Therefore, we see from (7) that a null occurs when

$$\frac{1}{nj\omega C}\left[-\frac{1}{\omega^2C^2}+\frac{2R}{nj\omega C}\right]+m\frac{R}{n}\left[R^2+\frac{2R}{nj\omega C}\right]=0. \tag{9}$$

Equating the real and imaginary parts to zero we have,

$$\omega^2=\left[\frac{1}{mR^2C^2}\right]\frac{2}{n} \tag{10}$$

and

$$\omega^2=\left[\frac{1}{mR^2C^2}\right]\frac{n}{2}. \tag{11}$$

Hence (10) and (11) are both satisfied simultaneously if

$$n=2 \tag{12}$$

and the null frequency ω_0 is given by

$$\omega_0=\frac{1}{\sqrt{m}}\frac{1}{RC}. \tag{13}$$

Thus, ω_0 may be varied by varying m, and no other adjustments are necessary.

The results of Ganguly[1] for a dual input symmetrical bridged-T network (Fig. 3) may also be obtained as a special case of the twin-T network. Letting

$$\begin{gathered} z_1=z_1'=1/(j\omega C); \qquad z_2=R \\ z_3=r+j\omega L; \qquad z_3'=0; \qquad z_4=\infty \end{gathered} \tag{14}$$

in (6), we get

$$e_0=\frac{z_2z_3+mz_1(z_1+2z_2)}{z_2z_3+z_1(z_1+z_3+2z_2)}. \tag{15}$$

Hence a null occurs when

$$z_2z_3+mz_1(z_1+2z_2)=0. \tag{16}$$

Equating the real and imaginary parts to zero, we have

$$\omega_0^2=\frac{2m}{LC} \tag{17}$$

$$R=\frac{L}{2Cr}. \tag{18}$$

These are the same as those derived by Ganguly.

M. N. S. Swamy
Dept. of Elec. Engrg.
Nova Scotia Tech. College
Halifax, Canada

A Tapped Electronically Variable Delay Line Suitable for Integrated Circuits

In electronic [1, p. 83] and sampled-data systems [2, p. 76] the use of delay lines is frequent and their design important. Often it is important to consider tapped delay lines, as in distributed amplifiers [3, p. 149] or adaptive communication systems [4, p. 25]. Here we show how with fixed taps a variable delay per section can be implemented. The theory rests upon that of a time-variable gyrator [5] and associated circuitry [6]. Possible applications are to the design of variable-gain distributed amplifiers and to the modeling of variable media, as the ionosphere for radar studies [7, p. 69].

Using the classical theory of delay line design [1, pp. 800–808], but based upon image admittances in place of image impedances, we obtain a delay line as a cascade of m-derived sections as shown in Fig. 1. Customarily for wide frequency usage $m=1.27$ for internal sections, while for matching to source and load $m=0.6$ for terminating sections, with $R=\sqrt{L/C}$. Ignoring the delay of the terminating sections the delay to the kth tap is approximately [1, p. 805]

$$t_k=km\sqrt{LC} \tag{1}$$

from which we determine

$$L=\frac{t_kR}{km}, \qquad C=L/R^2. \tag{2a}$$

From (2) we note that if we desire to keep C constant then the kth delay can be varied by simultaneously varying L and R such that L/R^2 remains constant, thus

$$L=\frac{1}{C}\left(\frac{t_k}{km}\right)^2 \tag{2b}$$

determines L for a given capacitor size C and delay time t_k.

To vary L we first observe that the circuit of Fig. 2(a), which is a special case of a previous circuit [6], and which has also been described by Holt and Taylor [8] in the constant parameter case, is described by

$$v_1=\frac{R_{b_1}}{R_{a_2}}v_2-R_{b_1}c\frac{dR_{b_2}i_2}{dt} \tag{3a}$$

$$i_1=-\frac{R_{b_2}}{R_{a_1}}i_2. \tag{3b}$$

Consequently the equality of Fig. 2(a) with 2(b) results when $R_{b_2}/R_{a_1}=R_{b_1}/R_{a_2}=1$ and the equivalent inductance can be varied by varying R_{b_1}. But R_{b_1} can be easily electronically varied [1], as can the other resistors R, in fact such that $C=L/R^2$ remains constant.

When m is set equal to its customary value of 1.27, the capacitors shunting the inductors will be *negative;* however, the capacitor π-network is always realizable by coupled capacitors [6]. The main sections are then realized by connecting the coupled capacitors in parallel with the circuit of Fig. 2(a), as shown in Fig. 3. A similar structure of course holds for the terminating half sections.

Several comments are worthwhile. First, Fig. 3 is imminently realizable by integrated circuit techniques. For such a realization the original gyrator [5] is most suitable for the lower-right gyrator because of the presence of the capacitor, while a modified bias structure under investigation should allow combining the two lower gyrators. However, the remaining two gyrators would preferably be of the more recent direct-coupled type [9], [10]. Since thousands of sections could be cascaded in integrated form the circuit should allow for rather large delays. In addition, it can be noted that where one is interested in delaying signals of audio frequencies [4], the gyrator realization should be considerably cheaper and smaller than the corresponding normal passive reciprocal realization. For instance to realize delays of about 0.1 ms per section of a line in the normal reciprocal passive circuit form requires several inductors of the order of 10–100 mh for a 1 kΩ image impedance. In gyrator form the entire circuit can be integrated, with considerable savings in size, weight, and cost.

If all that is desired is delay, without taps, then modern filter design can be applied to synthesize a ladder structure similar to Fig. 1 [11], and the inductor replacements of Fig. 2 made. By relaxing the $R_{b_2}=R_{a_1}$ constraint of Fig. 2(b), but choosing R_{a_1} very large, one can also obtain essentially reflectionless transmission. This results from the nonreciprocal nature of the resulting structure, or noting that $i_1=0$ results from $i_2\neq 0$ when $R_{a_1}\approx\infty$, (3b).

In actual fact, variation of the terminating resistors does not seem too critical, as indicated by previous types of practical variable delay lines [12]. Because the present variable gyrators [5], [9], [10] are limited in frequency response, the circuits for Fig. 3 are limited to

Manuscript received April 29, 1966. This work was supported by the Air Force Office of Scientific Research, Grant AF-AFOSR 337-63.

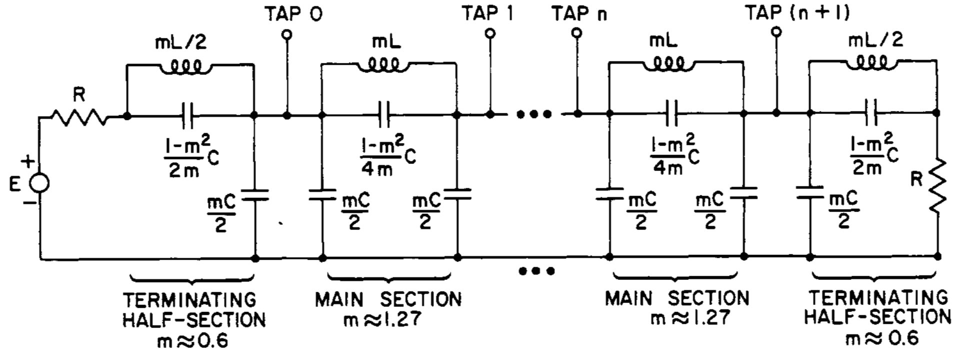

Fig. 1. Admittance designed delay line.

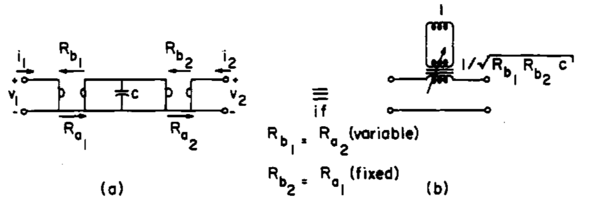

Fig. 2. Variable inductor realization.

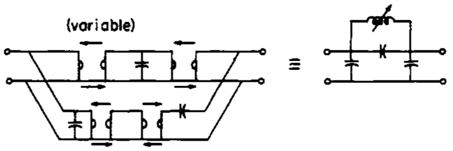

Fig. 3. Main section realization.

under 10 Mc/s at this date. Likewise it should be pointed out that (1) is really valid only for somewhat stationary values of L and thus for rapid and continuous variations in delay a new theory needs development.

The idea of converting to an admittance formulation stems from a similar use of admittances to obtain RC grounded gyrator realizations of all Darlington sectons by H. J. Orchard [13].

ACKNOWLEDGMENT

The authors are grateful for a motivating discussion with B. Widrow.

B. D. ANDERSON
D. M. BRADY
W. NEW
R. NEWCOMB
Stanford Electronics Labs.
Stanford, Calif.

REFERENCES

[1] J. Millman and H. Taub, *Pulse, Digital, and Switching Waveforms.* New York: McGraw-Hill, 1965.
[2] J. R. Ragazzini and G. F. Franklin, *Sampled-Data Control Systems.* New York: McGraw-Hill, 1958.
[3] J. M. Pettit and M. M. McWhorter, *Electronic Amplifier Circuits.* New York: McGraw-Hill, 1961.
[4] R. W. Lucky, "Techniques for adaptive equalization of digital communication systems," *Bell Sys. Tech. J.*, vol. 45, pp. 255–286, February 1966.
[5] W. New and R. Newcomb, "An integratable time-variable gyrator," *Proc. IEEE (Correspondence)*, vol. 53, pp. 2161–2162, December 1965.
[6] B. D. Anderson, W. New, and R. Newcomb, "Proposed adjustable tuned circuits for microelectronic structures," *Proc. IEEE (Letters)*, vol. 54, p. 411, March 1966.
[7] J. L. Torres, "Relativistic theory of wide-bandwidth radar," Stanford Electronics Labs., Stanford, Calif., Tech. Rept. 1602-1, March 1966.
[8] A. G. J. Holt and J. Taylor, "Method of replacing ungrounded inductors by grounded gyrators," *Electron. Letters*, vol. 1, p. 105, June 1965.
[9] W. New, D. Treter, and R. Newcomb, "A direct coupled variable gyrator," in preparation.
[10] T. N. Rao and R. Newcomb, "A digest coupled gyrator suitable for integrated circuits and time variation," in preparation.
[11] L. Storch, "Synthesis of constant-time-delay ladder networks using Bessel polynomials," *Proc. IRE*, vol. 42, pp. 1666–1675, November 1954.
[12] T. N. Rao and R. Newcomb, April 23, 1966, private discussion.
[13] H. J. Orchard, D. Sheahan, and R. Newcomb, April 5, 1964, private discussion.

Incoherent Source Optical Pumping of Visible and Infrared Semiconductor Lasers

Previously, optical pumping of semiconductor lasers has required the use of another laser to obtain sufficient power densities. This letter demonstrates that flashlamp pumping of semiconductor lasers is possible and reports laser action in the infrared and visible using InSb and CdS_xSe_{1-x} pumped with a flashlamp.

Optical pumping of semiconductor lasers was first achieved using a GaAs diode laser pump [1], [2]. However, the emission achieved using this method has been limited to the infrared. Extension to visible wavelengths was achieved by a two photon pumping of CdS using a high power ruby laser [3]. However, the overall efficiency and required pump powers of the two photon process limit its usefulness. Direct flashlamp pumping is simple, may be used at high repetition rates, extends the short wavelength limit beyond that obtainable by using diode lasers, and reduces the limitations of an intermediate laser regarding pulse length and power.

Although the emission of continuously operated xenon lamps peaks in the region of 0.8 and 1.0μ, the spectra of lamps pulsed to high current densities are shifted to the ultraviolet [4]. Using this shift, peak power densities in the wavelength interval of 0.6 to 0.2μ of 41 kW cm^{-2} sr^{-1} were obtained in a pulse of a few microseconds duration. Half of this power was in the spectral region between 0.4 and 0.2μ. Since heating of the samples can quench the laser emission, the rise time of the flashlamp was made sufficiently fast to obtain the threshold power densities in less than 0.5 μs.

In the present experiments the flashlamp was focused on one face of a sample and the opposite face was mounted on a copper heat sink. By making the thickness between these two faces comparable to or less than the diffusion length, losses in the material are reduced and greater excess carrier density may be obtained for a given incident power density. The thicknesses of the InSb samples were reduced to between 25 and 50μ. The CdS_xSe_{1-x} samples, supplied by Reynolds, were, as grown, in the neighborhood of 10 to 20μ thick. Two faces were cleaved perpendicular to the pumped face to form the cavity.

The evidence for laser emission includes all the commonly observed phenomena of line narrowing, clearly defined thresholds, and narrow beam angles. In the case of the infrared emission from InSb we obtained detailed mode structure and beam angles as low as 10°. Because of mode shifting due to thermal transients and a narrow spacing due to a high dispersion of the index of refraction, the mode structure in the CdS_xSe_{1-x} system is more difficult to resolve [5] and was not obtained. For a CdSe sample at 4°K with a 250μ cavity length, the observed emission narrowed from a spontaneous line about 50 Å wide to a stimulated emission line 4 Å wide, which is the width of the mode envelope observed by Hurwitz [5] for approximately the same cavity length. In the case of CdS_xSe_{1-x} the laser emission was observed to occur in brilliant filaments that suddenly emerge at threshold, and CdSe yielded beam angles in the range of 10° to 30°. Pulse lengths for the InSb laser radiation have thus far

Manuscript received May 23, 1966.