the corresponding vacuum tube circuit must be taken into account.

The effects of I_{CO_1} and $_{B1}$ variations may be reduced to negligible proportions by making the parallel resistance R_i of R_3 and R_4 sufficiently low. The input impedance may be increased by adding inductors in series to the input resistors.

The effects of ω_{α_2}, I_{CO_2} and β_2 spread are minimized by the circuit of Fig. 3. The solution adopted consists in isolating the timing network from TR_2 both during the first transient and during the pulse with the diode D_1 (it must be chosen for low leakage current and low storage charge) and clamping at rest the base of TR_2 with the diode D_2. The coupling between the collector of TR_1 and the base of TR_2

* Received by the PGCT, October 29, 1961.

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