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SENSITIVITY IMPROVEMENT OF INDUCTORLESS FILTERS*

Although RC–n.i.c. circuits have many advantages over conventional RLC circuits (size, weight and convenient design formulations), practical applications of n.i.c. circuits are limited by their high sensitivity to parameter changes in the active element. This high sensitivity results from the n.i.c. circuit-design technique of 'subtracting' relatively large element values of the circuit to obtain small effective values (Reference 1, p. 558). Therefore a minor variation in one of the large values greatly changes the small value, and the question of sensitivity becomes a matter of fundamental importance.

Several definitions of circuit sensitivity exist. If one considers only sensitivity of pole position to parameter change, the pole sensitivity is found to be almost independent of the RC–n.i.c. configuration used.[2] If, instead, the sensitivity of transfer functions is required, it is expected that the different definition of sensitivity also would not be affected by the n.i.c. configuration. Consequently, Orchard[3] has suggested, on a somewhat intuitive basis, that n.i.c. designs should be avoided when circuit sensitivity is of concern, and that conventional RLC designs should be used, with inductors replaced by the gyrator–capacitor method,[4,5] when inductorless designs are required. Here, by comparison of two realisations of the same transfer function, using identical resistors and capacitors, we show the validity of Orchard's observation for high-Qfactor equally terminated lowpass filters of second degree.

Consider the circuits of Figs. 1a and b, which are respectively described by

$$\frac{V_2}{V_1} = \frac{1}{(r_2c_1c_2)p^2 + \left(r_2c_1 + c_1 + r_2c_2 - \frac{c_2}{k}\right)p + \left(r_2 + 1 - \frac{1}{k}\right)} = \frac{1}{D_1} \quad \text{(1a)}$$

and
$$\frac{V_2}{V_1} = \frac{\sqrt{r_a}/(1+r_a)}{\left(\frac{r_ac_aL}{1+r_a}\right)p^2 + \left(\frac{r_ac_a + L}{1+r_a}\right)p + 1} = \frac{N_2}{D_2} \quad \text{(1b)}$$

In both cases, the transfer function

$$\frac{V_2}{V_1} = \frac{K}{p^2 + 2\zeta p + 1} \quad \text{(2)}$$

is obtained by a proper choice of element values; in particular, it is sufficient to choose

$$\left.\begin{aligned} r_2 &= k = 1 \\ c_1 &= \frac{1}{c_2} = \frac{1}{L} = \zeta \\ r_a &= \frac{1}{\zeta c_a} = \frac{1-\zeta^2}{\zeta^2} \end{aligned}\right\} \quad \text{(3)}$$

At this point, the inductor–transformer combination of Fig. 1b can be replaced by a capacitor–gyrator equivalence, as shown in Figs. 1c and d, where the gyration resistance γ can be arbitrarily chosen. If the impedance level for Fig. 1b is scaled by $1/r_a$, and $\gamma = \sqrt{(1-\zeta^2)}/\zeta^2$ is chosen, we finally arrive at the comparable circuits of Fig. 2, both having V_2/V_1 given by eqn. 2, with gain constants $K = 1$ and $\zeta\sqrt{(1-\zeta^2)}$, respectively. It should be observed that the two circuits of Fig. 2 have 'equal' element values, making a comparison significant.

At this point, the sensitivity of Fig. 2a with respect to k can be compared with that of Fig. 2b with respect to γ, by using eqns. 1 with $L = c\gamma^2$. Defining the transfer-function sensitivity as

$$S_x = \frac{\partial V_2/V_1}{\partial x}\,\frac{x}{V_2/V_1} \quad \text{(4a)}$$

we find that

$$S_k = -\frac{p+\zeta}{k\zeta D_1} \quad \text{(4b)}$$

$$S_\gamma = -\frac{2c\gamma^2 p(p+\zeta)}{(1+r_a)D_2} = -\frac{2p(p+\zeta)}{D_2} \quad \text{(4c)}$$

For frequencies of interest, near $p = j1$, we have, since $k = 1$ and $D_1 = D_2 = p^2 + 2\zeta p + 1$,

$$\left|\frac{S_k}{S_\gamma}\right|_{p=j1} = \frac{1}{2\zeta} \quad \text{(4d)}$$

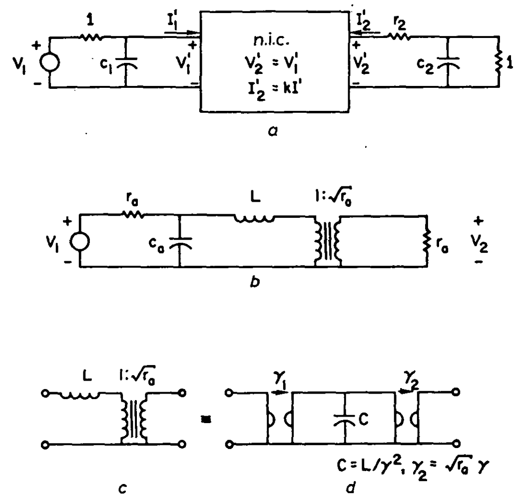

Fig. 1
a, b Basic second-order configurations
c, d Capacitor–gyrator inductor replacement

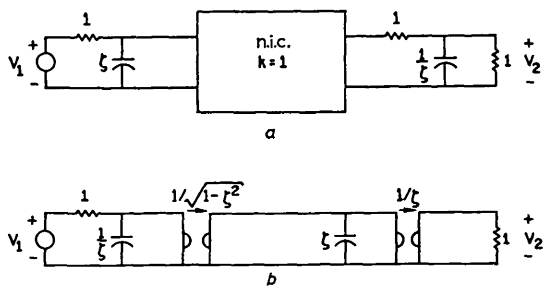

Fig. 2 *Final circuits for comparison*

In the high-Qfactor case, where $\zeta \ll 1$, we see that the sensitivity of the gyrator circuit (Fig. 2b) represents a significant improvement over the n.i.c. circuit (Fig. 2a). If we define the pole sensitivity of the pole p_j as (Reference 2, p. 211)

$$s_x = \frac{\partial p_j}{\partial x/x} \quad \text{(5a)}$$

we find, for the two poles of eqn. 2, that

$$s_k = -1/2\zeta \quad \text{(5b)}$$

* This work was supported by the US National Science Foundation under Grant GK 237.

$$s_Y = \frac{\zeta^5}{1 - \zeta^2}[1 \pm \sqrt{\{1 - (1/\zeta^2)\}}] \quad . \quad . \quad . \quad . \quad (5c)$$

$$\simeq \pm j\zeta^4 \text{ if } \zeta \ll 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5d)$$

and, again, Fig. 2*b* represents a significant improvement over Fig. 2*a*. Of course, this result is for a specific example, but it is of sufficient generality to indicate a meaningful philosophy for design; i.e. since well designed *RLC* filters have relatively small sensitivity in the passband,[3] and since gyrator–capacitor replacements of inductors scarely change this sensitivity (the gyrators essentially being passive), 'active' *RC* filters are best designed using gyrators, when sensitivity is of concern. Besides being of significance for customary filter design, this result should have considerable importance for integrated circuits, where adjustable gyrators appear readily available.[6]

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PROPOSED METHOD OF MEASURING GROUP-DELAY AND AMPLITUDE CHARACTERISTICS OVER WIDE BANDWIDTHS

An improved means of measuring the transfer characteristics of a microwave component is described. The method is applicable to frequencies other than those in the microwave band.

The measurement and adjustment of component transfer characteristics over a frequency range are facilitated by the display of these characteristics on an oscilloscope. The repetition rate of the display should be such as to make the result of any adjustment immediately apparent, and the availability of commercial sweep generators makes this practicable at microwave frequencies. This communication describes a possible method of displaying the group-delay characteristic over wide bandwidths.

The group delay of a component can be determined at a given frequency by applying a small perturbation, or modulation, to a carrier signal of this frequency and measuring the time taken for the modulation to traverse the component. If the modulation is sinusoidal, this time is conveniently measured by comparing the phases of the signal at the input and output terminals of the component, and relating the difference to the modulation frequency.

Existing equipment* operating on this principle gives good results when used at a fixed frequency, but it has disadvantages when displaying delay response over its relatively limited bandwidth (40 MHz at 4 GHz). One of these disadvantages lies in the use of superheterodyne techniques, which leads to difficulties in maintaining a constant difference frequency between the signal and local oscillators when these are being swept for display purposes. There is, in addition, the disadvantage that the delay displayed is that of the desired component plus the overall s.h.f. and i.f. system. The proposed method, shown in Fig. 1 and described below, overcomes these drawbacks.

The amplitude-modulated s.h.f. signal is sampled at the input and output of the test component by means of the two directional couplers and detectors. The first detector provides a feedback signal to the modulator, which maintains constant level at the input to the test component. If a modulator form of phase comparator is used, the output is a linear function of the difference of its input phases (provided that these are small), and it is relatively insensitive to the amplitude of one of its inputs. Thus the comparator gives an output proportional to the group delay of the test component and substantially independent of its attenuation. Since the input level to the test component is maintained constant, the output level from detector 2 will be a function of the attenuation of the component, and it can be displayed in place of, or alongside, the delay characteristic.

The measurement technique outlined here can be readily adapted to other than microwave frequency bands, and it is limited only by the availability of suitable sweep generators and means of modulation at the desired frequency. At microwave frequencies, the modulator may already exist in the

* TURNER, R. J., and LISNEY, D. L.: 'Equipment for the measurement of the group delay of waveguide networks in the frequency range 3·8–4·2 Gc/s', *Proc. IEE*, 1962, **109B**, Suppl. 23, p. 766

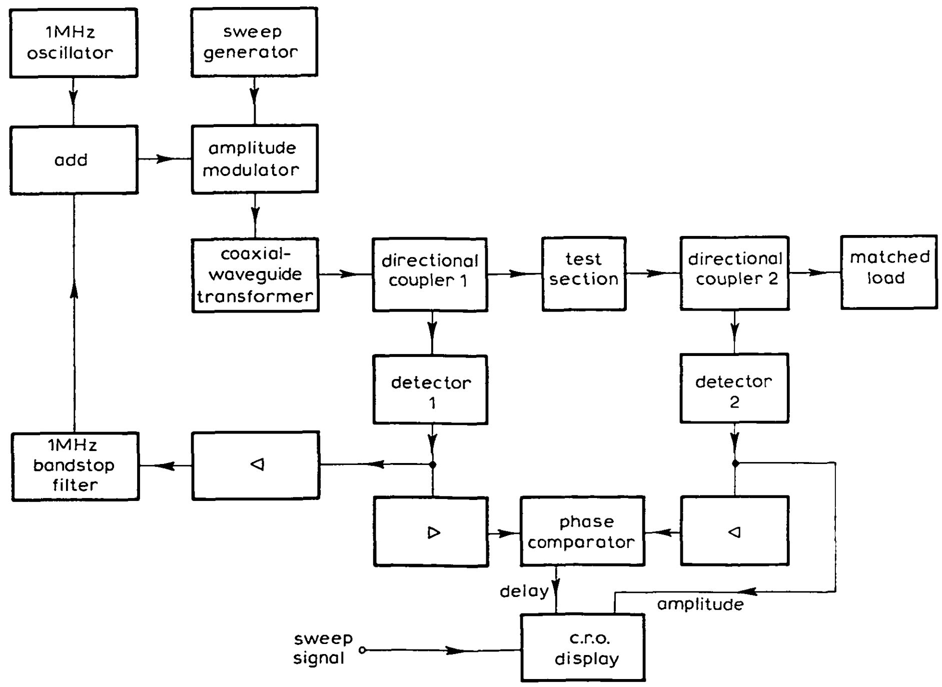

Fig. 1