Gyrator and State-Variable Results for Linear Integrated Circuits

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Summary

In a survey and summary manner various methods are discussed for the design of linear integrated circuits. Basic to the methods are the practical availability of integrated operational amplifiers and gyrators, for which the underlying theories of state-variable cascade synthesis methods are applicable.

1. Introduction

It is quite clear that integrated circuits will have considerable impact on future technology and even though the present impetus has come from the suitability of integrated devices for digital processes, it is also clear that their extensive use will occur in linear, continuous time processes. In fact, for processing done with linear devices, the availability and popularity of integrated operational amplifiers shows the practicability of the integrated concept. Here we investigate general theories which are available for the design of linear integrated circuits based upon either a complete integration of the structure or a partial integration of only basic mass produced components.

If one looks for basic building blocks suitable for integrated circuits one soon discovers that, besides resistors and capacitors, two of the most suitable elements are the operational amplifier and the gyrator. Associated with each one is a theory for design or synthesis, the one for operational amplifiers being that of state-variables while the one for gyrators being that of non-reciprocal synthesis. Alternative to both of these philosophies of design is a very simple one, that of element replacement in classical or tabulated designs. In all of these methods, which we will discuss below, the intention is to obtain a suitable structure for integration and hence without inductors and with a minimum number, if possible, of the relatively hard to obtain integrated capacitors. Of course there are other methods besides those which we will discuss, for example, negative impedance converters and multiloop feedback theories but those under consideration here have been chosen because of their very low sensitivity characteristics.

2. The Operational Amplifier and the Gyrator

The operational amplifier of interest is the differential one which ideally has infinite input impedance, zero output impedance and is described by a gain constant K through the equation

$$v_0 = K(v_+ - v_-) \qquad (2.1)$$

The device is symbolised as in fig. 1(a) where the various voltages for equation 2.1, measured with respect to ground, are indicated. Many commercial integrated operational amplifiers are available but fig. 2(a) shows a basic configuration.[1] In fig. 2(a) the input stage provides differential gain, with a degree of symmetry for compensation of temperature effects, while the output stage provides

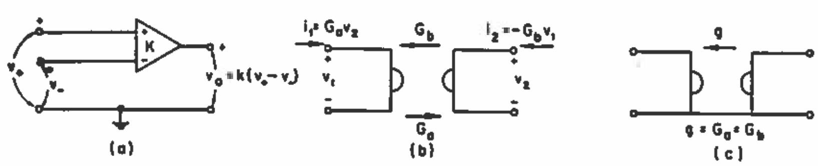

Figure 1.—Basic "gain" elements.
(a) Operational amplifier.
(b) Active gyrator.
(c) Passive gyrator.

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1. Kerwin, W. J., Huelsman, L. P. and Newcomb, R. W., "State-Variable Synthesis Insensitive for Integrated Circuit Transfer Functions", *I.E.E.E. J. of Solid-State Circuits*, (to be published).

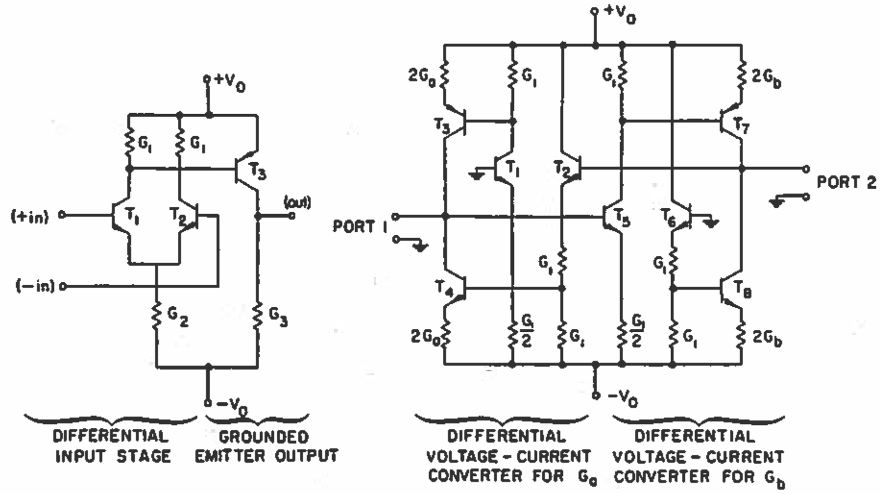
further gain and isolation. The output transistor, being pnp, causes minor concern for integration; it can be either made in lateral form[2] or replaced by an npn transistor and zener diode combination.

The active gyrator is the device symbolised in fig. 1(b) and described by the admittance matrix

$$\mathrm{Y} = \begin{bmatrix} 0 & G_a \\ -G_b & 0 \end{bmatrix} = \begin{bmatrix} 0 & G_a \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -G_b & 0 \end{bmatrix} \qquad (2.2)$$

where G_a and G_b are the forward and reverse conductances respectively and are assumed constant. When $G_a = G_b = g$ the gyrator is passive (the input energy equals the output energy) with economical practical realisations having a common input and output ground, as shown in fig. 1(c). The methods of constructing a gyrator are legion but those which incorporate negative resistance for cancellation purposes are of little practical use because of the sensitivity problems they introduce. The most useful method for realisation of an integrated gyrator is to design a current source for each of the two terms on the right of equation 2.2 (in differential (voltage to current converter) form to obtain desirable symmetry for temperature effects). Fig. 2(b) shows the device as it has been integrated;[3] by replacing the "differential" transistors, T_2 and T_5, by readily integratable m.o.s. transistors, higher quality gyrators can be obtained[4] (the quality of a gyrator can be defined as the maximum Q available for the equivalent inductor when the gyrator is capacitively loaded,[5] as shown in fig. 4(a)).

Several comments associated with fig. 2 are in order. Firstly, we observe that both structures are direct coupled, (avoiding the difficulty in integrating coupling capacitors), with considerable symmetry to allow for cancellation of effects due to temperature variation. Secondly, the gyrator structure basically uses more components than the simple operational amplifier; however this advantage of the latter is compensated, in practice, by a need for another stage to raise the gain to the very high values usually required. Thirdly, both devices are limited in high frequency response, to generally less than several megahertz, due to phase shift in the gyrator (which is a major problem[4]) and due to junction capacitance in the operational amplifier.



(a) (b)

Figure 2.—Suitable integration circuitry (conductance values are given for resistors).
(a) Operational amplifier.
(b) Gyrator.

The primary uses of the operational amplifier in this paper will be for forming integrators and summers, as shown in fig. 3.[1] Configurations using only one infinite gain amplifier are given with the grounded capacitor integrator shown because of the desire for capacitor grounding in integrated fabrication. Some of the useful equivalences involving the gyrator are shown in fig. 4, where it is seen that inductors and coupled coils can be simulated in various configurations, the coupling resulting from the (d.c.) transformer equivalence of fig. 4(c). Note that any sign for the mutual coupling in fig. 4(c) can be obtained and that the same technique applies to give coupled capacitors.[6] Since the gyrator gyration conductances are relatively easy to adjust by electronic means, for example, by varying the gate voltage of an m.o.s. integrated transistor,[7] the various simulated inductors and the mutual coupling, shown in fig. 4, can be adjusted by electronic means. Fig. 4(f) shows how it is possible to float one of the gyrator ports by using two completely grounded gyrators; note that fig. 4(f) is just fig. 4(c) with input and output terminals re-arranged.

2. Lin, H. C. et al., "Lateral Complementary Transistor Structure for the Simultaneous Fabrication of Function Blocks", *Proc. I.E.E.E.*, Vol. 52, No. 12, December 1964, p. 1491.

3. Chua, H. T. and Newcomb, R. W., "Integrated Direct-Coupled Gyrator", *Electronics Letters*, Vol. 3, No. 5, May 1967, p. 182.

4. Sheahan, D. F. and Orchard, H. J., "Integratable Gyrator Using M.O.S. and Bipolar Transistors", *Electronics Letters*, Vol. 2, No. 10, October 1966, p. 390.

5. Rao, T. N., Gary, P. and Newcomb, R. W., "Equivalent Inductance and Q of a Capacitor-Loaded Gyrator", *I.E.E.E. J. of Solid-State Circuits*, Vol. SC-2, No. 1, March 1967, p. 32.

6. Anderson, B. D., New, W. and Newcomb, R. W., "Proposed Adjustable Tuned Circuits for Micro-electronic Structures", *Proc. I.E.E.E.*, Vol. 54, No. 3, March 1966, p. 411.

7. New, W. and Newcomb, R. W., "An Integrable Time-Variable Gyrator", *Proc. I.E.E.E.*, Vol. 53, No. 12, December 1965, p. 2161.

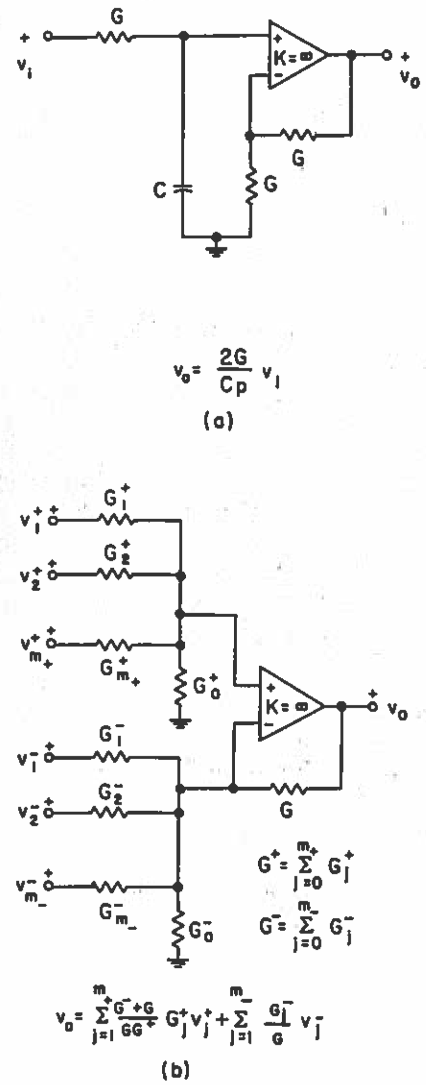


Figure 3.—Primary operational amplifier uses.
(a) Integrator.
(b) Summer.

3. State-Variable Results

One commonly met design problem is that of realising a given rational transfer function

$$\frac{V_2}{V_1}(p) = T(p) = \frac{b_{\delta+1}p^{\delta} + b_{\delta}p^{\delta-1} + \dots + b_1}{p^n + a_n p^{n-1} + \dots + a_1} \tag{3.1}$$

For example, such may come from design tables[8] or from a control system compensation specification.[9] Using the flow-graph of fig. 5[10] the transfer function is immediately obtained using the summers and integrators of fig. 3. In fact this realisation uses the minimum number of capacitors, δ, (all of which are grounded) this number being the degree of the transfer function T(p). In fig. 5 the state-variables $x_1 \dots x_\delta$ are labelled, while a simple scaling of transmittances allows any multiplier, 2G/C, to be used for the integrators.

However, if one attempts a direct realisation via fig. 5 of a high degree transfer function, it is found that the structure is somewhat sensitive to various element value changes. Consequently, it is advantageous to decompose the transfer function into the product of degree-one (or two) factors of the form

$$T(p) = \frac{b_2 p + b_1}{p + a_1} \tag{3.2a}$$

$$T(p) = \frac{b_3 p^2 + b_2 p + b_1}{p^2 + 2\zeta\omega_n p + \omega_n^2} \tag{3.2b}$$

where we have assumed a stable transfer function ($\zeta \geqslant 0$, $\omega_n > 0$, $a_1 \geqslant 0$). Fig. 5 can be applied to these degree-one and two transfer functions with a cascade connection giving the overall transfer function. Then, if we define the *sensitivity* of a quantity, X(p), with respect to a parameter x as

$$S_x^{X(p)} = \frac{x}{X}\frac{\partial X}{\partial x} \tag{3.3}$$

we find very low transfer function sensitivities; for example, the degree-two sections approximately have[1]

$$\left| S_K^{|T(j\omega_n)|} \right| \approx \frac{Q}{K} \tag{3.4}$$

where Q equals $\frac{1}{2}\zeta$ and K is any amplifier gain in the configuration.

8. Skwirzynski, J. K., "Design Theory and Data for Electrical Filters", van Nostrand (1965).
9. Truxall, J. G., "Automatic Feedback Control System Synthesis", McGraw-Hill (1955).
10. Kalman, R. E., "Mathematical Description of Linear Dynamical Systems", *J. of S.I.A.M. Control*, Series A, Vol. 1, No. 2, 1963, p. 162.

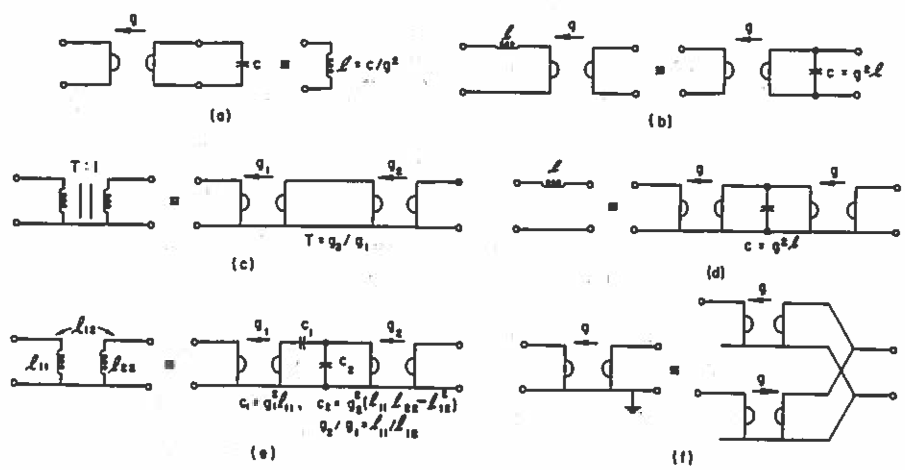


Figure 4.—Useful gyrator equivalences.

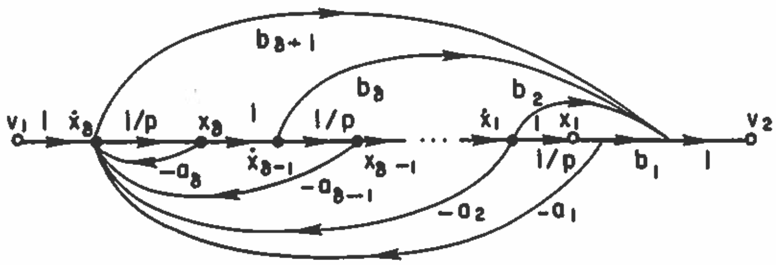


Figure 5.—Signal-flow graph for T(p).

We see then that this approach, which allows in fact a completely integrated structure, gives a simple means of obtaining convenient low sensitivity designs using readily available integrated amplifiers and a minimum number of capacitors. Although the techniques discussed up to this point covers most practical cases, it is of interest to look at the procedure in a more general framework, that of the state-variable minimal realisation for a multiple input/output transfer function matrix, **T**(p).

To develop the theory we consider as given an m × n rational transfer function matrix, **T**(p), which is assumed to have no pole at infinity. This transfer function matrix is to be considered as relating the Laplace transform, $\mathbf{V}_1(p)$, of the input voltage time-domain n-vector, $\mathbf{v}_1(t)$, to the Laplace transform, $\mathbf{V}_2(p)$, of the output voltage time-domain m-vector, $\mathbf{v}_2(t)$, through

$$\mathbf{V}_2(p) = \mathbf{T}(p)\mathbf{V}_1(p) \tag{3.5}$$

As we will describe, the techniques of modern system theory allow one to introduce a k-vector, **x**(t), called the *state*, for which there exist four constant matrices **A**, **B**, **C** and **D**, such that the following equations are satisfied ($\mathbf{1}_k$ is the k × k identity matrix)

$$\frac{d\mathbf{x}}{dt} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{v}_1 \tag{3.6a}$$

$$\mathbf{v}_2 = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{v}_1 \tag{3.6b}$$

$$\mathbf{T}(p) = \mathbf{D} + \mathbf{C}[p\mathbf{1}_k - \mathbf{A}]^{-1}\mathbf{B} \tag{3.6c}$$

These state-variable equations are of interest to us since they immediately yield a connection of integrators and summers, as shown in fig. 6 where each block represents a multiple input/output connection of the devices of fig. 3.[11]

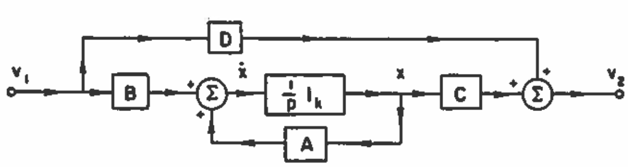


Figure 6.—State-variable equations block diagram.

In fig. 6 there are k integrators required and since each integrator uses one capacitor, we wish to find an **A**, **B**, **C** and **D** such that k is the minimum. This smallest value of k is known as the *degree*, $\delta[\mathbf{T}(p)]$, and is calculable beforehand directly from **T**(p)[12]; a determination of **A**, **B**, **C** and **D** satisfying equation 2.6 for $k = \delta[\mathbf{T}(p)] = \delta$ is called a *minimal realisation* for **T**(p).

Perhaps the simplest method of obtaining a minimal realisation follows the ideas almost simultaneously developed by Ho[13] and Youla.[14] Explaining the notation in the constructive procedure which follows, these theories show that a minimal realisation is explicitely calculated as

$$\begin{aligned} \mathbf{A} &= \mathbf{1}_{\delta, rm}\,\mathbf{P}\,\Omega\,\mathbf{S}_r\,\mathbf{Q}\,\tilde{\mathbf{1}}_{\delta, rn} \\ \mathbf{B} &= \mathbf{1}_{\delta, rm}\,\mathbf{P}\,\mathbf{S}_r\,\tilde{\mathbf{1}}_{n, rn} \\ \mathbf{C} &= \mathbf{1}_{m, rm}\,\mathbf{S}_r\,\mathbf{Q}\,\tilde{\mathbf{1}}_{\delta, rn} \\ \mathbf{D} &= \mathbf{T}(\infty) \end{aligned} \tag{3.7}$$

The right hand expressions for **A**, **B**, **C** and **D** are obtained from the following definitions and manipulations. Given an m × n rational **T**(p), with **T**(∞) finite and well-defined (yielding **D**), form the least common multiple of all denominators

$$g(p) = p^r + a_r p^{r-1} + \ldots + a_2 p + a_1 \tag{3.8a}$$

which serves to define r and the constant coefficients $a_1 \ldots\ldots a_r$. Also let

$$\Omega = \begin{bmatrix} 0 & \mathbf{1}_m & 0 & \ldots & 0 \\ 0 & 0 & \mathbf{1}_m & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & \mathbf{1}_m \\ -a_1\mathbf{1}_m & -a_2\mathbf{1}_m & -a_3\mathbf{1}_m & \ldots & -a_r\mathbf{1}_m \end{bmatrix} \tag{3.8b}$$

which is generalised *companion matrix*. By definition, $\mathbf{1}_{m, rm}$ is the m × rm matrix whose first m columns are the m × m identity matrix $\mathbf{1}_m$ and whose last (r − 1)m columns are zero; $\tilde{\mathbf{1}}_{n, rn}$ is the transpose after replacing m by n and similarly for $\mathbf{1}_{\delta, rm}$ and $\tilde{\mathbf{1}}_{\delta, rn}$ where δ, the degree of **T**(p), is further defined below. To determine $\mathbf{S}_r$, **T**(p) is expanded about p = ∞,

$$\mathbf{T}(p) = \mathbf{T}(\infty) + \frac{\mathbf{T}_0}{p} + \frac{\mathbf{T}_1}{p^2} + \ldots = \sum_{i=-1}^{\infty} \frac{\mathbf{T}_i}{p^{i+1}},\ \mathbf{T}_{-1} = \mathbf{T}(\infty) \tag{3.8c}$$

Then $\mathbf{S}_r$ is the (constant) generalised Hankel rm × rn matrix

11. Schwarz, R. J. and Friedland, B., "Linear Systems", McGraw-Hill (1965).
12. Newcomb, R. W., "Linear Multi-port Synthesis", McGraw-Hill (1966).
13. Ho, B. L., "On Effective Construction of Realisations from Input/Output Descriptions", Ph.D. Dissertation, Stanford University, March 1966.
14. Youla, D. C., "The Synthesis of Networks Containing Lumped and Distributed Elements", Proc. Brooklyn Polytechnic Symposium on Generalised Networks, April 1966, p. 289.

$$S_r = \begin{bmatrix} T_0 & T_1 & \cdots & T_{r-1} \\ T_1 & T_2 & \cdots & T_r \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ T_{r-1} & T_r & \cdots & T_{2r-2} \end{bmatrix} \quad (3.8d)$$

Finally **P**, **Q** and δ are determined by finding matrices **P** and **Q** to diagonalise S_r to 1_δ and zeros,

$$P\,S_r\,Q = \tilde{1}_{\delta,\,rn}\,1_{\delta,\,rm} \quad (3.8e)$$

which shows that δ is the rank of S_r.

Given one minimal realisation, $R = \{A, B, C, D\}$, for example as calculated in equation 3.7, then all other minimal realisations take the form $R_\delta = \{T_\delta^{-1}AT_\delta, T_\delta^{-1}B, CT_\delta, D\}$ where T_δ is an arbitrary $\delta \times \delta$ non-singular matrix.[13] Of course other than minimal realisations exist (that is, $k > \delta$ is possible for equation 3.6) but interest in them is small for integrated circuits since more than the minimum number of capacitors are required.

In summary, the construction of equation 3.8 yields an **A**, **B**, **C** and **D** through equation 3.7 to give a physical construct, fig. 6, using the summers and a minimal number of the integrators of fig. 3 for a multiple input/output configuration. In the most commonly met single input/output, degree-two case, the procedure applied to the transfer function of equation 3.2(b) yields

$$\begin{aligned} A &= \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \\ B &= \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ C &= [b_1 - \omega_n^2 b_3,\; b_2 - 2\zeta\omega_n^2 b_3] \\ D &= [b_3] \end{aligned} \quad (3.9)$$

and fig. 5 results as a special case of fig. 6.

An alternative but closely related method in the single input/output case is to use tabulated structures, of which there are many.[15–17] These circuits usually contain an excess number of capacitors with significantly higher sensitivities than those obtained with the state-variable concepts; hence they appear to be of limited use for integrated circuits. However, Kerwin has shown the value of derived distributed parameter structures.[18] For example, fig. 7, when used in conjunction with fig. 8 for the choice of C, K and R given ζ and ω_n, shows a means of obtaining a degree-two transfer function with a $j\omega$ axis zero (normalised to j1). As can be seen the structures use a very small number of components and are relatively easy to integrate; the uniformly distributed RC lines in fact take advantage of the inherent distributed nature of integrated circuits.

15. Sallen, R. P. and Key, E. L., " A Practical Method of Designing RC Active Filters ", *Trans. I.R.E.*, Vol. CT-2, No. 1, March 1955, p. 74.

16. Thiele, A. N., " The Design of Filters Using Only RC Sections and Gain Stages ", *Electronic Engineering*, Parts 1 and 2, Vol. 28, Nos. 335 and 336, January 1956, p. 31 and February 1956, p. 80.

17. Holt, A. G. J. and Sewell, J. I., " Table for the Voltage Transfer Functions of Single-Amplifier Double-Ladder Feedback Systems ", *Electronics Letters*, Vol. 1, No. 3, May 1965, p. 70.

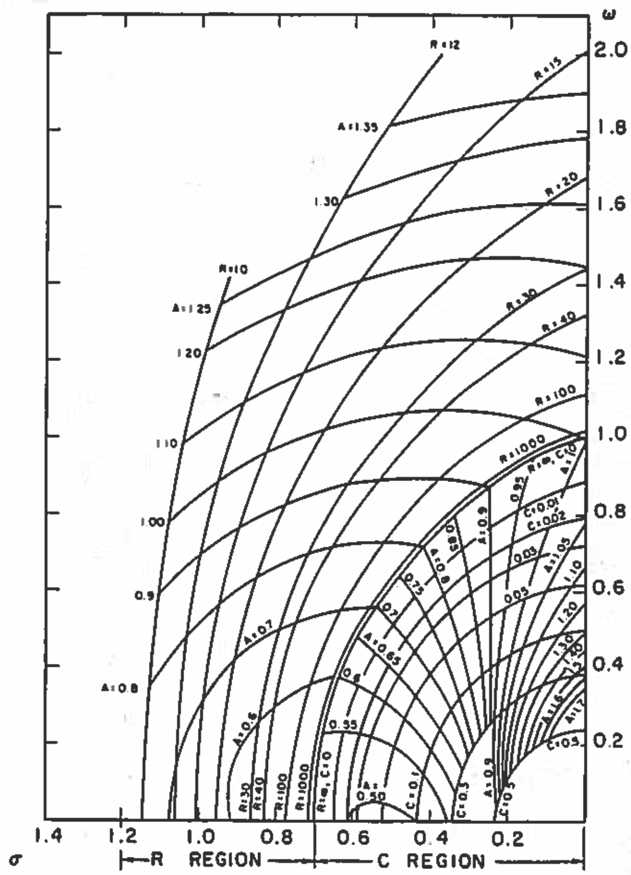


Figure 8.—Parameter choices for desired poles in fig. 7.

18. Kerwin, W. J., " Analysis and Synthesis of Active RC Networks Containing Distributed and Lumped Elements ", Ph.D. Dissertation, Stanford University, 1967.

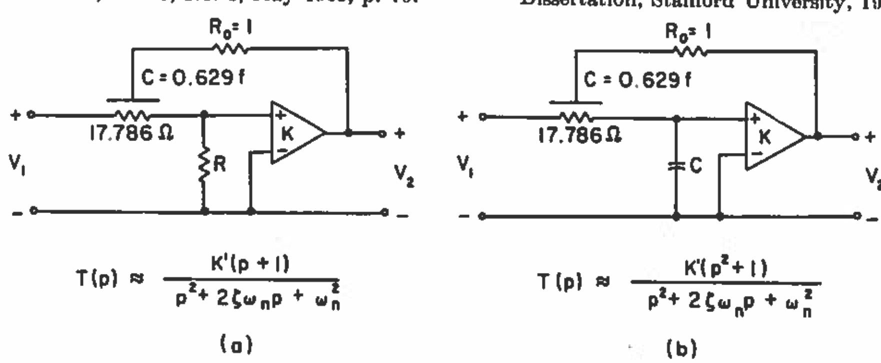


Figure 7.—Lumped distributed degree-two sections.
(a) Case $\omega_n > 1$.
(b) Case $\omega_n < 1$.

4. Element Replacements

A very simple means of design is to use tabulated or existing configurations which are known to perform well in practice. For example, there are extensive tables for desired filter characteristics.[8] However, to be useful for integrated or micro-electronic configurations, replacements must be made for inductors. With gyrators available such replacements can be made through the gyrator/capacitor inductor equivalents of fig. 4. There are, however, other means of inductor simulation suitable for integration[19, 20] and these may prove useful in specific situations. Here, though, we base the treatment on the gyrator because of its versatility.

As a first example we mention the simple replacement of inductors in oscillator and mixing circuits.[21] Thus fig. 9(a) shows a Colpitts oscillator[22] in which the resonating inductor has been replaced by the gyrator/capacitor equivalent of fig. 4(a). Similarly, by using the parametric amplifier effect obtained by time variation of an inductor, fig. 9(b) shows a means of realising a mixer by use of a gyrator with time-variable gyration conductance.

Perhaps of more interest is the use of gyrator/capacitor replacements of inductors in filter design. Using the equivalence of fig. 4(d), Holt and Taylor[23] have introduced the replacement concept into filter design with the practical advantages in terms of sensitivity being pointed out by Orchard.[24] Orchard's argument can be summarised as follows. Given a doubly resistive terminated lossless filter designed on an insertion loss basis, for example, as shown for the degree-three maximally flat case in Fig. 10(a),[25] we know that maximum power is transferred from source to load in the passband. We infer that in the filter any change (up or down) in any reactive component value, x, from its actual design value, x_0, can only serve to introduce a loss of power transferred; that is, the derivative of the insertion loss, $IL = |V_1/2V_2|^2$ with respect to x, evaluated at x_0, is zero. Thus, in the passband a well-designed doubly terminated filter has zero sensitivity with respect to any reactive component; precisely

$$S_x^{IL}\Big|_{x=x_0} = \frac{x}{IL}\frac{\partial IL}{\partial x}\Big|_{x=x_0} = -2S_x^{|V_2/V_1|}\Big|_{x=x_0} = 0 \qquad (4.1)$$

Consequently, a zero sensitivity design results by replacing an inductor by an equivalent, those of fig. 4(d) or 4(a) and 4(f) sufficing for the configurations of fig. 10(a) as shown in fig. 10(b) and 10(c). Since the inductors present in insertion loss designs are passive, one desires

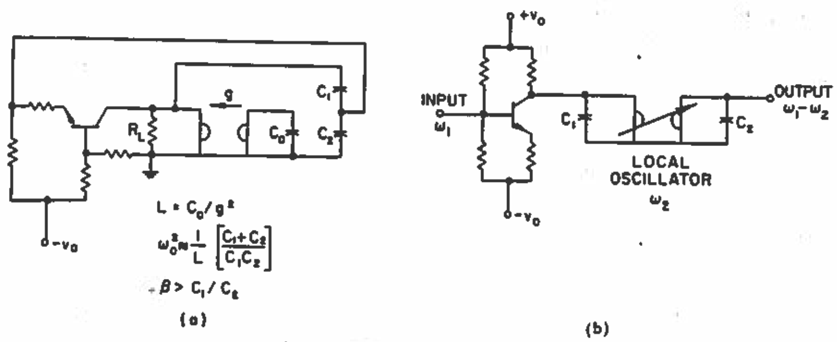


Figure 9.—Inductorless circuits.
(a) Colpitts oscillator.
(b) Mixing circuit.

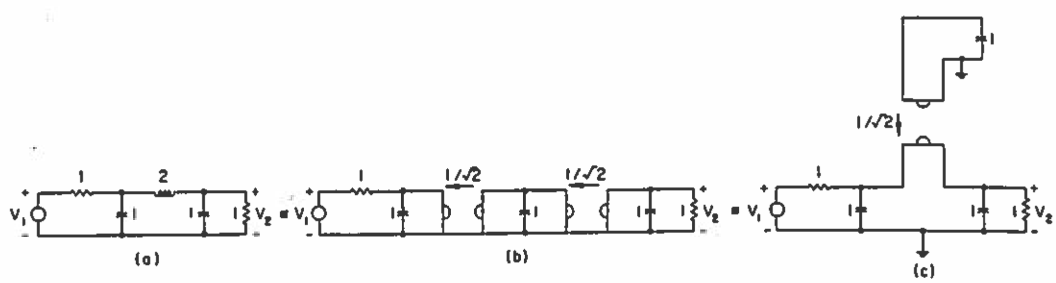


Figure 10.—Degree-three maximally flat insertion loss filter and replacements.

to use only passive integrated components in their simulation; hence, the interest in the use of passive gyrators constructed as in fig. 2(b) (in actual fact the introduction of a small amount of activity, as a result of excess phase

19. Riordan, R. H. S., "Simulated Inductors Using Differential Amplifiers", *Electronics Letters*, Vol. 31, No. 2, February 1967, p. 50.
20. Lampard, D. G. and Rigby, G. A., "The Application of a Positive Immittance Inverter to the Design of Integrated Selective Filters", *Proc. I.E.E.E.*, Vol. 55, No. 6, June 1967, p. 1101.
21. Anderson, B. D., New, W. and Newcomb, R. W., "Oscillators, Modulators and Mixers Suitable for Integrated Circuit Realisation", *Proc. I.E.E.E.*, Vol. 55, No. 3, March 1967, p. 438.
22. Walston, J. A. and Miller, J. R. (Editors), "Transistor Circuit Design", McGraw-Hill (1963).
23. Holt, A. G. J. and Taylor, J., "Method of Replacing Ungrounded Inductors by Grounded Gyrators", *Electronics Letters*, Vol. 1, No. 4, June 1965, p. 105.
24. Orchard, H. J., "Inductorless Filters", *Electronics Letters*, Vol. 2, No. 6, June 1966, p. 224.
25. van Valkenburg, M. E., "Introduction to Modern Network Synthesis", John Wiley & Sons (1960).

shift, for example, results in improved filter performance through increased inductor Q). In lumped form, high degree low-pass, band-pass and high-pass filters have been successfully constructed in this manner,[26, 27] it being only a matter of time before complete integration occurs.

We comment that a direct comparison with negative impedance converter circuits also shows the superiority of the gyrator approach.[28] However, especially in the band-pass case, the replacement method may use an excess number of capacitors, making the next technique to be discussed meaningful. Nevertheless, an alternate replacement method in terms of state variable modelling of the equations describing the filter has been shown to be of practical significance by Sheahan.[27]

5. Direct Gyrator/Capacitor Synthesis

By developing a synthesis technique which has all the properties of classical insertion loss design but which basically extracts gyrator/capacitor sections, a zero pass-band sensitivity configuration very suitable for integration can be obtained.[29]

One considers as given a positive-real admittance, y(p), this either being directly given or derived from a prescribed insertion loss characteristic through $|[y(j\omega) - 1]/[y(j\omega)+1]|^2 = 1 - 4|V_2(j\omega)/V_1(j\omega)|^2$ (where normalised unit source and load impedances have been assumed). The philosophy is to extract cascade Hazony sections (fig. 11) by an application of the Richards' transformation

$$y_\ell(p) = y(k)\,\frac{ky(k) - py(p)}{ky(p) - py(k)} \qquad (5.1)$$

developed for k a zero of the even part of y(p), that is, $y(k) = -y(-k)$. With such a choice for k the load admittance is positive-real (though with perhaps non-real

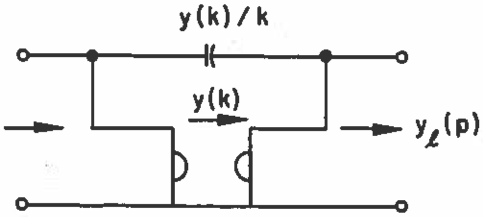


Figure 11.—Hazony section.

coefficients when k is complex) and of one degree less than y(p). The extraction is repeated on $y_\ell(p)$, using the complex conjugate k* if k is the first of a conjugate pair of even part zeros, until a degree zero (resistive) termination is reached. The complex valued sections obtained when k is not real can be combined as shown in fig. 12 to yield real-valued components. One finds for fig. 12

$$y_{\ell\ell}(p) = \frac{\left[\dfrac{|k|^2}{p^2}\dfrac{\operatorname{Im} ky(k)}{\operatorname{Im} ky^*(k)} + 1\right]y(p) - \dfrac{|y(k)|^2 \operatorname{Im} k^2}{p \operatorname{Im} ky^*(k)}}{\left[\dfrac{|k|^2}{p^2}\dfrac{\operatorname{Im} ky^*(k)}{\operatorname{Im} ky(k)} + 1\right] - \dfrac{y(p)}{p}\dfrac{\operatorname{Im} k^2}{\operatorname{Im} ky(k)}} \qquad (5.2)$$

By the angle constraint of positive-real functions, the elements for fig. 12(b) are passive, while they become indeterminate when k is imaginary, in which case the Brune section of fig. 12(c) results with

$$2a_+ = y'(k) + \frac{y(k)}{k}, \quad 2a_- = y'(k) - \frac{y(k)}{k} \qquad (5.3a)$$

$$y_{\ell\ell}(p) = \frac{\left[\dfrac{|k|^2 a_+}{p^2 a_-} + 1\right]y(p) - \dfrac{|y(k)|^2}{pa_-}}{\left[\dfrac{|k|^2}{p^2}\dfrac{a_-}{a_+} + 1\right] - \dfrac{y(p)}{pa_+}} \qquad (5.3b)$$

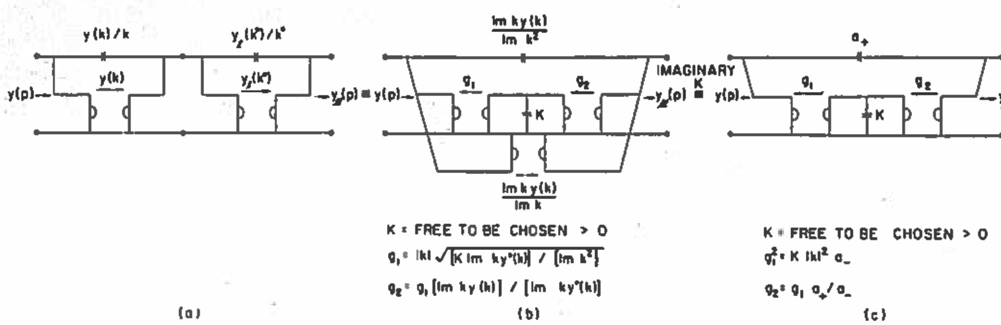


Figure 12.—Darlington and Brune sections.
(a) Complex Hazony section cascade.
(b) Non-reciprocal Darlington section.
(c) Non-reciprocal Brune section.

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where a_+ and a_- are positive by Takahasi's Theorem.[12] The cases of $y(k) = 0$ and $1/y(k) = 0$ are easily taken care of, by series or shunt capacitors.

In conclusion a filter or any rational positive-real y(p), can be synthesised by extracting Hazony sections, some of which may need to be combined into non-reciprocal Darlington or Brune sections to obtain real-valued components. The method uses the minimum number, $\delta[y(p)]$, of capacitors (as well as a minimum number of resistors); all capacitors can be grounded by an equivalence similar to those in fig. 4.[30]

6. Discussion

Three practical methods of design using readily integrated components have been discussed. The state-variable method uses operational amplifiers and a minimum number of capacitors. It can yield highly selective circuits with relatively small sensitivities to component variations, these latter arising in integrated circuits through mask inaccuracies or temperature variations. However, the state-variable method uses a large total number of components. The element replacement method takes advantage of standard designs and is very effective when only portions, such as the active components, of a circuit are integrated. In some cases, for example some band-pass filters, the replacement technique may not yield optimum designs, especially for complete integration where a minimum number of capacitors are desired. As a consequence, the third technique of passive non-reciprocal synthesis offers distinct advantages for complete integration when the ideas can be applied, since zero transfer function magnitude sensitivity can be obtained in a cascade structure using a minimum number of capacitors.

When more than a single input and output is desired the state-variable theory offers an attractive approach. Yet, as can be seen from the scalar case, a development in terms of degree-one matrix factors would be useful for low sensitivity structures. For the synthesis of time-variable multi-ports, such a factorisation has been given[31] for quasi-lossless scattering matrices with the structures suitable for integration, on small modification, in terms of time-variable gyrators. If, however, the prescribed description is an $n \times n$ admittance matrix, $T(p) = Y(p)$, the realisation of equation 27 can be used to define an $(n + \delta)$-port coupling network (constant) admittance matrix

$$Y_c = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} D & -C \\ B & -A \end{bmatrix} \qquad (6.1a)$$

The formula $Y = y_{11} - y_{12}(y_{22} + y_l)^{-1}y_{21}$ shows that equation 3.6(c) holds when Y_c is loaded in $\delta = k$ unit capacitors at its final δ ports, $y_l = p1_\delta$.[32] By a proper choice of the transformation, T_δ, the modified realisation

$$\hat{Y}_c = \begin{bmatrix} D & -CT_\delta \\ T_\delta^{-1}B & -T_\delta^{-1}AT_\delta \end{bmatrix} \qquad (6.1b)$$

can be guaranteed positive-real if the original given $Y(p)$ is positive-real,[33] allowing a completely passive synthesis in terms of integrated resistors, capacitors and gyrators. Further, if one wishes to find all equivalents, perhaps to obtain an optimum layout, then a general theory is available starting with the state-variable realisation of equation 3.7.[34]

The importance of gyrators and matrix specifications can be seen from the fact that many filter design problems require a certain input impedance, a certain forward transfer function and perhaps a certain output impedance. The classical filters are of course all reciprocal but the ability to use gyrators means that we can design passive non-reciprocal filters as easily as reciprocal ones, realising, for example, a zero reverse transfer function. Moreover, the non-reciprocal ones may be generally more useful. Thus, energy reflections due to mismatch are more easily avoided or, in a communication link for example, one person may be able to talk to another without the latter being able to talk back.

The Bott-Duffin procedure indicates that there can be advantages in considering non-minimal realisations in certain contexts. Thus, even though at this point minimal realisations seem the most appropriate for integrated circuits, we should not overlook the theory of non-minimal realisations;[34] some hitherto undiscovered advantage to non-minimal capacitor structures may exist.

There are of course other valuable techniques for design, which, however, are in general more specialised than we have mentioned here. Among those useful for low sensitivity designs are those associated with the concepts of phantom zeros,[35] zero slope phase shift configurations[36] and general feedback structures. The concepts discussed in this paper as well as others will be covered in detail in a forthcoming book.[37]

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