Correspondence

Equivalent Inductance and Q of a Capacitor-Loaded Gyrator

Abstract—**Taking into account the nonzero input and output impedances of practical gyrators, the Q and input inductance of a capacitor-loaded gyrator are calculated. It is shown that, at the expense of inductance, arbitrarily large Q can be obtained.**

One of the most promising schemes of simulating an inductor for use in integrated circuits appears to be the gyrator shunt loaded at its far end by a suitable capacitor.[1],[2] Since all practical circuits used to realize a gyrator in integrated form[3]–[6] differ from the ideal in several ways, it is of importance to know the behavior of inductors that can be practically obtained by this method. Here a simple equivalent circuit for the nonideal gyrator is used to predict the realizable Q and equivalent real frequency inductance of the simulated inductor.

The ideal gyrator is described by its 2-port admittance matrix

$$Y_g = \begin{bmatrix} 0 & \dfrac{1}{R_a} \\ -\dfrac{1}{R_a} & 0 \end{bmatrix} \tag{1a}$$

where R_a is the gyration resistance. At low and medium frequencies, an adequate representation for a practical gyrator is

$$Y = \begin{bmatrix} \dfrac{1}{R_1} & \dfrac{1}{R_a} \\ -\dfrac{1}{R_b} & \dfrac{1}{R_2} \end{bmatrix}. \tag{1b}$$

Terminating port 2 in a capacitor C gives the equivalent circuit shown in Fig. 1. The input impedance $z_{\text{in}}(p)$ is readily found as

$$z_{\text{in}}(p) = \frac{R_1R_aR_b[1 + pCR_2]}{R_1R_2 + R_aR_b + pCR_2R_aR_b} \tag{2}$$

from which the real frequency behavior is determined. Thus,

$$\begin{aligned} z_{\text{in}}(j\omega) &= \frac{R_1R_aR_b}{[R_1R_2 + R_aR_b]^2 + [\omega CR_2R_aR_b]^2} \\ &\quad \cdot \{[R_1R_2 + R_aR_b + \omega^2C^2R_2^2R_aR_b] + j\omega CR_1R_2^2\} \\ &= R_{\text{eq}}(\omega) + j\omega L_{\text{eq}}(\omega). \end{aligned} \tag{3a}$$

Considering the real and imaginary parts of (3a), the equivalent series resistance and inductance, as shown in Fig. 2, are

$$R_{\text{eq}}(\omega) = \frac{R_1R_aR_b}{R_1R_2 + R_aR_b} \left[\frac{1 + \dfrac{\omega^2C^2R_2^2R_aR_b}{R_1R_2 + R_aR_b}}{1 + \left(\dfrac{\omega CR_2R_aR_b}{R_1R_2 + R_aR_b}\right)^2} \right] \tag{3b}$$

$$L_{\text{eq}}(\omega) = \frac{CR_1^2R_2^2R_aR_b}{[R_1R_2 + R_aR_b]^2} \left[\frac{1}{1 + \left(\dfrac{\omega CR_2R_aR_b}{R_1R_2 + R_aR_b}\right)^2} \right]. \tag{3c}$$

Manuscript received August 5, 1966. This work was supported by the National Science Foundation under Grant GK-237.

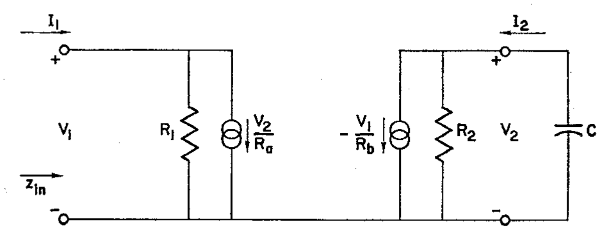

Fig. 1. Equivalent circuit of a practical gyrator at low and medium frequencies.

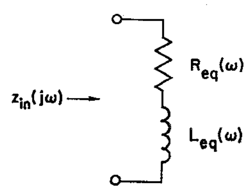

Fig. 2. A representation for $z_{\text{in}}(j\omega)$ of Fig. 1.

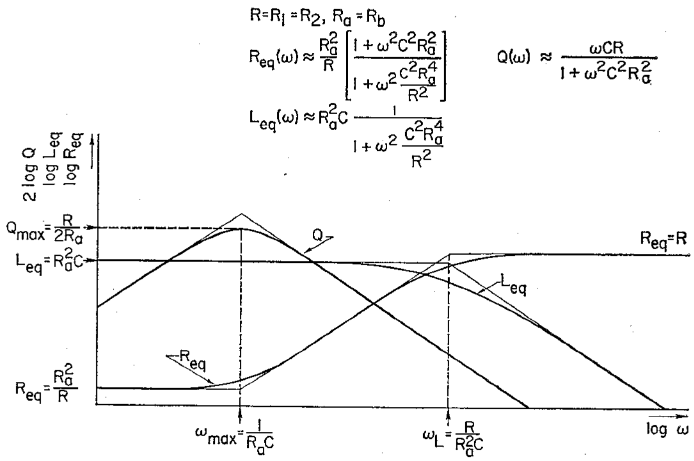

Fig. 3. Plots of R_{eq}, L_{eq}, and Q as functions of ω.

If we calculate Q, the quality factor of the inductor, in the usual way as Terman,[7] page 30,

$$Q(\omega) = \frac{\omega L_{\text{eq}}(\omega)}{R_{\text{eq}}(\omega)}. \tag{4}$$

Equation (3) gives

$$Q(\omega) = \frac{\omega CR_1R_2^2}{R_1R_2 + R_aR_b + \omega^2C^2R_2^2R_aR_b}. \tag{5}$$

The quality factor then reaches its maximum value at

$$\omega_{\max} = \frac{1}{CR_2}\sqrt{1 + \frac{R_1R_2}{R_aR_b}} \tag{6a}$$

and this maximum is given by

$$Q_{\max} = \frac{R_1R_2}{2[R_1R_2 + R_aR_b]}\sqrt{1 + \frac{R_1R_2}{R_aR_b}}. \tag{6b}$$

For many applications it is true that

$$R_1 = R_2 = R, \qquad R_a = R_b \tag{7a}$$

and practically

$$R \gg R_a. \tag{7b}$$

This last condition is justifiable in practice since generally R is of the order of megohms while R_a is of the order of a thousand ohms or less. Under the assumptions of (7) we have

$$\omega_{\max} = 1/CR_a, \qquad Q_{\max} = R/2R_a, \tag{8}$$

which agrees with the result quoted by Orchard.[2] Under the assumptions of (7), (3) and (5) are evaluated and plotted in Fig. 3, which with somewhat obvious coordinate changes is equally valid for the more general equations (3) and (5). It should be observed that the frequency of maximum Q is lower than the cutoff frequency of the inductance, and that the Q can be made arbitrarily large—(6b) with $R_a \rightarrow 0$—at the expense of the value of the inductance. Note that considering $R_2 \neq R_1$ allows for capacitor dissipation if it is of importance. At high frequencies, one also needs to investigate the negative resistance effects, which were ignored here. These are introduced by capacitance effects in the gyration conductances,[5] due to capacitive effects in the transistors. It is also of interest to note that $z_{\text{in}}(j\omega)$ behaves more like a coiled-wire inductor than an ideal inductor.

Experimental results have been obtained [8] for a gyrator with measured values of $R = R_1 = R_2 = 300\text{k}\Omega$, $R_a = R_b = 2.4\text{k}\Omega$, and $C = 0.0026$ farad. From (8) one expects $f_{\max} = \omega_{\max}/2\pi = 250\text{kHz}$ and $Q_{\max} = 62.5$ while measured values are $f_{\max} = 280\text{kHz}$ and $Q_{\max} = 58$.

T. N. Rao
P. Gary
R. W. Newcomb
Stanford Electronics Labs.
Stanford, Calif.

References

[1] B. D. Anderson, W. New, and R. Newcomb, "Proposed adjustable tuned circuits for microelectronic structures," *Proc. IEEE (Letters)*, vol. 54, p. 411, March 1966.

[2] H. J. Orchard, "Inductorless filters," *Electronics Letter*, vol. 2, pp. 224–225, June 1966.

[3] W. New and R. Newcomb, "An integrable time-variable gyrator," *Proc. IEEE (Correspondence)*, vol. 53, pp. 2161–2162, December 1965.

[4] T. N. Rao and R. Newcomb, "A direct coupled gyrator suitable for integrated circuits and time-variation," *Electronics Letters*, vol. 2, pp. 250–251, July 1966.

[5] D. F. Sheehan, "Realization of a transistorized gyrator and its application to low-frequency synthesis," Stanford University, Stanford, Calif., Rept. EE291, June 1966.

[6] I. H. Hawley, Jr., "A gyrator realization using operational amplifiers," in "Papers on integrated circuit synthesis," (compiled by R. Newcomb and T. Rao), Stanford Electronics Labs., Stanford, Calif., Rept. TR 6560-5, SEL-66-058.

[7] F. E. Terman, *Electronic and Radio Engineering*. New York: McGraw-Hill 1955.

[8] P. Gary, "An integrable direct coupled gyrator," in "Papers on integrated circuit synthesis" (complied by R. Newcomb and T. Rao), Stanford Electronics Labs., Stanford, Calif., Rept. TR 6560-4, SEL-66058.