There is, however, a capacitance C_{DS} associated with R_{DS}, and under certain circumstances this can lead to degradation of the signal. For example, if the photocurrent is due to a chopped light beam, then the resultant trapezoidal signal across R_{DS} can be severely integrated by C_{DS}, a phenomenon which becomes worse as R_{DS} increases. However, by proper choice of chopping frequency this effect can be minimized.

Since such electro-optical instruments are legion, there is a clear requirement for a P-channel FET designed to have a wide range of R_{DS}, where R_{DS} at $V_{GS}=0$ is not less than 10 kilohms. In this context, the actual value of the pinch-off voltage is of little importance.

An experiment based on the accompanying sketch used a Siliconix 2N3113, which provided a gain range of about 1½ decades with a minimum R_{DS} of 20 kilohms.

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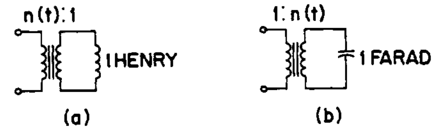

Fig. 1. Networks which may lack scattering matrices.

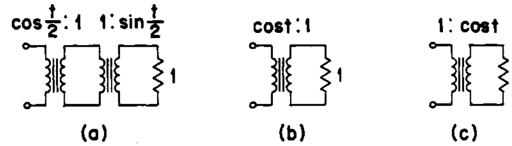

Fig. 2. Networks which lack immittance matrices.

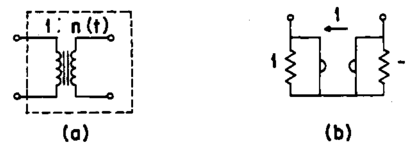

Fig. 3. Networks which lack scattering matrices.

Degenerate Networks

In the study of network theory the description of networks by means of network matrices (e.g., the impedance matrix, the scattering matrix) has proved exceedingly useful. For linear, passive, time-invariant networks it is known that if an impedance or admittance matrix exists, then so does a scattering matrix (see Youla et al. [1], p. 123); however, the converse is not true, as seen by the ideal transformer. In this letter, by considering linear passive but time-variable elements, we exhibit some networks which are describable by impedance or admittance matrices but not by a scattering matrix. For completeness we discuss various other degenerate networks, tabulating in Table I a set of networks not describable by one or more of the impedance, admittance, or scattering matrix.

TABLE I

Summary of Degenerate Networks

Network	$s(t, \tau)$	$z(t, \tau)$	$y(t, \tau)$
Transformer-coupled inductor, Fig. 1(a)	—	$n(t)\delta'(t-\tau)n(\tau)$	—
Transformer-coupled capacitor, Fig. 1(b)	—	—	$n(t)\delta'(t-\tau)n(\tau)$
Cascade transformer-coupled resistor, Fig. 2(a)	$\cos t\delta(t-\tau)$	—	—
Transformer-coupled resistor, Fig. 2(b)	Exists	$\cos^2 t\delta(t-\tau)$	—
Transformer-coupled resistor, Fig. 2(c)	Exists	—	$\cos^2 t\delta(t-\tau)$
Transformer viewed from one port, Fig. 3(a)	—	—	—
Nullator, Fig. 3(b)	—	—	—
Short circuit	$-\delta(t-\tau)$	0	—
Open circuit	$\delta(t-\tau)$	—	0
Time-invariant transformer	Exists	—	—

In Fig. 1(a) is shown a network which possesses the impedance matrix [2]

$$z(t, \tau) = n(t)\delta'(t-\tau)n(\tau). \tag{1}$$

Here δ' is the derivative of the unit impulse.

Manuscript received February 11, 1966. This work was sponsored by the Air Force Office of Scientific Research under Contract AF-AFOSR 337-63.

The scattering matrix of this network can be calculated [3] to be

$$s(t, \tau) = \delta(t-\tau) - \phi(t)\psi(\tau)u(t-\tau) \tag{2a}$$

where u is the unit step function and the function ϕ and ψ are related by

$$\psi(t) = \frac{\phi(t)}{\int_t^\infty \phi^2(\lambda)d\lambda + c} \tag{2b}$$

(c being a non-negative constant), and $\phi(t)$ and $n(t)$ are related by

$$\frac{1}{n(t)} = \frac{\phi(t)}{\sqrt{2\left\{\int_t^\infty \phi^2(\lambda)d\lambda + c\right\}}}. \tag{2c}$$

From these equations it immediately follows that

$$\frac{1}{n^2(t)} = \frac{\phi(t)\psi(t)}{2}. \tag{3}$$

Now (1) is well defined if we assume smooth variations in $n(t)$, even if we let $n(t)=0$ for isolated values of t, or over an interval. But (3) shows that at least one of ϕ and ψ behaves in a discontinuous fashion when $n(t)$ goes to zero, and consequently the network cannot possess a (well-defined) scattering matrix if $n(t)$ ever vanishes.

Fig. 1(b) shows the dual network, which possesses an admittance matrix but no scattering matrix if $n(t)$ ever vanishes.

The admittance of the network of Fig. 1(a) is given formally as

$$y(t, \tau) = \frac{1}{n(t)} u(t-\tau) \frac{1}{n(\tau)}. \tag{4}$$

This will not be defined—even as a distributional kernel ([4], sec. 2)—precisely when the scattering matrix is not defined. Notice that a dual result holds for the network of Fig. 1(b).

The use of time-variation allows us to construct other examples of pathological networks which have no time-invariant parallel. For a one-port linear time-invariant network of lumped passive elements, impedance and admittance matrices (which are in fact functions) always exist, save in the case of the open and short circuit. However,

1) $s(t, \tau)=\cos t\delta(t-\tau)$, realized in Fig. 2(a), has neither an impedance nor an admittance matrix
2) $z(t, \tau)=\cos^2 t\delta(t-\tau)$, realized in Fig. 2(b), has no corresponding admittance matrix, but $s=(\cos^2 t+1)^{-1}(\cos^2 t-1)\delta(t-\tau)$
3) $y(t, \tau)=\cos^2 t\delta(t-\tau)$, realized in Fig. 2(c), has no corresponding impedance matrix, but $s=(\cos^2 t+1)^{-1}(1-\cos^2 t)\delta(t-\tau)$.

For completeness, we mention two other pathological networks which have been discussed elsewhere [4], [5]. Fig. 3(a) shows a transformer with only one port accessible; when $n(t)$ goes to zero at some finite time, this one-port network possesses no scattering matrix [4]. The network of Fig. 3(b) is the nullator, first introduced by Tellegen [6]; it has $v=i=0$, where v and i are the port voltage and current. As shown by Newcomb [5] and Carlin and Youla [7], p. 908, this network has no scattering matrix.

Table I summarizes the preceding material. A dash indicates lack of existence of a given matrix, where any necessary assumptions for pathological behavior are assumed made [e.g., $n(t)=0$ for some t]. We include for completeness the short circuit, the open circuit, and the standard time-invariant transformer.

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Bridge-*T* and Ladder Gyrators

Gyrators are realized by electronic circuitry using active network elements like pentodes [1], transistors [2]–[4], and operational amplifiers [5]. In all these methods, the approach is to realize the gyrators by having $y_{12}=-y_{21}$ and then compensating suitably to obtain $y_{11}=0=y_{22}$.

In this letter, it is shown that the gyrators can be simulated by utilizing Bridge-*T* or ladder networks in which certain relationships between the arms have to be satisfied so that compensation becomes unnecessary.

The starting point in this method is the negative gyrator [6] defined by the *ABCD* matrix

$$\begin{Vmatrix} 0 & B \\ -1/B & 0 \end{Vmatrix}. \tag{1}$$

When such a negative gyrator is either preceded by an NIC of the current inversion type or followed by an NIC of the voltage inversion type, a positive gyrator results. Hence, the problem reduces to the determination of relationships between the different elements to get a negative gyrator.

For the Bridge-*T* network shown within dotted lines of Fig. 1, the *Y*-matrix will be (conversion to ABCD matrix can be done, if necessary)

$$\begin{Vmatrix} -\frac{1}{Z_4}+\frac{Z_2+Z_3}{|Z|} & -\frac{1}{Z_4}-\frac{Z_3}{|Z|} \\ -\frac{1}{Z_4}-\frac{Z_3}{|Z|} & -\frac{1}{Z_4}+\frac{Z_1+Z_3}{|Z|} \end{Vmatrix} \tag{2}$$

where $|Z|=Z_1Z_2+Z_2Z_3+Z_3Z_1$. Either Z_4 or Z_3 can be made negative and y_{11} and y_{22} made zero. Both y_{11} and y_{22} will simultaneously be zero when $Z_1=Z_2$, thereby resulting in a symmetrical Bridge-*T* net-

Manuscript received February 4, 1966.

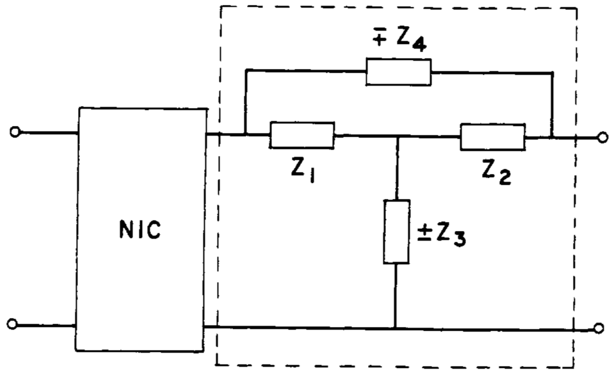

Fig. 1. The bridge-*T* gyrator.

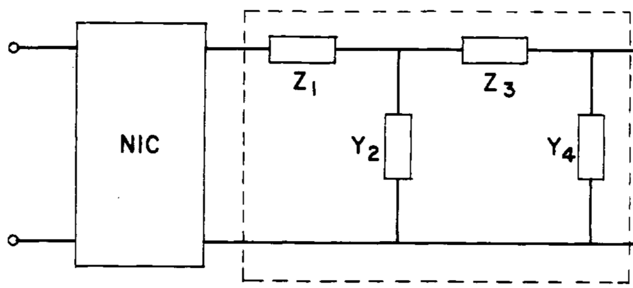

Fig. 2. The ladder gyrator.

work. This obviates the necessity for compensating elements, giving the following results.

a)
$$\frac{1}{Z_4}=\frac{Z_2+Z_3}{Z_2^2+2Z_2Z_3} \tag{3}$$

if Z_4 is negative, giving the gyrational admittance as

$$Y_g=\frac{1}{Z_2+2Z_3} \tag{4}$$

or b)

$$Z_3=\frac{Z_2(Z_2+Z_4)}{Z_4+2Z_1} \tag{5}$$

if Z_3 is negative, giving

$$Y_g=-\frac{Z_4+2Z_1}{Z_1Z_4}. \tag{6}$$

If an unsymmetrical network is used, compensation becomes necessary. Either y_{11} or y_{22} may be equated to zero. If y_{11} were equated to zero, the gyrational admittance will be $Z_2/|Z|$ and the compensation required will be $-(Z_1-Z_2)/|Z|$. If y_{22} were equated to zero, the gyrational admittance will be $Z_1/|Z|$ and the compensation required is $-(Z_2-Z_1)/|Z|$.

For the ladder network shown within dotted lines of Fig. 2, the *ABCD* matrix will be

$$\begin{Vmatrix} 1+Z_1(Y_2+Y_4)+Z_3Y_4+Z_1Y_2Z_3Y_4 & (Z_1+Z_3)+Z_1Y_2Z_3 \\ (Y_2+Y_4)+Y_2Z_3Y_4 & 1+Y_2Z_3 \end{Vmatrix} \tag{7}$$

The parameter D will be zero, when

$$Y_2Z_3=-1 \tag{8}$$

and, hence, the parameter A will be zero, when

$$Z_4=\frac{Z_3^2}{Z_1+Z_3} \tag{9}$$

giving a gyrational impedance of Z_3. Therefore, the individual ele-