cal systems. We propose here a method for treating the study of electromagnetic wave propagation through magneto-plasmas by applying a modification of his method which is appropriate to the microwave region of the spectrum.

The method of Jones is based upon the assumption that the components of the electric field vector of the light wave leaving the system are linear functions of the electric field components of the wave entering the system. That is, if E_1 and E_2 are the complex components of a wave entering the system, then the output components of the wave, E_1' and E_2', are given by

$$\begin{bmatrix} E_1' \\ E_2' \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}$$

or

$$E' = ME,$$

where M is a matrix characterizing the system.

Systems which act to alter the relationship between orthogonal components of an electromagnetic wave in the microwave region might usefully be characterized by such a matrix. A plasma in the presence of a dc magnetic field might be considered as such a system.

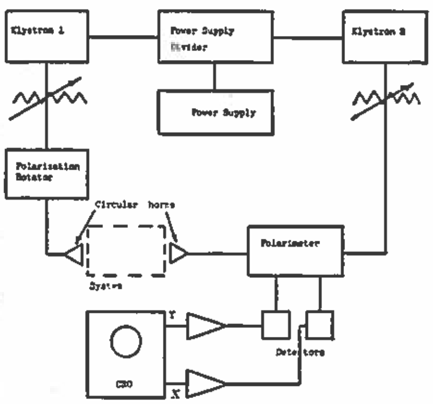

Fig. 1—Polarimeter microwave circuit.

A method for measuring the matrix has been proposed[2] for application in the optical regions. The ease with which the components of the electric field can be separated in a microwave circuit makes the measurement of the matrix M simpler at microwave frequencies.

By factoring out m_{22} three polarization measurements using a microwave polarimeter[3] will determine the matrix M to within the complex constant m_{22}. These measurements consist of measuring the output polarization for three differently oriented linearly polarized input signals. A block diagram of the microwave circuit is shown in Fig. 1. The complex constant m_{22} can be determined from a conventional attenuation and phase-shift measurement. This is done by orienting the rectangular guide of the receiving horn such that only the E_2' component is received when the input signal to the system is linearly polarized in the X_2 direction. In this way the matrix M can be uniquely determined.

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[2] *Ibid.*, see vol. 37.
[3] P. J. Allen and R. D. Tompkins, "An instantaneous microwave polarimeter," PROC. IRE, vol. 47, pp. 1231–1237; July, 1959.

A Decoding Procedure for Double-Error Correcting Bose-Ray-Chaudhuri Codes*

A two-error correcting Bose-Ray-Chaudhuri Code[1] with m information bits has as Syndromes corresponding to a single error in the $(r+1)$th bit position the vector (x^r, x^{3r}), where x is the primitive element of the $GF(2^m)$ field. A double-error would give rise to the syndrome $(x^r+x^s, x^{3r}+x^{3s})$. If we call the two components of this vector m and n respectively, we have

$$n = (x^r + x^s)(x^{2r} + x^{r+s} + x^{2s})$$

$$m^3 = (x^r + x^s)(x^{2r} + x^{2s}),$$

or

$$1 + \frac{n}{m^3} = \frac{x^{r+s}}{x^{2r} + x^{2s}} = \frac{x^{2r+(s-r)}}{x^{2r}(1 + x^{2(s-r)})}$$

$$= \frac{x^{s-r}}{1 + x^{2(s-r)}} = f(s - r) \ \ \text{(say)}.$$

It will be noticed that

$$f(s - r) = f(2^m - 1 - s + r),$$

so that a table containing only $2^{m-1}-1$ entries would set up a one-to-one correspondence between $1+(n/m^3)$ and $s-r$.

Also, $m = x^r(1+x^{s-r})$, so that the error positions r and s can both be obtained by the given manipulation of m and n.

As an example, the double-error correcting (15, 7) Bose-Ray-Chaudhuri code has as single error syndromes,

	1000 (1)	1000 (1)
	0100 (x)	0001 (x^3)
	0010 (x^2)	0011 (x^6)
*	0001 (x^3)	0101 (x^9)
	1100 (x^4)	1111 (x^{12})
	0110 (x^5)	1000 (1)
	0011 (x^6)	0001 (x^3)
*	1101 (x^7)	0011 (x^6)
	1010 (x^8)	0101 (x^9)
	0101 (x^9)	1111 (x^{12})
	1110 (x^{10})	1000 (1)
	0111 (x^{11})	0001 (x^3)
	1111 (x^{12})	0011 (x^6)
	1011 (x^{13})	0101 (x^9)
	1001 (x^{14})	1111 (x^{12})

and the correspondence between $s-r$ and $x^{s-r}/1+x^{2(s-r)}$ is as follows:

$s-r$	$\frac{x^{s-r}}{1+x^{2(s-r)}}$
1	x^8
2	x
3	x^5
4	x^2
5	1
6	x^{10}
7	x^4

Let there be an error in the positions shown by asterisks in the syndrome matrix so that the resulting syndrome, being the sum of the corresponding rows, is

$$1100(m = x^4)0110(n = x^5),$$

yielding

$$1 + \frac{n}{m^3} = 1 + \frac{x^5}{x^{12}} = 1 + x^8 = x^2,$$

whence we have from the table of correspondence $s-r=4$, *i.e.*, there is a distance of four bits between the error positions.

Again,

$$x^r = \frac{m}{1 + x^4} = \frac{x^4}{x} = x^3,$$

that is, $r=3$ yielding the error positions as 4 and 8.

In the case of single errors, $1+(n/m^3)$ will yield 0, so that the position of the error can be found from m above.

Unlike the Peterson Decoding Method,[2] this method cannot be easily generalized to a larger number of errors. However, in the special case of double-error correcting codes, the method is advantageous in involving no successive trials.

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* Received by the IRE, July 17, 1961.
[1] R. C. Bose and D. K. Ray-Chaudhuri, "On a class of error correcting binary group codes," *Information and Control*, vol. 3, pp. 68–79; 1960.
[2] W. W. Peterson, "Error Correcting Codes," Mass. Inst. Tech. Press, Cambridge, and John Wiley and Sons, Inc., New York, N. Y.; 1961.

Nonrealizability of the Complex Transformer*

In these PROCEEDINGS Carlin[1] has given a passive (and real) realization for the complex transformer at a single real frequency, $p_0 = j\omega_0$. The question arises if such a transformer can be realized at a single complex frequency p_0 with Re $p_0 > 0$. This question is of importance because of the recent uses of such a device in the various branches of electrical engineering. In this connection

* Received by the IRE, July 10, 1961.
[1] H. J. Carlin, "On the physical realizability of linear non-reciprocal networks," PROC. IRE, vol. 43, pp. 608–616; May, 1955. (See p. 611.)

Edelmann[2] has used them for visualizing the various transformations of power engineering. Bello[3] has used them to interpret the energy functions of nonreciprocal systems, and Belevitch[4] has incorporated them in an ingenious extension of the Brune synthesis to nonreciprocal networks. We will show that *Carlin's result does not extend to p_0 with* Re $p_0>0$.

Let a superscript asterisk denote complex conjugation and a superscript tilde denote matrix transposition. A device described by

$$V_2 = NV_1$$
$$I_1 = -\tilde{N}^* I_2, \tag{1}$$

with the V's and I's n-vectors and N an $n\times n$ complex constant matrix, is called a complex transformer $2n$-port (here we assume that N is not real). By properly loading such a device by an n-port of admittance matrix Y_2, we can obtain an input admittance of

$$Y_1 = \tilde{N}^* Y_2 N. \tag{2}$$

Further we have for the complex transformer itself

$$\tilde{V}_2^* I_2 + \tilde{V}_1^* I_1 = 0. \tag{3}$$

Interpreting the components of the V's and I's as phasors, this last equation states that the average power input in the sinusoidal steady state is zero. Thus, the complex transformer has several of the useful properties of the real transformer.

We can now apply the result of Desoer and Kuh[5] which states that a network is passive at p_0 with Re $p_0\geq 0$, if and only if, $q_-(p_0)\geq 0$. For our purposes it is more convenient to use a generalization of q_- defined by

$$Q_-(V, I, p) = \begin{cases} 1/2[\tilde{V}^* I + \tilde{I}^* V] \\ \quad -(\sigma/|p|)\,|\tilde{V}I| & \text{if } \omega \neq 0 \\ 1/2[\tilde{V}^* I + \tilde{I}^* V] & \text{if } \omega = 0. \end{cases} \tag{4}$$

Here $|\ |$ denotes the absolute value. In Newcomb,[6] it is shown that the inequality $Q_-(V, I, p_0)\geq 0$ must hold for every V and I if the network is to be passive at p_0. By using (3) and its complex conjugate transpose, we obtain for the complex transformer

$$Q_-(V, I, p) = \begin{cases} -(\sigma/|p|)\,|\tilde{V}I| & \text{if } \omega \neq 0 \\ 0 & \text{if } \omega = 0. \end{cases} \tag{5}$$

Clearly, if $\sigma_0>0$, $\omega_0\neq 0$, then $Q_-(V, I, p_0)<0$ and the network is not passive at p_0. As a consequence the complex transformer can't be realized at p_0 by passive, real elements.

It should be observed that $Q_-(V, I, j\omega)=0$ which, because of the synthesis method of Newcomb,[7] coincides with Carlin's result. Further for $\sigma_0>0$, but $\omega=0$, the device doesn't have real elements and can't be realized at a fixed frequency $p_0=\sigma_0$ even though $Q_-(V, I, \sigma_0)=0$. An alternate derivation of the "activity" of the complex transformer is found in Newcomb.[8]

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[2] H. Edelmann, "Über die Anwendung von Übertragermatrizen in untersuchungen auf dem Netzmodell," *Arch. elekt. Übertragung*, vol. 11, pp. 149–158; April, 1957.

[3] P. Bello, "Extension of Brune's energy function approach to the study of LLF networks," IRE Trans. on Circuit Theory, vol. CT-7, pp. 270–280; September, 1960. (See p. 273.)

[4] V. Belevitch, "On the Brune process for n-ports," IRE Trans. on Circuit Theory, vol. CT-7, pp. 280–296; September, 1960. (See p. 284.)

[5] C. A. Desoer and E. S. Kuh, "Bounds on natural frequencies of linear active networks," *Proc. of the Polytechnic Inst. of Brooklyn Symp.*, vol. X, pp. 415–436; 1960. (See p. 425.)

[6] R. W. Newcomb, "Synthesis of Networks Passive at p_0," University of California, Berkeley, ERL Rept., Series 60, Issue No. 317; September, 1960. (See p. 2.)

[7] *Ibid.*, p. 26.

[8] *Ibid.*, p. 25.

Tunnel-Diode Super-Regenerative Parametric Motor*

In an effort to harness the stimulance of a diode into rotary motion while maintaining the contact-free operation of the parametric motor, the author has carried out measurements on tunnel diodes and built a number of electro-mechanical devices.[1–3] The basic case of resistive load and maintained stability is shown in Fig. 1(a). If the load con-

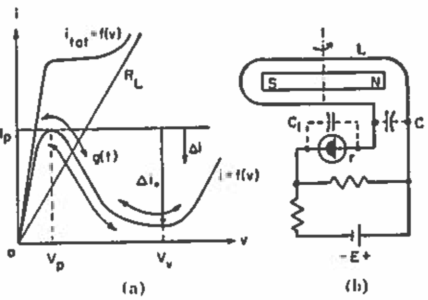

Fig. 1—(a) Tunnel-diode characteristics with possible variational bias ranges. (b) A simple scheme for turning tunnel-diode stimulance into rotary motion.

sists of a tuned circuit of proper inductance and capacitance, and if the dissipation of the system is sufficiently reduced, supersonic or high audio-frequency oscillations can be obtained, provided that the bias voltage is made to exceed the peak-point value V_p, but not the valley-point value V_v. The circuitry with diode and load in series connection is then largely that shown in Fig. 1(b), for which the steady-state frequency to a first approximation may be written

$$f_0 \doteq \frac{1}{2\pi}\sqrt{\frac{1}{LC'} - \frac{g^2}{C_1 C'}}, \tag{1}$$

where $C'=C+C_1$. If we now make the bias voltage variational and periodic around a chosen point, such as the peak point or the valley point, the tunnel diode will never settle down to the value f_0 in (1); on the contrary, it produces a Fourier spectrum of frequencies. Swinging into the negative-slope region in Fig. 1(a), we encounter a growing transient wave train with an instantaneous amplitude associated with the downwards current deflection Δi. The wave-train instantaneous amplitude $i(t)$ is in the interval of interest almost proportional to Δi, so that we may write to a first approximation $i(t)\doteq K[1-\exp(-t/T)]$, where T is the time constant pertaining to the supersonic wave-train envelope. We observe that the generated stimulance is far from constant, taking the form of the time function

$$g(t) = g + \sum_{p=1}^{\infty} g_{p\max}\cos(p2\pi ft + \phi_p). \tag{2}$$

The net effect of a periodic bias voltage is a varying negative conductance, going through a maximum, the subsequent quenching action that makes the magnetic field appear and vanish in a periodic fashion, and the ensuing supersonic growing and decaying wave train of variational frequency. Utilizing these facts for the accomplishment of rotary motion, we may now formulate a simple question: how can we make an alternating field in an inductor do mechanical work in such a manner that a periodic induced EMF and bias voltage results, all within the realms of the source power, secured from the negative-conductance slope of the tunnel diode? The simple answer appears in Fig. 1(b). If here the inductor contains a spinning magnet NS, the pulses of magnetic field will maintain the magnet in rotation, with the induced EMF of the spinning magnet providing the periodic bias voltage. This new electric motor has only three essential parts: the inductor L with its winding capacitance C, the rotor-magnet NS, and the tunnel diode.[4] The speed of the motor is governed by the time constant T of the wave-train envelope, and the speed is therefore quite constant. Due to the quenching action the operation is said to be superregenerative; the response on a CRO is quite typical for that of a super-regenerative device.

Parametric action appears in more than one respect. For an elementary system that may be represented by a single mesh with inductance and capacitance, loss resistance R, and slope-achieved stimulance R_N, there is a considerable change in inductance due to the rotation of the magnet, as is partly evident from the extensive change in pitch of the supersonic oscillation. A first approach integro-differential equation may therefore be written in the form

$$L\frac{di}{dt} + \left(\frac{dL}{dt} + R_n + R\right)i + \Gamma\int i\,dt = e(t). \tag{3}$$

* Received by the IRE, July 20, 1961.

[1] H. E. Stockman, "Parametric oscillatory and rotary motion," Proc. IRE (Correspondence), vol. 48, pp. 1157–1158; June, 1960.

[2] H. E. Stockman, "Parametric variable-capacitor motor," Proc. IRE (Correspondence), vol. 49, pp. 970–971; May, 1961.

[3] H. E. Stockman, "Electric field motor" and "Tunnel diode electromechanical movement," U. S. Patent applications of January 26, 1961, and July 12, 1961.

[4] Made by Sine-Ser Co., Waltham, Mass.