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A MINIMAL CAPACITOR CASCADE SYNTHESIS FOR INTEGRATED CIRCUITS*

by

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ABSTRACT

Using a theory of nonreciprocal cascade synthesis based upon removal of even part admittance zeros a synthesis of resistively terminated filters is developed with reference to integrated realizations. The resulting structure is a cascade of capacitor-gyrator sections, with all gyrators grounded, and using a minimum number of capacitors all of which can be chosen equal.

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A problem that's basic to man
Is how integration's performed,
For the race is to yet crystalize;
The designer is similarly faced
But in circuits can here realize.

I. INTRODUCTION

Although fabrication and design techniques are fairly well-developed for the basic building blocks (transistors, resistors, capacitors, etc.) used in integrated circuits [1], there are relatively few synthesis techniques available which are particularly pertinent to integrated circuits. The problems of linear integrated circuit synthesis are of course associated with obtaining inductorless configurations, thus most of the results of active RC synthesis [2] appear at first sight to be applicable. However, a closer look [3] reveals that, as with lumped circuits, active RC techniques turn out to be uneconomical because of the tolerances involved. Recognizing this latter fact in the lumped case, Orchard [4] has proposed that the most practical of the inductorless synthesis techniques should result by making gyrator-capacitor replacements of inductors [5], [6] in "classical" passive designs. This practicality is expected since good passive designs are relatively insensitive to changes in element values and the introduction of the gyrator does little to change this insensitivity [7].

Because of the availability of gyrators for integrated circuits [8], [9], [10], [11] the idea of inductor replacement by gyrator-capacitor equivalents seems attractive for integrated circuits. However, mere replacements can often be quite wasteful of elements, so that it seems more appropriate to develop new techniques which naturally fit the integrated philosophy. Consequently, we develop here a cascade RC-gyrator synthesis appropriate to the structures useful for integrated circuit synthesis. In particular this synthesis employs three terminal gyrators with a common ground, (the only configuration physically available), and uses the minimum possible number of capacitors. If so desired, all capacitors can be chosen identical, by an appropriate parameter choice in the gyrators. Similarly an arbitrary ratio of terminating resistors can be accomodated.

We assume as given a rational positive-real input admittance $y(p)$.

Customarily we would be more interested, for example, in the synthesis of a prescribed transfer voltage ratio magnitude $|V_2(j\omega)/V_s(j\omega)|^2$, under given resistive load and source terminations, as shown in Fig. 1. But, following standard theories [12, p. 425] $y(p)$ can be obtained from $|V_2/V_s|$ through

$$\left|\frac{y(j\omega) - G_1}{y(j\omega) + G_1}\right|^2 = 1 - 4\,\frac{G_2}{G_1}\left|\frac{V_2(j\omega)}{V_s(j\omega)}\right|^2 \tag{1}$$

where G_1 and G_2 are the source and load conductances. Equation (1) assumes that $1 \geq 4G_2|V_2/V_s|^2/G_1$ for all real ω, but, if such is not the case a constant gain amplifier can be inserted at the output at least in the cases of interest when $V_2(j\omega)/V_s(j\omega)$ is finite for all real ω. In a similar manner, if transducer gain, or the insertion power ratio, is prescribed $y(p)$ can be obtained.

Given then $y(p)$ the philosophy is to realize zeros of the even part of $y(p)$ by extracting factors from $y(p)$ by means of a cascade of capacitor grounded-gyrator sections of degree one or two. The development follows that of Hazony [13, pp. 130-135] but with emphasis on a structure of primary interest to integrated circuits.

II. PRELIMINARIES

As a preliminary to the synthesis, we review results to be used in the presented cascade method, these results also serving to define the notation used. We begin by considering a general 2-port described by its admittance matrix $\underset{\sim}{Y}(p)$ and its chain (or transmission) matrix $\underset{\sim}{\mathcal{Y}}(p)$, which, with variables as illustrated in Fig. 1, are defined by

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \underset{\sim}{Y}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}, \quad \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \underset{\sim}{\mathcal{Y}}\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \tag{2a}$$

and related through

$$\underset{\sim}{Y} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} \mathcal{Y}_{22}\mathcal{Y}_{12}^{-1} & \mathcal{Y}_{21}-\mathcal{Y}_{22}\mathcal{Y}_{12}^{-1}\mathcal{Y}_{11} \\ -\mathcal{Y}_{12}^{-1} & \mathcal{Y}_{12}^{-1}\mathcal{Y}_{11} \end{bmatrix} \tag{2b}$$

$$\underset{\sim}{\mathcal{Y}} = \begin{bmatrix} \mathcal{Y}_{11} & \mathcal{Y}_{12} \\ \mathcal{Y}_{21} & \mathcal{Y}_{22} \end{bmatrix} = \begin{bmatrix} -y_{21}^{-1}y_{22} & -y_{21}^{-1} \\ y_{12}-y_{11}y_{21}^{-1}y_{22} & y_{11}y_{21}^{-1} \end{bmatrix} \tag{2c}$$

For example, a gyrator is described by

$$\underset{\sim}{Y} = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} \tag{3a}$$

where g, the gyration conductance, is a real parameter. If such a gyrator is cascaded with an inductor as shown in Fig. 2a, multiplication of chain matrices and conversion to $\underset{\sim}{Y}$ through (2a) gives the equivalence with Fig. 2b, which is described by

$$\underset{\sim}{Y} = \begin{bmatrix} 0 & g \\ -g & pg^2\ell \end{bmatrix} \tag{3b}$$

If, as shown in Fig. 3, a load of admittance $y_\ell(p)$ is connected at port two of a general 2-port, then one finds, through $y = y_{11}-y_{12}(y_\ell+y_{22})^{-1}y_{21}$, that the input admittance $y(p)$ is given by

$$y = \frac{\Delta_y + y_{11}y_\ell}{y_\ell + y_{22}} \qquad (4a)$$

where $\Delta_y = y_{11}y_{22} - y_{12}y_{21}$. Solving for y_ℓ gives

$$y_\ell = \frac{\Delta_y - y_{22}y}{y - y_{11}} \qquad (4b)$$

For example the gyrator connection of Fig. 4 yields

$$y = g^2/y_\ell \; , \; y_\ell = g^2/y \qquad (5)$$

which shows that a preliminary gyrator extraction can be used to change impedances into admittances as well as inductors into capacitors.

To continue, we let a subscript asterisk, $_*$, denote replacement of p by $-p$ (Hurwitz conjugation), a superscript tilde, $\tilde{}$, denote matrix transposition, and define the even part of a function $y(p)$ through

$$2 \text{ Ev } y = y + y_* \qquad (6a)$$

It is important to note that if $z = 1/y$ then

$$2 \text{ Ev } y = yy_* \text{ Ev } z \qquad (6b)$$

Thus zeros of the even part of z are generally zeros of the even part of y and vice versa.

If $\underset{\sim}{Y}(p)$ is the admittance matrix of a lossless 2-port then [13, p. 155]

$$\underset{\sim}{Y} = -\tilde{\underset{\sim}{Y}}_* \qquad (7)$$

For a lossless 2-port loaded as in Fig. 3, (4a) gives, since $\Delta_y = \Delta_{y*}$

$$\text{Ev } y = \frac{-y_{12}y_{21} \text{ Ev } y_\ell}{(y_\ell+y_{22})(y_{\ell*}+y_{22*})} \qquad (8a)$$

while in a similar manner (4b) gives

$$\text{Ev } y_\ell = \frac{-y_{12}y_{21} \text{ Ev } y}{(y-y_{11})(y_*-y_{11*})} \tag{8b}$$

For comparison purposes we note, from $G_2V_2 = -I_2$ and $G_1V_s = I_1 + G_1V_1$, that for Fig. 1

$$\frac{V_2}{V_s} = \frac{-G_1y_{21}}{[y_{11}+G_1][y_{22}+G_2] - y_{12}y_{21}} \tag{8c}$$

Since zeros of y_{21} appear in the numerator of (8a), (8b), and (8c) we conclude that zeros of $V_2(p)/V_s(p)$, zeros of transmission, are generally zeros of Ev y(p). Consequently, recalling (1), a synthesis of $|V_2(j\omega)/V_s(j\omega)|$ can proceed by a cascade synthesis of y(p) based upon the removal of zeros of Ev y. Equation (8b) shows that original zeros of Ev y not extracted in a given cascade section remain available for extraction in later sections, thus giving the designer some freedom in the choice of the final structure through the order chosen for even part zero removal. We note, from (6a), that the extraction of a right half plane zero of Ev y(p) also removes a left half plane zero of Ev y(p). However, since we are treating nonreciprocal synthesis right half plane zeros of transmission need not be left half plane zeros of transmission or vice versa, necessitating the development of separate extractions for left and right half plane zeros.

III. BASIC SECTION - GENERAL REAL ZEROS

The 2-port of Fig. 5 represents the section upon which cascade synthesis is based. Its advantages for integrated circuitry lie in the grounding of the gyrator and the presence of a single capacitor.

By adding admittance matrices the 2-port of Fig. 5, called a basic section for y at $p = k$, is found to be

$$\underset{\sim}{Y} = \frac{y(k)}{k} \begin{bmatrix} p & -p-k \\ -p+k & p \end{bmatrix} \tag{9a}$$

where for now k is a positive and real number. Applying (2c) we find, the chain matrix as

$$\underset{\sim}{\mathcal{Y}} = \frac{1}{p-k} \begin{bmatrix} p & k/y(k) \\ ky(k) & p \end{bmatrix} \tag{9b}$$

If this basic section is used as the 2-port in Fig. 3, then (4b) shows, since $\Delta_y = y^2(k)$,

$$\frac{y_\ell(p)}{y(k)} = \frac{ky(k) - py(p)}{ky(p) - py(k)} \tag{10}$$

At this point we observe that $y_\ell(p)$, as given in (10), is positive-real when $y(p)$ is since (10) defines a Richards' transformation [15, p. 779]. Further, if k is chosen as a zero of $\mathrm{Ev}\, y(p)$ then the degree of $y_\ell(p)$ is one less than that of $y(p)$ [15, p. 779]. That is, if we denote the degree [16, p. 543] by $\delta[\ ]$ then

$$\delta[y_\ell(p)] = \delta[y(p)] - 1 \quad \text{if} \quad \mathrm{Ev}\, y(k) = 0 \tag{11}$$

We further observe that when k satisfies $y(k) = -y(-k)$ then so does $-k$, and, hence, (11) still holds since $y_\ell(p)$ in (10) is still positive-real. Thus, the section shown in Fig. 5 is realizable with the capacitor positive valued when k is either a positive-or a negative-real zero of Ev $y(p)$.

We conclude that given a rational positive-real $y(p)$, if k is a real finite nonzero zero of Ev $y(p)$ we can extract the gyrator-capacitor section of Fig. 5 to obtain the load admittance $y_{\ell}(p)$ as given by (10). This extraction uses one capacitor and decreases the degree of the admittance by one. Since connection of a realization for $y_{\ell}(p)$ as a load on the basic section yields $y(p)$ at the input, a repetition of the process yields a cascade synthesis. However, for all-inclusive synthesis we need to investigate other than real finite and nonzero zeros of transmission.

IV. GENERAL COMPLEX ZEROS

To proceed to the more general case let us make a second extraction of a basic section for y_ℓ at $p = k_1$, this to follow a basic section for y at $p = k$, as shown in Fig. 6. Equation (10) still holds, even though we are going to allow complex k, and in a similar way the load for the second section is

$$\frac{y_{\ell\ell}(p)}{y_\ell(k_1)} = \frac{k_1 y_\ell(k_1) - p y_\ell(p)}{k_1 y_\ell(p) - p y_\ell(k_1)} \tag{12}$$

After substituting (10) into (12) and considerable, but straightforward manipulation, one finds

$$y_{\ell\ell}(p) = \frac{\left[\frac{kk_1}{p^2}\frac{ky(k) - k_1 y(k_1)}{ky(k_1) - k_1 y(k)} + 1\right] y(p) - \frac{y(k)y(k_1)}{p}\frac{k^2 - k_1^2}{ky(k_1) - k_1 y(k)}}{\left[\frac{kk_1}{p^2}\frac{ky(k_1) - k_1 y(k)}{ky(k) - k_1 y(k_1)} + 1\right] - \frac{y(p)}{p}\frac{k^2 - k_1^2}{ky(k) - k_1 y(k_1)}} \tag{13}$$

Similarly, by multiplying chain matrices as given by (9b) and converting to $\underset{\sim}{Y}$ through (2b), one finds for the 2-port of Fig. 6, again after suitable manipulation,

$$\underset{\sim}{Y} = p\frac{ky(k) - k_1 y(k_1)}{k^2 - k_1^2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{1}{p}\frac{kk_1[ky(k_1) - k_1 y(k)]}{k^2 - k_1^2}\begin{bmatrix} 1 & -\frac{y_\ell(k_1)}{y(k)} \\ -\frac{y_\ell(k_1)}{y(k)} & \left[\frac{y_\ell(k_1)}{y(k)}\right]^2 \end{bmatrix}$$

$$+ \frac{ky(k) - k_1 y(k_1)}{k - k_1}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \tag{14}$$

At this point we let k be a nonreal zero of Ev y, with Re $k > 0$ temporarily. Then we note that $y_\ell(p)$ is positive [17, p. 278] but not real. Still (12) holds and thus y_ℓ is one degree less than y. But if k is a zero of Ev y then so is the complex conjugate k^* since Ev y has real coefficients when y is positive-real. Thus, by (8b), k^* is a zero of Ev y_ℓ (which does not have real coefficients). Choosing

$$k_1 = k^* \qquad (15)$$

then guarantees that $y_{\ell\ell}$ of (12) must decrease in degree by one below that of y_ℓ, or

$$\delta[y_{\ell\ell}(p)] = \delta[y(p)] - 2 \qquad (16)$$

Further, by the above arguments $y_{\ell\ell}$ is positive. Using (15) in (13) we find, recalling the positive-real constraint $y(k^*) = y^*(k)$,

$$y_{\ell\ell}(p) = \frac{\left[\frac{|k|^2}{p^2}\frac{\text{Im } ky(k)}{\text{Im } ky^*(k)} + 1\right]y(p) - \frac{|y(k)|^2}{p}\frac{\text{Im } k^2}{\text{Im } ky^*(k)}}{\left[\frac{|k|^2}{p^2}\frac{\text{Im } ky^*(k)}{\text{Im } ky(k)} + 1\right] - \frac{y(p)}{p}\frac{\text{Im } k^2}{\text{Im } ky(k)}} \qquad (17)$$

which shows that $y_{\ell\ell}$ has real coefficients; $y_{\ell\ell}$ is then positive-real and of degree two less than y.

But with k complex the gyrators and capacitors of Fig. 6 become complex valued and hence unrealizable. Fortunately, however, Fig. 6 can be replaced by a physically obtainable structure by evaluating its admittance matrix, (14) in terms of $k_1 = k^*$

$$\underset{\sim}{Y} = p\frac{\text{Im } ky(k)}{\text{Im } k^2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{|k|^2}{p}\frac{\text{Im } ky^*(k)}{\text{Im } k^2}\begin{bmatrix} 1 & -\frac{\text{Im } ky(k)}{\text{Im } ky^*(k)} \\ -\frac{\text{Im } ky(k)}{\text{Im } ky^*(k)} & \left[\frac{\text{Im } ky(k)}{\text{Im } ky^*(k)}\right]^2 \end{bmatrix}$$

$$+ \frac{\text{Im } ky(k)}{\text{Im } k}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \qquad (18)$$

This admittance is realized by the two reactive element circuit of Fig. 7, which by a somewhat involved argument [13, p. 137] is seen to have positive inductance and capacitance. Figure 7 is therefore a realizable equivalent for the cascaded basic sections of Fig. 6. Figure 7 represents a general nonreciprocal Darlington section developed on an admittance basis [13, p. 134]. It should be observed that $y_{\ell\ell}$ of (17) remains unchanged and $\underset{\sim}{Y}$ of (18) is replaced by its transpose when k is replaced by its negative. Thus zeros of transmission with $\operatorname{Re} k < 0$ can equally well be extracted by this method.

To make Fig. 7 useful for integrated circuits one notes that the inductor-transformer sub-2-port can be replaced by the convenient capacitor grounded-gyrator equivalence of Fig. 8 [5], [6]. The equivalence of Fig. 8 is easily checked by noting that a transformer of turns ratio $m:1$ is equivalent to a cascade of two gyrators as shown in Fig. 9 with $m = g_1/g_2$. Figure 9 is readily checked by comparing the chain matrices for a) and b), the latter being obtained by chain matrix multiplication. Letting the capacitance of Fig. 8 be free to be chosen in any convenient manner as K, Fig. 2 applies to fix $g_1^2 = K/\ell$, thus giving the element values of Fig. 8.

In summary, when k, with $\operatorname{Re} k \neq 0$, is a zero of the even part of y the degree two extraction of Fig. 7 can be made, with the replacement of Fig. 8 allowing a capacitor grounded-gyrator network suitable for integrated circuit implementation. The capacitance K can be chosen to insure that all capacitors have identical value, as will later be discussed.

V. IMAGINARY ZEROS

If k is purely imaginary the results of the last section become indeterminate. However, a Brune type structure can be obtained by differentiating numerators and denominators of the various terms in (14). With $k_1 = k^* = -k \neq 0$ satisfying $y(k_1) = -y(k)$ we find, on defining

$$2a_+ = y'(k) + \frac{y(k)}{k}, \quad 2a_- = y'(k) - \frac{y(k)}{k} \tag{19a}$$

where the prime denotes differentiation, that

$$Y = pa_+\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{|k|^2}{p}a_-\begin{bmatrix} 1 & -\frac{a_+}{a_-} \\ -\frac{a_+}{a_-} & \left[\frac{a_+}{a_-}\right]^2 \end{bmatrix}. \tag{19b}$$

In this case Y is symmetric and positive-real since a_+ and a_- are positive by Takahasi's Theorem [18, p. 58]. Equation (19b) is realized by Fig. 7, without the gyrator and with a suitable change in element values; the replacement of Fig. 8 therefore applies to give a realization suitable for integrated circuits. The final structure, which is practically useful for obtaining imaginary axis zeros of transmission, is shown in Fig. 10. As in Fig. 8 we insert a parameter $K > 0$ which can freely be chosen, say to equalize capacitance values, while requiring the turns ratio, from (19b), to equal the gyration conductance ratio, $a_+/a_- = g_1/g_2$.

Performing the same differentiations of numerators and denominators on (13) we find that Fig. 10 has

$$y_{\ell\ell}(p) = \frac{\left[\frac{|k|^2}{p^2}\frac{a_+}{a_-} + 1\right]y(p) - \frac{|y(k)|^2}{pa_-}}{\left[\frac{|k|^2}{p^2}\frac{a_-}{a_+} + 1\right] - \frac{y(p)}{pa_+}} \tag{20a}$$

which, following previous arguments, is positive-real and of degree two less than $\delta[y(p)]$.

In the cases where $y(k) = 0$ these previous arguments run into some difficulty, as witness (10), but, nevertheless, the final results remain valid and correspond to removing poles at $\pm k$ from $z = 1/y$. Under this condition, $y(k) = 0$, we note that $2a_+ = 2a_- = y'(k)$, which is still positive by Takahasi's Theorem, such that $g_1 = g_2$ in Fig. 10. Consequently, the transformer in an equivalent similar to Fig. 8 is 1:1; hence Fig. 10 is identical to the capacitor, $c = a_+$, in parallel with an inductor, $\ell = 1/|k|^2 a_-$. Further, if $y(k) = 0$, (20a) gives

$$y_{\ell\ell}(p) = \frac{y(p)y'(k)[p+|k|^2]}{[p^2+|k|^2]y'(k) - 2py(p)} \tag{20b}$$

and, since $y(p) = (p+k)y'(k) + \ldots = (p-k)y'(k) + \ldots,$ one can check that $(p+|k|^2)^2$ cancels from numerator and denominator such that $\delta[y_{\ell\ell}] = \delta[y] - 2$, as expected. It should also be pointed out that if $z(k) = 0$ then a preliminary gyrator can be extracted, by (15), to turn the impedance into an admittance (for which $y(k) = 0$).

VI. ZEROS AT ZERO AND INFINITY

Zeros of transmission, or zeros of Ev y, at zero and, especially, infinity require somewhat special attention since the pertinent cascade sections may not have an admittance matrix, or the zeros may come from the denominator of (8c). Thus it is important to note that such zeros are zeros of $y(p)$ or its inverse $z = 1/y$. If $p = 0$ is a zero of y it is a pole of z and can be extracted as a series capacitor, Fig. 11a). If $p = 0$ is a zero of z it can be made a pole of z by a gyrator extraction and then the pole can be removed by a series capacitor, as shown in Fig. 11b). In Fig. 11b) the gyration conductance g is arbitrary and can be chosen to obtain a convenient capacitance value. If $p = \infty$ is a zero of z, it is a pole of y and can be removed by a shunt capacitor, Fig. 12a). Finally a preliminary gyrator extraction can change a zero of y at infinity into a pole of y yielding again a shunt capacitor extraction, Fig. 12b) where, as before, any convenient value of g can be chosen. Note the applicability of Fig. 2 to this extraction.

VII. SUMMARY OF PROCEDURE

At this point we can summarize the procedure. Given $y(p)$ one removes, in any desired order, the zeros of $\mathrm{Ev}\, y(p)$ through the sections of Figs. 4 through 12. If $y(p)$ has been derived from a filter characteristic for a circuit as shown in Fig. 1, then removals are made of zeros of $\mathrm{Ev}\, y(p)$ which are zeros of transmission, it being observed that the negative of the zero of $\mathrm{Ev}\, y(p)$ will automatically be extracted in the process. Each removal need use only grounded-gyrators and capacitors and each section reduces the degree of the admittance by the number of capacitors used. Repetition of the procedure eventually leads to a zero degree function, which is a resistor. Hence the final structure is a cascade of capacitor grounded-gyrator sections terminated in a resistor and using the minimum number, $\delta[y]$, of capacitors [19]. The structure as such is ideal for integrated circuits where it is of special interest to note that all capacitors can be made identical by appropriate admittance normalizations.

VIII. NORMALIZATION

To introduce another parameter at almost any stage in the procedure a gyrator of gyration conductance g can be extracted as in Fig. 4. By (5) this converts an admittance to an impedance and scales the impedance level. Since zeros of $\mathrm{Ev}\; y$ are generally zeros of $\mathrm{Ev}\; z$, (6b), synthesis can still proceed from the admittance as described to this point. However, the inserted parameter allows the scaling of one capacitor per section while a free choice of K in Figs. 8 and 10 allows the scaling of the second capacitor when two are present. In the case of Figs. 11a) and 12a), a preliminary gyrator extraction requires a second gyrator extraction with Fig. 9 showing that the two gyrators are merely a transformer, clearly allowing capacitor scaling. In conclusion, we see that by a simple preliminary gyrator extraction per section, all capacitors can be chosen of equal capacitance.

To illustrate the details let us normalize the Brune section of Fig. 10 such that both capacitors are of unit capacitance. As shown in Fig. 13, we first extract the gyrator, such that we wish to apply the theory to $y_\ell = g^2/y$. Since k is assumed an imaginary zero of $\mathrm{Ev}\; y$ it is also an imaginary zero of $\mathrm{Ev}\; y_\ell$ and Fig. 10 applies directly, upon inserting subscript ℓ's, to give a Brune section pertaining to y_ℓ. Since $y_\ell'(k) = -g^2y'(k)/y^2(k) > 0$, we have

$$2a_{\ell+} = -\frac{g^2}{y^2(k)}[y'(k) - \frac{y(k)}{k}] = -\frac{g^2}{y^2(k)}2a_- \qquad (21a)$$

$$2a_{\ell-} = -\frac{g^2}{y^2(k)}[y'(k) + \frac{y(k)}{k}] = -\frac{g^2}{y^2(k)}2a_+ \qquad (21b)$$

$$\frac{g_{\ell_2}}{g_{\ell_1}} = \frac{a_{\ell-}}{a_{\ell+}} = \frac{a_+}{a_-} = \frac{g_1}{g_2} \qquad (21c)$$

$$g_{\ell_1}^2 = K|k|^2 a_{\ell-} \qquad (21d)$$

Choosing unit capacitors requires $a_{\ell+} = K = 1$ or

$$g^2 = -y^2(k)/a_- \tag{22a}$$

$$g_{\ell_1}^2 = |k|^2 a_{\ell-} = -|k|^2 \frac{g^2 a_+}{y^2(k)} = |k|^2 \frac{a_+}{a_-} \tag{22b}$$

The final section is as shown in Fig. 13.

Such normalizations are especially convenient for integrated circuits, since identical elements are quite preferable for circuit layout and electronically adjustable gyrators are readily obtainable [9]. In actual fact one could set up standard sections, as Fig. 10 for real frequency zeros of transmission, which can be electronically adjusted, by gyrator variation, to obtain desired zeros of transmission (and with fixed capacitors). An interesting application, for example, would be to the synthesis of variable bandwidth filters.

IX. EXAMPLES

Here we consider three examples which illustrate almost all the points of the theory.

Example 1 (Real Zeros): The admittance

$$y(p) = \frac{3p^2 + 12p + 9}{9p^2 + 25p + 19} \tag{23a}$$

has zeros of $Ev\ y(p) = 9[3p^4-18p^2+19]/[81p^4-2183p^2+361]$ at $p = \pm 2.1524$, and ± 1.1692. Choosing, arbitrarily, or perhaps for a desired zero of transmission,

$$k = -2.1524 \tag{23b}$$

yields

$$y(k) = -.4255$$

and, from (10),

$$y_\ell(p) = 1.2766 \frac{p + 1.2521}{3.8299p + 4.1813} \tag{23c}$$

Repeating with $k = +1.1692$, which has $y_\ell(k) = .3570$ and $y_{\ell\ell}(p) = .3333$ yields the circuit of Fig. 14, which then illustrates the extraction of positive and negative real zeros of $Ev\ y$ to produce transmission zeros at these frequencies.

Example 2 (Complex Zeros): The admittance

$$y(p) = \frac{3p^2 + 12p + 9}{6p^2 + 13p + 10} \tag{24a}$$

has zeros of $Ev\ y(p) = 18[p^4-4p^2+5]/[36p^4-49p^2+100]$ at $p = \pm[1.4553 \pm j.3436]$. Arbitrarily choosing

$$k = 1.4553 - j.3436 \tag{24b}$$

requires the choice of $k_1 = k^* = 1.4553 + j.3436$ and yields $y(k) = .7864 + j.0271$ for which we have $k^2 = 2 - j1$, $\text{Im}\, ky(k) = -.2308$, $\text{Im}\, ky^*(k) = -.3096$, and $|y(k)|^2 = .6192$. Equation (17) is then $y_{\ell\ell}(p) = .5000$. Figure 7, combined with Fig. 8, where $K = .2308$ is chosen to give identical capacitors, is then as shown in Fig. 15.

Example 3 (Imaginary Zeros): Let it be desired to design a low-pass, doubly-terminated, third-order Cauer filter with an effective passband ripple of .18db. Such a filter is dimensioned by standard techniques by Saal and Ulbrich [20, p. 304] from which we can derive, for (1), either

$$y(p) = \frac{2p^3 + .992009p^2 + 1.035852p + .276463}{.992009p^2 + .489186p + .276463} \tag{25a}$$

or y as the reciprocal of the right side of (25a). This low-pass filter has a zero of transmission at infinity, and by construction there are zeros of transmission at $p = \pm j1.912730$. Proceeding from (25a) the zero of transmission at infinity (pole of y at ∞) is easily extracted yielding

$$y_1(p) = y(p) - 2.01611p = \frac{.005756p^2 + .478472p + .276463}{.992009p^2 + .489186p + .276463} \tag{25b}$$

Then $y_1(p)$ has a zero of its even part at

$$k = k_1^* = -k_1 = j1.912730 \tag{25c}$$

from which we find $y_1(k) = -j.272959$, $y_1'(k) = .1662409$ giving, by (19a) applied to y_1, $a_+ = .011768$, $a_- = .154474$, and, by (20a), $g_{\ell\ell}(p) = .005803$. Figure 16 then results from Fig. 10, with $K > 0$ any desired value. The convenient cascade structure of the synthesis which uses a minimum number of capacitors should be observed; note that one reactive element is saved over the conventional design [20, p. 306].

Since all the capacitors of Fig. 16 are not of equal size it is of interest to illustrate a normalization procedure. We could insert a gyrator after the left hand capacitor of Fig. 16 and apply (21) to obtain any desired values, by a choice of K and g, for the right hand two capacitors. If the left hand capacitor is desired to be changed in value it can be preceeded by a transformer, which also scales all impedance levels to the right and which can be realized by Fig. 9. A more appealing approach, since it saves one gyrator, is to begin with

$$y(p) = \frac{.992009p^2 + .489186p + .276463}{2p^3 + .992009p^2 + 1.035852p + .276463} \qquad (26)$$

(the inverse of (25a) which yields the same $|V_2(j\omega)/V_s(j\omega)|$). This allows the extraction of Fig. 12b) with g a parameter which can be freely chosen. At this point the remaining admittance is $g^2 y_1(p)$, where $y_1(p)$ is given by (25b). Another gyrator can then be extracted to finally yield Fig. 17 with the element values found by applying (21) with g replaced by g_n. Note, then, that with the three parameters g, g_n, and k free to be chosen one can obtain any desired values of capacitance, in particular all capacitors can be chosen equal and of any size readily available. If the final load resistor is desired to be equal to the source resistance this can be also obtained by inserting a gyrator before G_2 (since a gyrator loaded in a resistor is another resistor).

Figures 16 and 17 are normalized such that a cut-off frequency of $\omega_c = .59863$ results with a zero of transmission at $\omega_a = 1.91273$ and a one ohm source resistance. By standard frequency and impedance level scaling any desired cut-off frequency and source resistance can of course be obtained.

X. DISCUSSION

By revisizing, and extending portions of, the cascade synthesis of Hazony a synthesis of driving-point admittances, and, with these, voltage-transfer filters, is obtained. Because the synthesis uses in a natural way only capacitive reactive elements, and of these a minimum number all of which can be chosen identical, the method is ideally suited to the synthesis of linear integrated circuits. Of special interest are the sections of Figs. 10 and 11 which yield real frequency $(p = j\omega)$ zeros of transmission, such zeros being the ones of most interest in usual filter designs.

The techniques rest upon the availability of gyrators in integrated form. But, when one places the restriction that all gyrators have a common ground (for the purpose of biasing transistors in available gyrator realizations), gyrators in integrated form seem readily available at least for frequencies below the middle megacycle range. Indeed one is fortunate in having electronically adjustable gyrators making it possible to easily accomodate the identical capacitors so convenient for integrated use.

If one investigates the calculations involved one sees that for any really useful filter the numbers need to be carried to many significant figures. Yet this is common to any accurate filter design and points to the advantages of a computerized society. It should be observed, however, that if $y(p)$ is originally lossless, then $y(p) + y(-p) \equiv 0$ and all p are zeros of the even part. Consequently, any convenient k can be chosen for the development with $k = 0$ or ∞ leading to the standard Cauer forms upon capacitor-gyrator replacements of inductors. In other than the lossless case the main freedom available to the designer is the choice of order of removal of the sections, with a judicious choice worth searching for.

The method given is of course valid only for scalar $y(p)$. If it is desired to synthesize matrices then it may be well-worth developing the present n-port scattering matrix cascade methods [17], [21] to fit integrated circuits.

Likewise, since recent results in time-variable synthesis [22] call heavily upon the time-variable gyrator, which should be readily available in integrated form, extensions of the theory under present investigation should yield practical means for time-variable circuit design.

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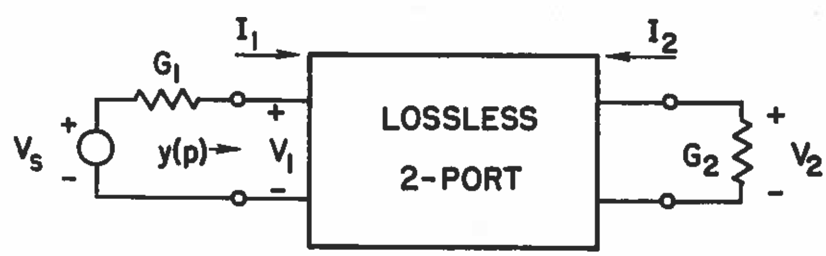


Fig. 1. Terminated Structure For V_2/V_s

Fig. 2. Useful Gyrator Equivalence

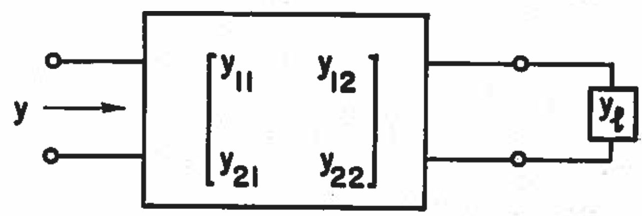


Fig. 3. Determination of y From y_ℓ

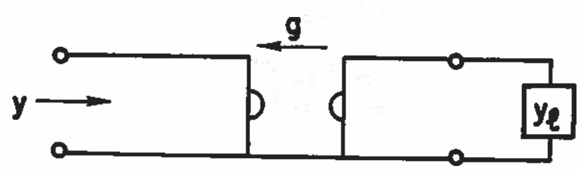


Fig. 4. Cascade Gyrator Removal

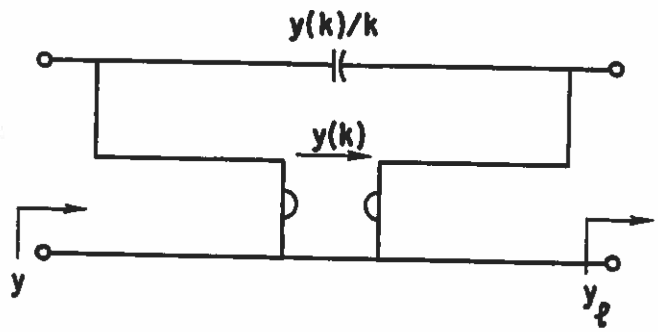


Fig. 5. Basic Cascade Section

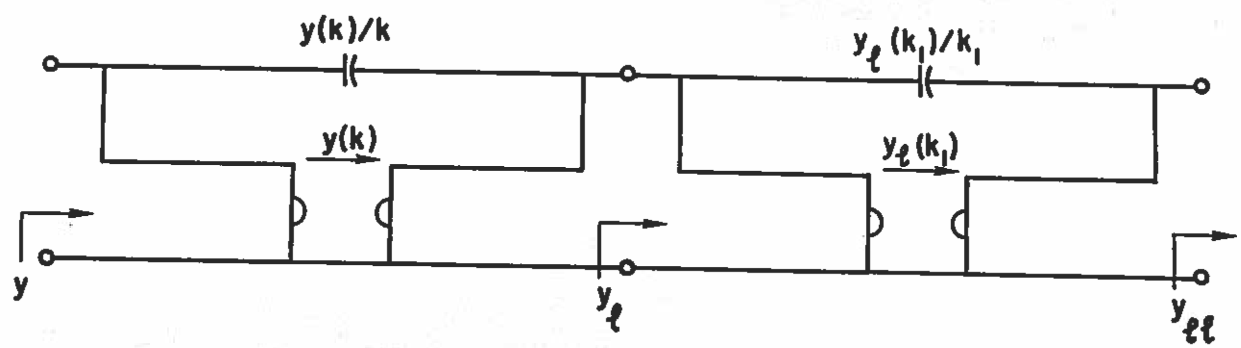


Fig. 6. Basic Section Cascade For Complex k

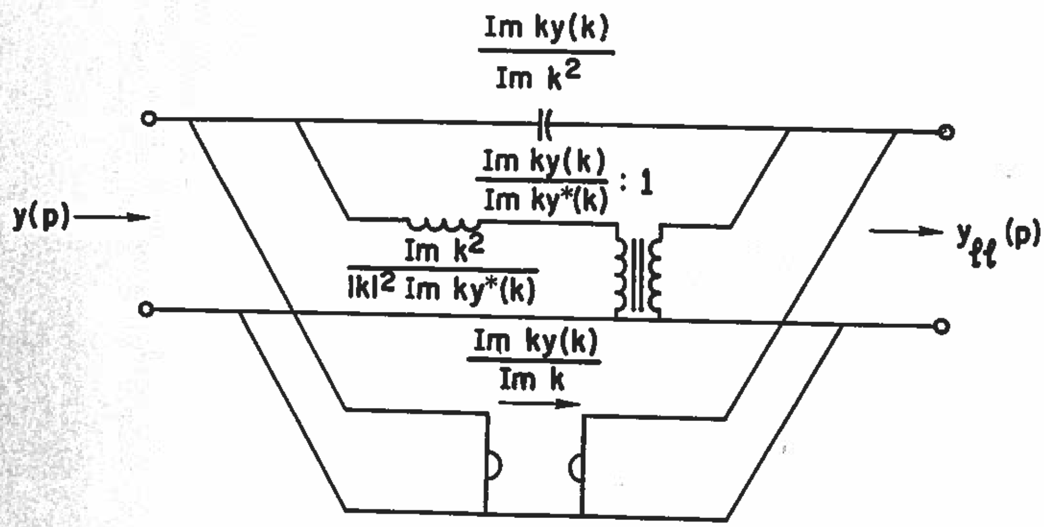


Fig. 7. Realizable Equivalent For Fig. 6

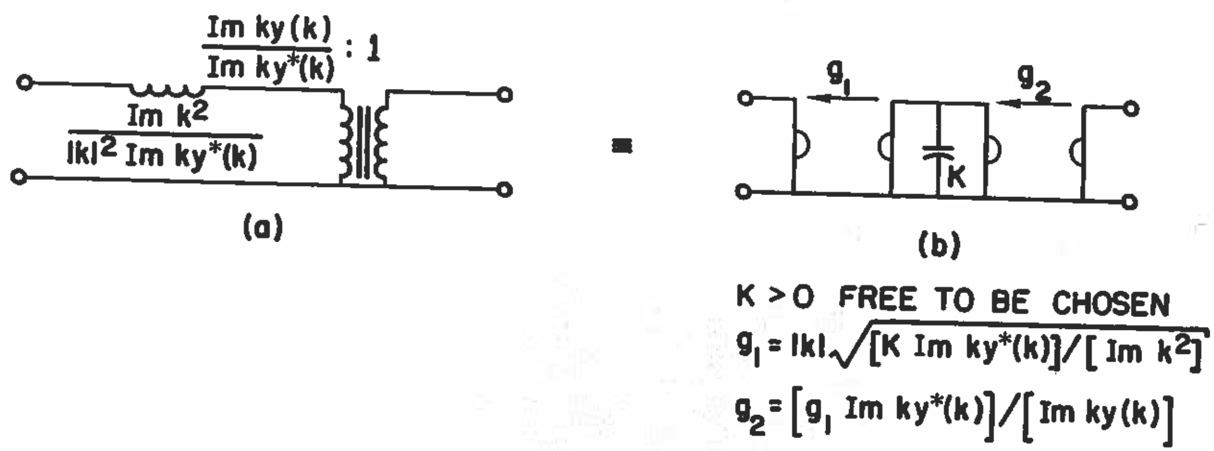


Fig. 8. Grounded Gyrator Replacement For Inductor and Transformer of Fig. 7

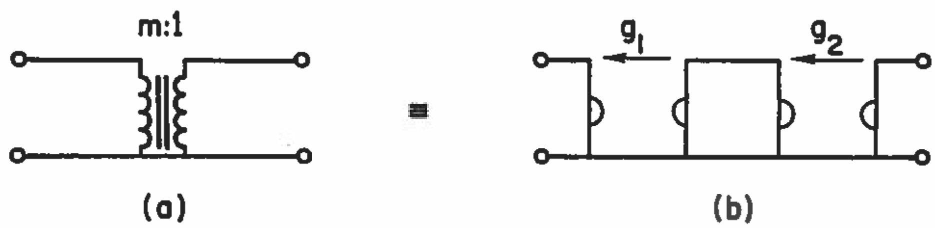
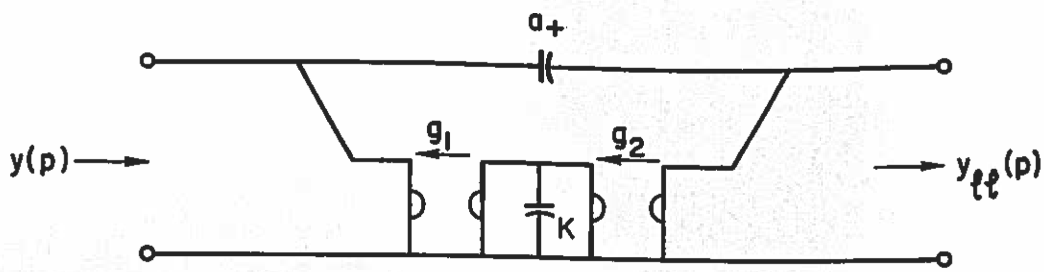


Fig. 9. Transformer Gyrator Realization, $m = g_1/g_2$



K > 0 FREE TO BE CHOSEN

$$g_1^2 = K|k|^2 a_- \; ; \; g_2 = g_1 a_- / a_+$$

$$2a_+ = y'(k) + \frac{y(k)}{k} \; ; \; 2a_- = y'(k) - \frac{y(k)}{k}$$

Fig. 10. Brune Type Section For Imaginary k

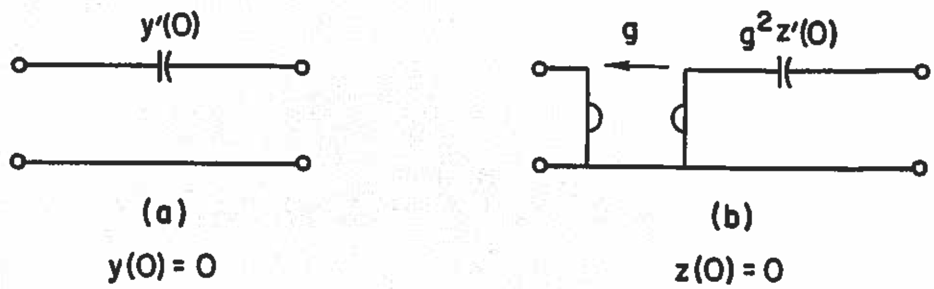


Fig. 11. Extraction of (Transmission) Zeros at Zero

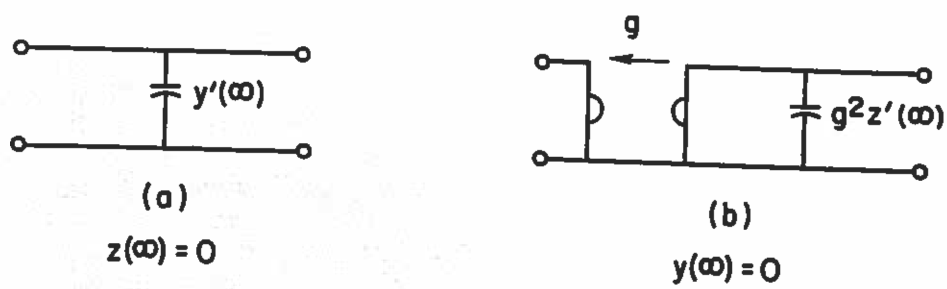


Fig. 12. Extraction of (Transmission) Zeros at Infinity

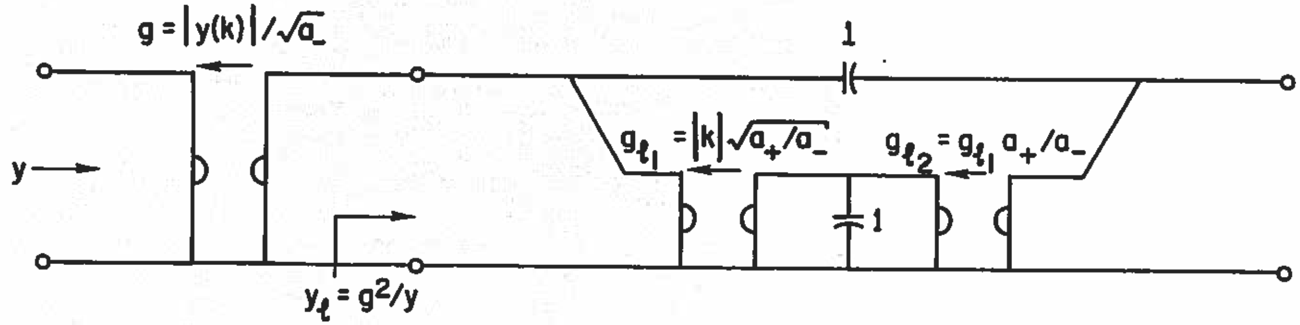


Fig. 13. Normalization of a Brune Section to Unit Capacitors

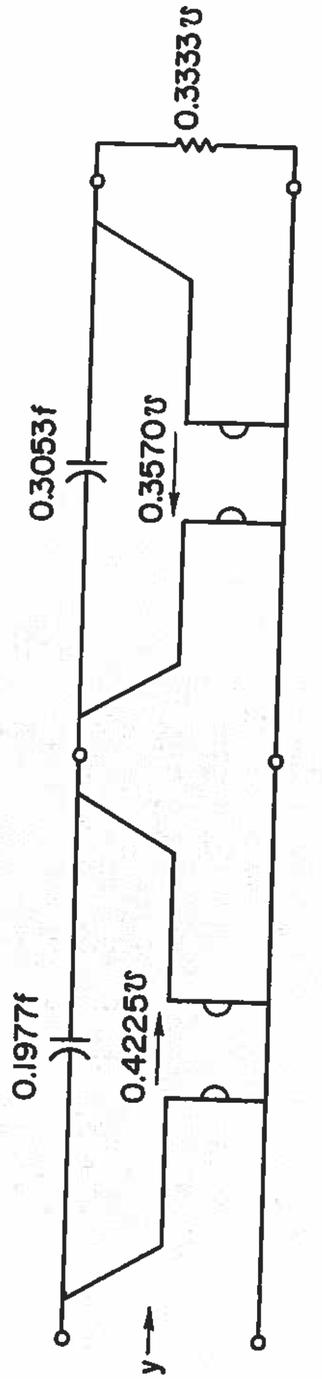


Fig. 14. Real Zero Example Circuit

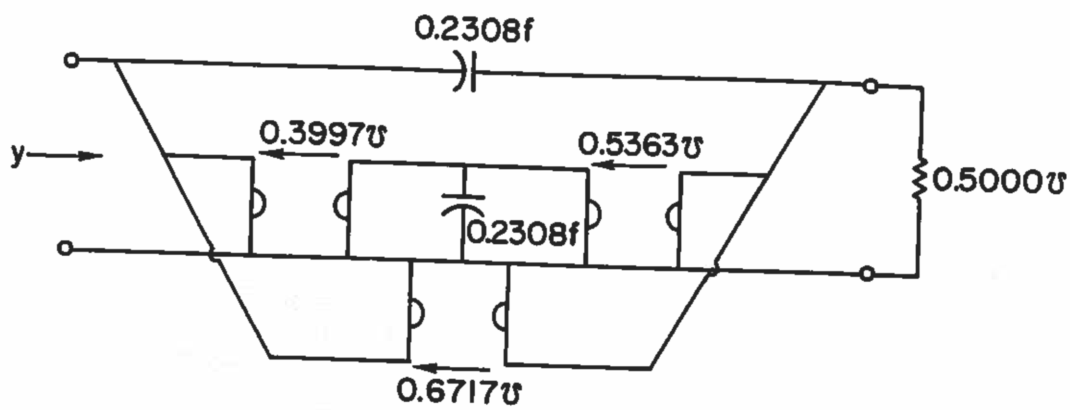


Fig. 15. Complex Zero Example Circuit

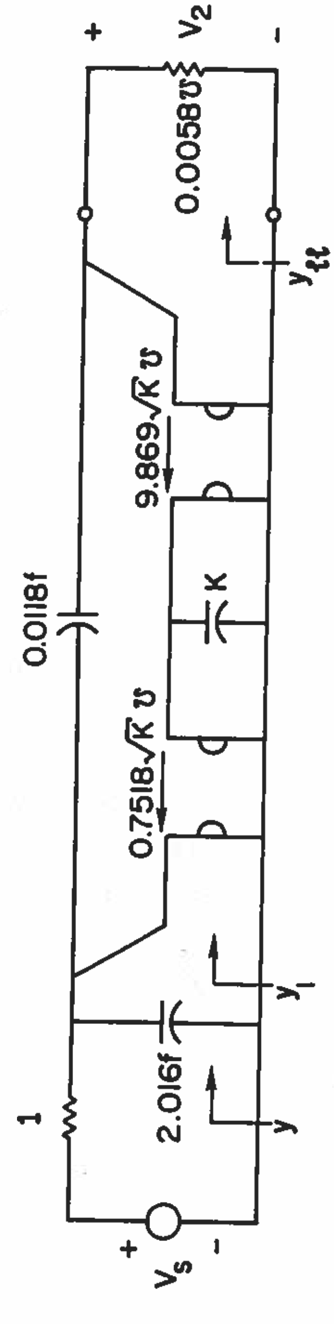
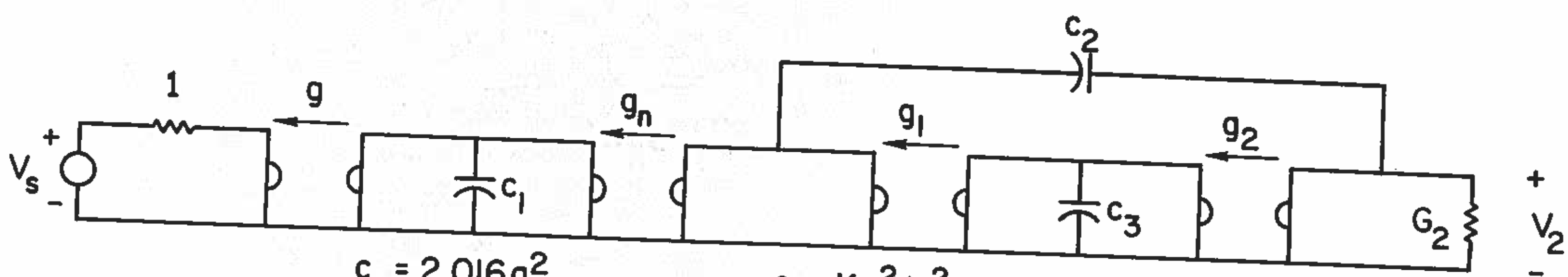


Fig. 16. Realization of a Third-Order Cauer Filter



$c_1 = 2.016 g^2$

$c_2 = 2.073 g_n^2/g^2$

$c_3 = K g_n^2/g^2$

$g_1^2 = 0.5778 K g_n^4/g^4$

$g_2 = 0.0761 g_1$

$G_2 = 172.3 g_n^2/g^2$

Fig. 17. Equivalent Realization of Fig. 16 With Arbitrary Capacitance Values