IEEE JOURNAL OF SOLID-STATE CIRCUITS, VOL. SC-2, NO. 3, SEPTEMBER 1967

State-Variable Synthesis for Insensitive Integrated Circuit Transfer Functions

WILLIAM J. KERWIN, SENIOR MEMBER, IEEE, LAWERENCE P. HUELSMAN, SENIOR MEMBER, IEEE, AND ROBERT W. NEWCOMB, MEMBER, IEEE

Abstract—Using state-variable flow graphs and simple operational configurations suitable for integration, a theory for insensitive transfer function realization in terms of integrated circuits is discussed. The theory emphasizes the decomposition into second-order systems that are developed, following state-space concepts, with special reference to their sensitivity which is shown to be very low for high operational amplifier gains.

I. Introduction

For LINEAR integrated circuits the availability of operational amplifiers seems well recognized.[1]-[4] But, though their use in obtaining low sensitivity designs has been advantageously considered,[5] and RC operational amplifier synthesis methods exist,[6],[7] the availability of simple but highly theoretical state-variable results for practical design seems to have been overlooked until recently.[8] To be sure, several methods applicable to RC operational amplifier synthesis exist,[9]-[18] but generally these are rather restrictive or use an excessive number of capacitors. Here, with slight modifications, some standard but modern systems theory techniques (Kalman,[19] p. 177) are applied to obtain insensitive designs for voltage transfer functions. The resulting structures use a minimum number of grounded capacitors and relatively few resistors and transistors, all of which are in an easily integrated form.

II. Flow Graph

The theory is begun by considering as given a transfer function of the form

$$T(p) = \frac{a_{n+1}p^n + a_n p^{n-1} + \cdots + a_1}{p^n + b_n p^{n-1} + \cdots + b_1}. \qquad (1)$$

That is, $T(p)$ is the transfer function of a single input, $u(t)$, single output, $y(t)$, differential system with no pole at infinity; $T(p) = \mathcal{L}[y]/\mathcal{L}[u]$ with $\mathcal{L}[\ ]$, the Laplace transform, and all a_i and b_i real constants. The introduction of state variables x_i leads to the signal flow graph of Fig. 1(a), which is a standard one in the theory of systems (Kalman,[19] p. 177). Fig. 1(a) uses positive gain integrators, but negative gain integrators can be

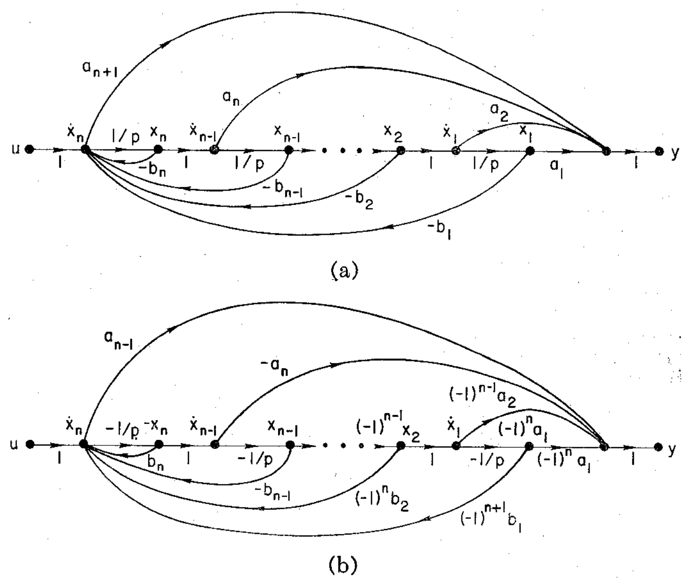


Fig. 1. Signal flow graph realizing a general transfer function with (a) positive and (b) negative gain integrators.

introduced by appropriate changes in sign, as shown in Fig. 1(b). For generality, the constant a_{n+1} has been inserted at infinity. The transfer function for the graph can easily be checked by applying Mason's rule (Mason and Zimmermann,[20] p. 104). Each element of the graph can be realized by an integrator or a summer (with appropriate scalings), both of which can be obtained in integrated form using operational amplifiers, resistors, and capacitors. In actual fact, the unit transmittances of Fig. 1 can be omitted in any physical realization since they are only present in the graph for purposes of illustration. Likewise, nonunity coefficients for physical integrators can be incorporated by scaling outgoing branches.

If the basic flow graph configuration of Fig. 1(b) is used to implement this circuit, a common network design problem is encountered; namely, that the roots of high-order polynomials can become very sensitive to the coefficients, such that very high accuracy is required in the resistive components that determine these coefficients. For this reason the most practical procedure is to factor the transfer function into first- and second-order terms. For first- and second-order factors this high accuracy is not required and a cascade of the realizations of such factors yields the transfer function in insensitive form. Of course, the basic flow graph holds for each factor separately though the first-order terms are most easily obtained with passive RC networks.

Manuscript received January 9, 1967; revised June 29, 1967. This work was sponsored by the AF Office of Scientific Research, Office of Aerospace Research, USAF Contract F44620-67-C-0001, the National Science Foundation Grant NSF GK-237, and the Office of Naval Research Contract NONR 225(83).

W. Kerwin and R. Newcomb are with Stanford Electronics Laboratories, Stanford, Calif.
L. Huelsman is with the Dept. of Elec. Engrg., University of Arizona, Tucson, Ariz.

III. Operational Amplifier

A very convenient and economical differential-input operational amplifier is shown in Fig. 2, where, if voltages are subscripted according to their point of measurement (with respect to ground),

$$e_C = K(e_B - e_A). \tag{2}$$

The amplifier of Fig. 2 can be conveniently considered as the cascade of a differential-input section (with the "negative" input at A and the "positive" input at B) and a grounded emitter stage of current gain β. Typical values for the circuit are $R_a = 10$ kΩ, $R_b = 50$ kΩ, $R_c = 20$ kΩ, $\beta = 60$, $V_0 = 15$ V, and (measured) $K \approx 1500$.

The gain K of amplifier can be roughly determined. First let point A be grounded, in which case the emitter-base resistance (Thiele,[11] p. 82) $r_{\epsilon_1} \approx 0.026/I_{e_1}$ of T_1 is effectively in parallel with R_b. The emitter current I_{e_2} or T_2 is determined by the base-emitter drop V_{be_3} of T_3; $I_{e_2} = V_{be_3}/R_a \approx 0.5/R_a$. Thus, the emitter-base resistance of T_2, $I_{e_2} \approx 0.5/R_a$, $I_{e_1} \approx V_0/R_b$. Hence the emitter-base resistance of T_2, $r_{\epsilon_2} \approx 0.026/I_{e_2}$, determines the signal current in the collector of T_2 since r_{ϵ_2} is larger than R_b in parallel with r_{ϵ_1} (with which it is in series). Likewise, the effective signal resistance in the collector of T_2 is R_a in parallel with the equivalent-input base-emitter resistance βr_{ϵ_3} of T_3, where β is the beta of T_3 and $r_{\epsilon_3} \approx 0.026/I_{e_3}$ (where $I_{e_3} \approx V_0/R_c$); this parallel combination is about βr_{ϵ_3}. Thus, the voltage gain of the first (differential-input) stage is about $-\beta r_{\epsilon_3}/r_{\epsilon_2}$. To obtain the gain of the output stage it is noted that the emitter signal current is determined by dividing the base-emitter voltage by r_{ϵ_3}; hence, the gain is $-R_c/r_{\epsilon_3}$. Combining, we have

$$K \approx \beta R_c/r_{\epsilon_2} = \frac{\beta}{0.052}\frac{R_c}{R_a}. \tag{3a}$$

Grounding point B in place of A yields the same K within about a 0.9 multiplier. Note, however, that if a feedback resistor R_f is applied between points A and C, as for the adders and integrators to be described, then R_c must be replaced in (3a) and (3b) by the parallel combination of R_c and the effective R_f.

A calculation using transistor parameters in more detail gives

$$K \approx \frac{\beta}{r_{\epsilon_1} + \frac{1}{2}r_{\epsilon_2}} \cdot \frac{R_a R_c}{R_a + \beta r_{\epsilon_3}}. \tag{3b}$$

Equation (3b) agrees with (3a) when $R_a > \beta r_{\epsilon_3}$ and $r_{\epsilon_1} \approx \frac{1}{2} r_{\epsilon_2}$; that is, (3a) can be used when $I_{e_1} \approx 2I_{e_2}$.

The only difficulty in integration of the amplifier of Fig. 2 comes from the presence of both *p-n-p* and *n-p-n* transistors. However, integration of the circuit can be accomplished by using a lateral *p-n-p* transistor (with a complementary *n-p-n* to restore the β) to obtain a satisfactory *p-n-p* transistor.[22] Alternatively, *n-p-n* transistors can be used exclusively if Zener diodes are incorporated to restore the dc bias conditions. Of course, any one of the many commercial integrated operational amplifiers can be used. However, in using a commercial unit care must be taken to insure a stable amplifier configuration, especially for the following summers that often require near unity feedback. It should also be checked, in particular, that the input impedance is adequate.

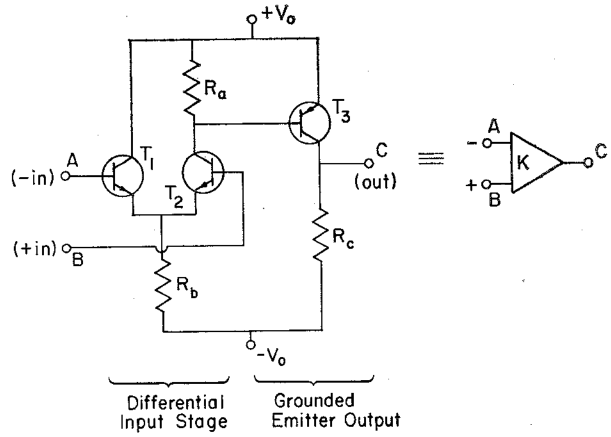


Fig. 2. The basic operational amplifier.

IV. Integrators and Summers

The operational amplifier of Fig. 2 can then be used to form the integrators, summers, and gain blocks needed for the signal flow graph of Fig. 1. For an integrator one can use the somewhat standard (Ryder,[23] p. 229) circuit of Fig. 3(a) which for large K is described by

$$e_i \approx -RCpe_0, \qquad p = d/dt. \tag{4a}$$

Because of the negative sign, this integrator is most convenient for implementing the flow graph of Fig. 1(b). However, for integrated circuits it is often more convenient to have capacitors grounded, in which case Fig. 3(b) can be used.[24] For large K, Fig. 3(b) is described by

$$e_i \approx \frac{R_f C}{2} pe_0 \tag{4b}$$

in which case the flow graph of Fig. 1(a) is the most appropriate.

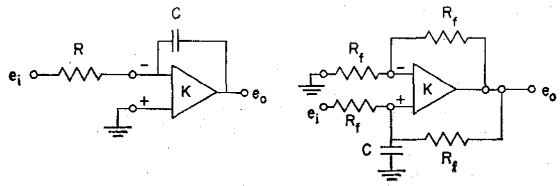
The summer is shown in Fig. 4. If we consider admittances $G_{i\pm} = 1/R_{i\pm}$ then we find, using mathematical induction on straightforward but detailed calculations,

$$e_0 \approx \sum_{i=1}^{m+} \frac{G_{i+}}{G_+}[1 + R_f G_-]e_{i+} - \sum_{i=1}^{m-} R_f G_{i-} e_{i-} \tag{4c}$$

where G_+ and G_- are the parallel combinations of the conductances at the positive and negative inputs

$$G_+ = \sum_{i=1}^{m+} G_{i+}, \qquad G_- = \sum_{i=1}^{m-} G_{i-}. \tag{4d}$$

We comment that degenerate results can occur if all signals to one of the summer amplifier inputs are absent, in which case we would insert a resistor from the input to ground; this simply augments G_+ or G_- in (4c) and (4d).



(a) (b)

Fig. 3. Negative (a) and positive (b) gain integrators.

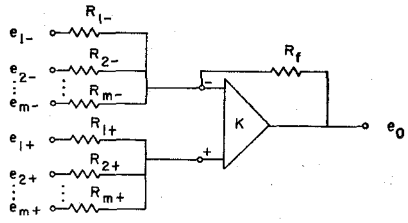
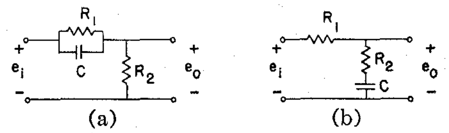


Fig. 4. Summer.



(a) (b)

Fig. 5. Passive first-order systems, (a) lead and (b) lag.

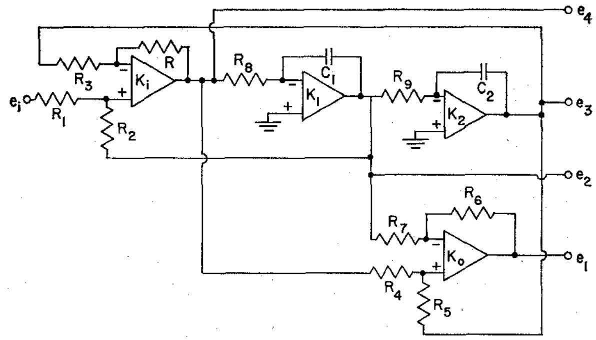


Fig. 6. General second-order function schematic.

Using these integrators and summers, the flow graph of Fig. 1 is easily implemented, and, in fact, uses at most n capacitors and $3(n + 2)$ transistors (from $n + 2$ operational amplifiers). Depending upon whether Fig. 3(a) or 3(b) is used, either $[n + 3(n + 2) + 2(n + 2)] = 6n + 10$ or $[n(3 + 1) + 3(n + 2) + 2(n + 2)] = 9n + 10$ resistors are used. These numbers are for a general nth-order system. However, many practical functions of degree n do not require the complete system (that is, some coefficients may be zero). For example, for a lowpass (all-pole)or for a highpass (all zeros at zero) function, no output summer is necessary.

V. Second-Order Realizations

Since any polynomial with real coefficients can be factored into first- or second-degree terms that also have real coefficients, $T(p)$ can be realized as a cascade of first- or second-order systems, each of which can be realized as previously described. In actual fact first-order sections can be realized to within a gain constant by standard passive RC lead–lag networks as shown in Fig. 5 (Kuo,[25] p. 345). The lead network of Fig. 5(a) is described by

$$T(p) = \frac{R_1R_2Cp + R_2}{R_1R_2Cp + R_1 + R_2}, \tag{5a}$$

and the lag network of Fig. 5(b) has

$$T(p) = \frac{R_2Cp + 1}{(R_1 + R_2)Cp + 1}. \tag{5b}$$

Note, however, that only the lag structure has a grounded capacitor.

As a consequence, it is necessary to be concerned here only with the realization of the general second-order transfer function. The circuit configuration for this function is shown in Fig. 6, using, for simplicity of drawing, the single resistor integrator of Fig. 3(a). The resulting transfer functions of interest are

$$\frac{\mathfrak{L}[e_1]}{\mathfrak{L}[e_i]} = T_1(p) = \frac{R_2(R + R_3)R_5(R_6 + R_7)}{(R_1 + R_2)R_3(R_4 + R_5)R_7} \cdot \left[\frac{R_8C_1R_9C_2p^2 + \frac{(R_4 + R_5)R_6}{R_5(R_6 + R_7)}R_9C_2p + \frac{R_4}{R_5}}{R_8C_1R_9C_2p^2 + \frac{R_1(R + R_3)}{(R_1 + R_2)R_3}R_9C_2p + \frac{R}{R_3}}\right] \tag{6a}$$

$$\frac{\mathfrak{L}[e_2]}{\mathfrak{L}[e_i]} = T_2(p) = -\frac{R_2(R + R_3)}{(R_1 + R_2)R_3} \cdot \left[\frac{R_9C_2p}{R_8C_1R_9C_2p^2 + \frac{R_1(R + R_3)}{(R_1 + R_2)R_3}R_9C_2p + \frac{R}{R_3}}\right] \tag{6b}$$

and

$$\frac{\mathfrak{L}[e_3]}{\mathfrak{L}[e_i]} = T_3(p) = \frac{R_2(R + R_3)}{(R_1 + R_2)R_3} \cdot \left[\frac{1}{R_8C_1R_9C_2p^2 + \frac{R_1(R + R_3)}{(R_1 + R_2)R_3}R_9C_2p + \frac{R}{R_2}}\right]. \tag{6c}$$

Note that though T_3 results from T_1 by letting R_4 and R_7 become infinite, T_2 is not a special case of T_1. Using T_1, T_2, and T_3 any second-order pole-zero pattern can be obtained, except that right half-plane zeros require sign reversal of the T_1 numerator p coefficient by connecting R_7 to the junction of R_4 and R_5 and replacing the output summer with an emitter follower. It is also of interest to note that at the resonant frequency

$$\omega_0 = \sqrt{\frac{R}{R_8C_1R_9C_2R_3}} \tag{7}$$

for which $R_8C_1R_9C_2p^2 + R/R_3 = 0$, we have

$$T_2(j\omega_0) = -R_2/R_1. \tag{6d}$$

If we also normalize by letting $R_1 = R_3 = R_5 = R_6 = R_8C_1 = R_9C_2 = 1$, then we obtain the simple and useful forms

$$T_1(p) = \frac{R_2(1+R)(1+R_7)}{(1+R_2)(1+R_4)R_7}\left[\frac{p^2 + \frac{1+R_4}{1+R_7}p + R_4}{p^2 + \frac{1+R}{1+R_2}p + R}\right] \tag{6e}$$

$$T_2(p) = -\frac{R_2(1+R)}{(1+R_2)}\left[\frac{p}{p^2 + \frac{1+R}{1+R_2}p + R}\right] \tag{6f}$$

$$T_3(p) = \frac{R_2(1+R)}{(1+R_2)}\left[\frac{1}{p^2 + \frac{1+R}{1+R_2}p + R}\right] \tag{6g}$$

with $\omega_0 = \sqrt{R}$ and $T_2(j\omega_0) = -R_2$. These expressions for T_1 and T_2 show which parameters are important for obtaining a desired pole-zero pattern.

VI. Sensitivity of Second-Order Realizations

At this point it will be determined how insensitive the previously mentioned realizations are to parameter changes. Since changes in the magnitude of transfer functions are of primary importance, we define the *system sensitivity* to changes in the value of any parameter x[26] as

$$S_x^{|T(p)|} = \frac{x}{|T(p)|}\frac{\partial |T(p)|}{\partial x}. \tag{8}$$

Since the sensitivity is a (real) function of frequency, it will be investigated at the frequency of most interest, ω_0, the resonant frequency. For most high Q cases, ω_0 is the frequency of maximum signal output and, hence, variations at ω_0 are usually maximum.

After lengthy but straightforward calculations, we obtain

$$S_R^{|T_3(j\omega_0)|} = S_{R_3}^{|T_3(j\omega_0)|} = 0 \tag{9a}$$

$$-S_{R_1}^{|T_3(j\omega_0)|} = S_{R_2}^{|T_3(j\omega_0)|} = 0. \tag{9b}$$

In addition to the very desirable lack of sensitivity to changes in R or R_3, the total effect of R_1 and R_2 approximates zero if, as would be the usual case, they have similar temperature coefficients since their effects are equal and opposite. This cancellation is particularly attractive since the individual sensitivities themselves are not large.

Since this method of synthesis is based on factoring the transfer function polynomial into first- and second-order terms, the degree of accuracy attained is determined by how closely the desired pole positions p_0 are approached. We will, therefore, look at the sensitivity of pole position to changes in the components and the amplifier gains. *Pole position sensitivity* to an element x will be defined as

$$S_x^{p_0} = \frac{d\sigma_0/\sigma_0}{dx/x} + j\frac{d\omega_0/\omega_0}{dx/x}. \tag{10}$$

This definition is similar to the one used by Blecher[27] with the real part normalized to σ_0 as he does, except that the imaginary part has also been normalized to ω_0.

This pole position sensitivity to R [for the poles of (6e) through (6g)] is

$$S_R^{p_0} = \frac{R}{1+R} + j\frac{\frac{R}{2}\left[1 - \frac{1+R}{2(1+R_2)^2}\right]}{R - \frac{1}{4}\left(\frac{1+R}{1+R_2}\right)^2}, \tag{11a}$$

which for $R = 1$ and $R_2 \to \infty$ (high Q) becomes

$$S_R^{p_0} = \tfrac{1}{2}(1 + j1). \tag{11b}$$

The sensitivity to R_1 and R_2 for $R = 1$ is

$$S_{R_1}^{p_0} = \frac{R_2}{R_1+R_2} + j\left(\frac{R_1}{2R_1+R_2} - \frac{R_1}{R_1+R_2}\right)$$

$$S_{R_2}^{p_0} = -\frac{R_1+R_2}{2R_1+R_2} + j\frac{R_1^2(R_1+R_2)}{(2R_1+R_2)^3\left[1 - \left(\frac{R_1}{2R_1+R_2}\right)^2\right]}, \tag{12a}$$

which for $R_1 = 1$ and $R_2 \to \infty$ reduce to

$$S_{R_1}^{p_0} = 1, \qquad S_{R_2}^{p_0} = -1. \tag{12b}$$

The sensitivity to R_3 for $R_3 = 1 = R$ and $R_2 \to \infty$ is

$$S_{R_3}^{p_0} = -\tfrac{1}{2}(1 + j1). \tag{12c}$$

Note that all of these sensitivities are comparable with those found in (9).

As can be seen from (6a), (6b), and (6c), the pole positions are *independent* of R_4, R_5, R_6, and R_7, and we therefore have zero pole position sensitivity to each of these parameters. The system sensitivity is, of course, not zero since the position of the zeros is affected by these parameters in the completely general case; however, these are usually minor effects and depend in detail on the specific function synthesized, so will not be discussed further here. For the case of the lowpass, all-pole, second-order function, (6c) (for which the output summer, K_0, can be eliminated), we then have zero system sensitivity to R_4, R_5, R_6, and R_7 since they are no longer used. For the highpass case we let $R_4 \to 0$ and $R_6 \to 0$ in (6a). This is equivalent to taking the output from e_4 and eliminating the output summer, K_0. Again we have zero sensitivity to R_4, R_5, R_6, and R_7. If consideration of the effects of R_8, R_9, C_1, and C_2 is restricted to their effect on the pole positions for the same reasons as given previously, it is found that the sensitivity of the magnitude of the denominator to each of these parameters is

$$S_{R_8}^{|D(j\omega_0)|} = S_{C_1}^{|D(j\omega_0)|} = 0 \tag{13a}$$

$$S_{R_9}^{|D(j\omega_0)|} = S_{C_2}^{|D(j\omega_0)|} = 1. \tag{13b}$$

Investigating the sensitivity of the resonant frequency of the system to variations in R_8, R_9, C_1 and, C_2, it is found from

$$S_x^{\omega_0} = \frac{x}{\omega_0}\frac{\partial \omega_0}{\partial x},$$

$$S_{R_8}^{\omega_0} = S_{R_9}^{\omega_0} = S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} = -\tfrac{1}{2}. \tag{14}$$

In summary, it can be seen that the sensitivity to the passive components is always equal to or less than 1, and most importantly that these sensitivities are essentially *independent* of the system Q.

If the sensitivity of the system to amplifier gain change is now considered, by using the exact expression for the integrators [Fig. 3(a)],

$$T(p) = \frac{-K}{1 + (1 + K)RCp}, \tag{15}$$

it is found that, for large K_1 and K_2, the denominators for (6) are given to a high degree of approximation by

$$D(p) = R_8C_1R_9C_2p^2 + \left[\frac{1}{K_1} + \frac{1}{K_2}\left(\frac{R_8C_1}{R_9C_2}\right) + \frac{R_1(R + R_3)}{R_3(R_1 + R_2)}\right]R_9C_2p + \frac{R}{R_3}. \tag{16a}$$

Identifying coefficients through

$$D = \alpha p^2 + \beta p + \gamma, \tag{16b}$$

it is found that

$$S_{K_1}^{|D(j\omega_0)|} = -\frac{QR_9C_2}{K_1\sqrt{\alpha\gamma}}, \qquad Q = \frac{\sqrt{\alpha\gamma}}{\beta} \tag{17a}$$

where Q is the quality factor for $T = 1/D$. Since $\sqrt{\alpha\gamma} = \sqrt{R_8C_1R_9C_2R/R_3}$, it is seen that $R_9C_2 \approx \sqrt{\alpha\gamma}$ in the usual cases where $R_8C_1 = R_9C_2$ and $R \approx R_3$, thus

$$S_{K_1}^{|D(j\omega_0)|} \approx -\frac{Q}{K_1}. \tag{17b}$$

For the all-pole second-order system we thus obtain

$$S_{K_1}^{|T_3(j\omega_0)|} \approx +\frac{Q}{K_1}. \tag{17c}$$

As there is no difference in D between K_1 and K_2, when $R_8C_1 = R_9C_2$, we immediately also have

$$S_{K_2}^{|T_3(j\omega_0)|} \approx \frac{Q}{K_2} \approx -S_{K_2}^{|D(j\omega_0)|}. \tag{17d}$$

The pole position sensitivities to K_1 and K_2 are [for the conditions of (6e)–(6g)]

$$S_{K1}^{p_0} = -\frac{Q}{K_1} + j\frac{1}{4QK_1[1 - (1/4Q^2)]}$$
$$S_{K_2}^{p_0} = -\frac{Q}{K_2} + j\frac{1}{4QK_2[1 - (1/4Q^2)]}, \tag{18}$$

which are essentially the same as the system sensitivities of (17c) and (17d). If a typical amplifier gain of $K_1 \approx K_2 \approx 1500$ used to realize a $Q = 500$ system is considered, we obtain

$$S_{K_1}^{|T_3(j\omega_0)|} \approx S_{K_1}^{p_0} \approx \tfrac{1}{3} \approx S_{K_2}^{p_0} \approx S_{K_2}^{|T_3(j\omega_0)|}. \tag{19}$$

This can be compared to a controlled source,[28],[29] or NIC realization which would have a sensitivity to gain change or NIC conversion factor of 1000 or more! Thus, there is a 3000 to 1 difference between the pole position sensitivity using these state-variable techniques and the minimum pole position sensitivity of $2Q$ reported (Blecher,[27] p. 87) for NIC circuits.

In addition to the extreme improvement in sensitivity, this method of RC active synthesis has absolute stability when ideal amplifiers are used, since neither a change in the passive elements nor in the amplifier gains can cause the poles to move into the right half plane. It should also be noted that in the case of the very useful bandpass second-order function, (6b) including the effect of K_1 and K_2, it is found that the center frequency–voltage ratio is still $T_2(j\omega_0) = -R_2/R_1$ and, thus, it is seen that the amplifier gains have no effect. In the cases of second-order functions which are 1) lowpass (with all zeros at infinity), 2) highpass (with all zeros at zero), or 3) bandpass (with one zero at zero and one at infinity), only three amplifiers are required and the outputs are e_3, e_4, and e_2 (of Fig. 6), respectively.

VII. Conclusions

Using state-space systems theory results, Fig. 1, we have shown how any degree n transfer function can be realized by integrated circuit structures. For this a simple differential amplifier and corresponding summers and integrators that use only grounded capacitors have been introduced (Figs. 2–4).

An alternate, which we have wished to emphasize, to using the complete structure of Fig. 1 is to break the transfer function into degree one or two factors and cascade the resulting sections. These sections can be realized as special cases of Fig. 1 or by the almost identical structures of Figs. 5 and 6. As seen in Section VI, the sensitivity of structures obtained by these techniques is extremely low. In particular, it has been shown that the general second-order transfer function can be obtained with a minimal number of capacitors, all of which can be grounded, in a low sensitivity RC active configuration with nearly unlimited Q capability without increasing the sensitivity to the passive components and only appreciably increasing the sensitivity to the active elements at Q's greater than 1000. If D of (16a) is considered, it can be seen that the maximum Q is given, when $R_1 = 0$, by

$$Q_{\max} = \frac{K_1K_2}{K_1 + K_2}. \tag{20}$$

Thus, $Q_{\max} = 750$ in a typical case with amplifier gains of 1500. Preliminary experimental results using $R = R_1 = R_3 = 20\ \text{k}\Omega$, $R_2 = 6.8\ \text{M}\Omega$, $R_8 = R_9 = 20\ \text{k}\Omega$, and $C_1 = C_2 = 1600$ pF, verify (6c) with the denominator of (16a). These measurements have yielded a measured Q of 141 with $f_0 = 4933$ Hz and a voltage gain of 282 at f_0, while (6c) gives a predicted Q of 139 with $f_0 = 4970$ Hz at a voltage gain of 278 at f_0. Qualitatively, experimental results seem to verify the sensitivity results of the paper.

It is worth pointing out that the paper could have been written without the use of the word "state-space." However, it was the systems theory use of state-space concepts which led us to the results, and it seems well worth pointing this out. Further, the results and techniques are really only special cases of much deeper state-space ones.[8] Since these latter results, as applied to multi-input multi-output systems, should have considerable practical significance in the future, it seems profitable to fit the specific theory here presented in the general framework.

Finally, we mention that this and similar theories have been developed[30],[31] for circuits containing integrated circuit type RC transmission lines where some advantages result in terms of the total number of elements required.

Acknowledgment

The authors wish to acknowledge the assistance of Mary Ellen Terry in the care and presentation of the manuscript, and also that of their students and colleagues who have constructively commented on the results.

References

[1] W. J. Fowler, "A look at linear integrated circuits," *Electronic Ind.*, vol. 24, pp. 64–69 and 194, September 1965.

[2] C. L. Heizman and D. G. Paterson, "An integrated operational amplifier," *Sperry Engrg. Rev.*, pp. 35–39, Fall 1965.

[3] G. W. Haines, "An integrated stabilized gain block," *IEEE Internat'l Conv. Rec.*, pt. 7, pp. 296–307, 1966.

[4] S. P. Emmons and H. W. Spence, "Very low-drift complimentary semiconductor network dc amplifiers," *IEEE J. Solid-State Circuits*, vol. SC-1, pp. 13–18, September 1966.

[5] R. C. Foss and B. J. Green, "Q factor, Q stability and gain in active filters," *Electronics Letters*, vol. 2, pp. 99–100, March 1966.

[6] J. S. Brugler, "RC synthesis with differential input operational amplifiers," in "Papers on integrated circuit synthesis," (compiled by R. Newcomb and T. Rao), Stanford Electronics Labs., Stanford, Calif., Tech. Rept. 6560-4, June 1966.

[7] W. J. Kerwin, "RC active networks," in "Papers on integrated circuit synthesis," (compiled by R. Newcomb and T. Rao), Stanford Electronics Labs., Stanford, Calif., Tech. Rept. 6560-4, June 1966.

[8] R. W. Newcomb and B. D. O. Anderson, "State variable results for minimal capacitor integrated circuits," Stanford Electronics Labs., Stanford, Calif., Tech. Rept. 6558-14, October 1966.

[9] C. C. Shumard, "Design of high-pass, low-pass and band-pass filters using R-C networks and direct-current amplifiers with feedback," *RCA Rev.*, vol. 11, pp. 534–564, December 1950.

[10] R. P. Sallen and E. L. Key, "A practical method of designing RC active filters," *IRE Trans. Circuit Theory*, vol. CT-2, pp. 74–85, March 1955.

[11] A. N. Thiele, "The design of filters using only RC sections and gain stages," *Electronic Engrg.*, pts. 1 and 2, vol. 28, pp. 31–36, January 1956; pp. 80–82, February 1956.

[12] E. F. Good, "A two-phase low-frequency oscillator," *Electronic Engrg.*, pts. 1 and 2, vol. 29, pp. 164–169, April 1957; pp. 210–213, May 1957.

[13] M. J. Somerville and G. H. Tomlinson, "Filter synthesis using active RC networks," *J. Electronics and Control*, vol. 12, pp. 401–420, May 1962.

[14] A. G. J. Holt and J. I. Sewell, "Table for the voltage transfer functions of single-amplifier double-ladder feedback systems," *Electronics Letters*, vol. 1, pp. 70–71, May 1965.

[15] A. N. Thiele, "Filters with variable cut-off frequencies," *Proc. IREE* (Australia), vol. 26, pp. 284–300, September 1965.

[16] H. C. Pande and R. S. Shukla, "Synthesis of transfer functions using an operational amplifier," *Proc. IEE* (London), vol. 112, pp. 2208–2212, December 1965.

[17] A. G. J. Holt and J. I. Sewell, "Active RC filters employing a single operational amplifier to obtain biquadratic responses," *Proc. IEE* (London), vol. 112, pp. 2227–2234, December 1965.

[18] P. R. Geffe, "Make a filter out of an oscillator," *Electronic Design*, vol. 10, pp. 56–58, May 10, 1967.

[19] R. E. Kalman, "Mathematical description of linear dynamical systems," *J. SIAM Control*, ser. A, vol. 1, no. 2, pp. 162–192, 1963.

[20] S. J. Mason and H. J. Zimmermann, *Electronic Circuits, Signals, and Systems*. New York: Wiley, 1960.

[21] R. L. Walker, *Introduction to Transistor Electronics*. Belmont, Calif.: Wadsworth, 1966.

[22] H. C. Lin, T. B. Tan, G. Y. Chang, B. Van der Leest, and N. Formigoni, "Lateral complementary transistor structure for the simultaneous fabrication of functional blocks," *Proc. IEEE*, vol. 52, pp. 1491–1495, December 1964.

[23] J. D. Ryder, *Engineering Electronics*. New York: McGraw-Hill 1957.

[24] G. J. Deboo, "A novel integrator," in *Electronic Design* (to appear).

[25] B. C. Kuo, *Automatic Control Systems*. Englewood Cliffs, N. J.: Prentice-Hall, 1962.

[26] L. P. Huelsman, *Theory of Active RC Circuits*. New York: McGraw-Hill, to appear.

[27] F. H. Blecher, "Application of synthesis techniques to electronic circuit design," *IRE Trans. Circuit Theory* (Special Suppl.), vol. CT-7, pp. 79–91, August 1960.

[28] W. J. Kerwin and L. P. Huelsman, "The design of high performance active RC band-pass filters," *IEEE Internat'l Conv. Rec.*, pt. 10, March 1966.

[29] W. J. Kerwin, "An active RC elliptic function filter," *IEEE Bull.* (Los Angles Council Publication), April 1966.

[30] W. J. Kerwin, "Analysis and synthesis of active RC networks containing distributed and lumped elements," Ph.D. dissertation, Stanford University, Stanford, Calif., August 1967; and Stanford Electronics Labs., Stanford, Calif., Tech. Rept. 6560-14, August 1967.

[31] E. B. Kozemchak, "State-variable synthesis of distributed RC network transfer functions," in "Papers on integrated circuit synthesis-II," (complied by R. Newcomb and R. de Oliveira), Stanford Electronics Labs., Stanford, Calif., Tech. Rept. 6560-13, June 1967.