Design of a Hemispherical Antenna Array for Magnetic Field Control in the Brain

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Abstract— **A hemispherical antenna array consisting of 42 elements operating at a frequency of 1.8 GHz was proposed and simulated. The array elements are distributed on a hemisphere to be placed on a human's head. By considering the problem as a boundary valued problem and finding the electric and magnetic field inside the brain, a beamforming technique was used that made use of the orthogonality property of the *transverse electric* and *transverse magnetic* fields in producing a delta like function beampattern in order to direct the magnetic field to the area of interest within the brain to cause magnetic deep brain stimulation.**

Index Terms— **Parkinson's disease (PD), Deep brain stimulation (DBS), Hemispherical array, Boundary valued problem (BVP), Beamforming, Magnetic intensity.**

I. INTRODUCTION

Parkinson's disease (PD) is a type of a disorder which is characterized by tremors, muscle rigidity, suffering motor skills, problems in speech and other functions. It is caused by a degenerative disorder of the brain's nerve signals.

Deep brain stimulation (DBS) is a method used to treat the tremors that occur in PD patients as described in [1]. The process is accomplished by surgically inserting a pulse generator in the chest that is connected to two conducting electrodes. The electrodes are placed in the specific area of the brain that causes the tremor abnormality. The generator produces an electric pulse that is applied to the brain through the electrodes. This electric pulse regulates the electric activity of the brain and prevents tremors.

The device has many disadvantages since it must be inserted surgically and the battery of the device must be replaced every 2 to 7 years, hence another surgery. Furthermore, the device is very sensitive to external magnetic fields, so any device in the proximity of the stimulator that generates a magnetic field may cause the stimulator to shut down without the patient knowing about it [1].

The idea behind this paper is to design an antenna array that will be placed on the patient's head. This array will be able to focus the magnitude of the magnetic field intensity to the area of interest in the brain. The array may possibly offer a noninvasive approach to treat the tremors that are caused by Parkinson's disease. This is done in this case by magnetic stimulation instead of electric stimulation.

II. THEORETICAL ANALYSIS

A. Problem Statement

The design of the hemispherical array was done by solving an electromagnetic boundary valued problem in which the human head was modeled as a three layered hemisphere like the one shown in Fig. 1. Of course modeling the head as a perfect hemisphere is an assumption, but a necessary one to enable the use of the spherical coordinate system. The antenna elements are of course fixed onto a helmet which is in turn placed on the head but in the analysis it is assumed that the antennas are placed right above the head to reduce the number of layers and hence simplify the analysis. It is seen from the figure that there are four regions marked as (0), (1), (2) and (3). Region (0) is the outside of the head (i.e. free space), region (1) is the skin, region (2) is the skull and finally region (3) is the brain. The antennas are going to be placed on top of the skull surface and they will cover a hemispherical surface of radius c.

Since the human head is approximated to be spherical [6], it is therefore obvious that the boundary value problem will be solved by utilizing the spherical coordinate system. According to [2, ch.6-1], any electromagnetic problem with spherical symmetry can be solved using the electric vector potential F_r and the magnetic vector potential A_r in spherical coordinates. Any electromagnetic wave can be decomposed into two orthogonal sets of functions which are the *transverse electric (TE)* F_r and the *transverse magnetic (TM)* A_r waves. Both vector potentials are solutions to the Helmholtz equation [2, ch.6-1] and can be used to find the fields inside region (3) (the brain). Following the theoretical analysis of [2, ch.6-1], one can solve a boundary-valued problem (BVP) in spherical coordinates. With the boundary conditions given by equations (1)-(6), one can solve for the fields inside the brain.

$$\hat{r}\times(\vec{E}_1-\vec{E}_0)=0 \quad \text{for } r=c \qquad (1)$$

$$\hat{r}\times(\vec{E}_2-\vec{E}_1)=0 \quad \text{for } r=b \qquad (2)$$

$$\hat{r}\times(\vec{E}_3-\vec{E}_2)=0 \quad \text{for } r=a \qquad (3)$$

$$\hat{r}\times(\vec{H}_1-\vec{H}_0)=\vec{J}_i(\theta,\varphi) \quad \text{for } r=c \qquad (4)$$

$$\hat{r}\times(\vec{H}_2-\vec{H}_1)=0 \quad \text{for } r=b \qquad (5)$$

$$\hat{r}\times(\vec{H}_3-\vec{H}_2)=0 \quad \text{for } r=a \qquad (6)$$

Where H_j and E_j represent the magnetic intensity (H-field) and electric field in the region (j) and J_i is the current density of the i-th antenna element given by equation (7) ([3] was used to get (7)). So it is clear from equation (4) that the antennas are introduced in the boundary conditions like in [4].

$$\vec{J}_i(\theta,\varphi)=\frac{I_iL}{c}[\hat{\theta}\delta(\theta-\theta_i)\{\delta(\varphi-\varphi_i+\frac{\Delta\varphi}{2})+\delta(\varphi-\varphi_i-\frac{\Delta\varphi}{2})\}$$
$$+\hat{\varphi}\delta(\varphi-\varphi_i)\{\delta(\theta-\theta_i-\frac{\Delta\theta}{2})+\delta(\theta-\theta_i+\frac{\Delta\theta}{2})\}] \qquad (7)$$

Where I_i is the current value of the i-th antenna (controlled by the electronics), L is the dimension of the loop, θ_i & φ_i is angular position of the center of the i-th loop antenna, δ is the delta function and $\Delta\theta=\Delta\varphi=L/c$.

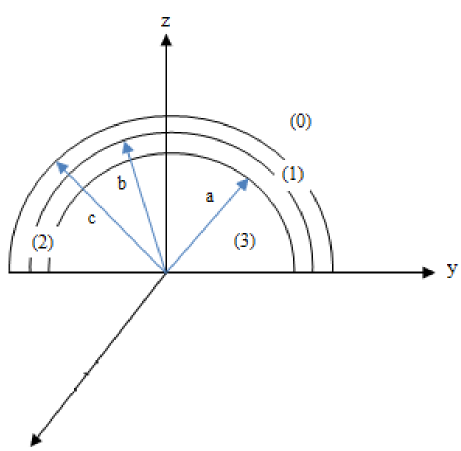

Fig. 1. A simple model of a human's head as seen from the front. There are four regions denoted by (0), (1), (2) and (3) which represent free space, the skull and brain respectively. The radius of the brain is a, while b denotes the radius from the center of the brain to the outer part of the skull and c for skin where the antenna elements are going to be placed.

Solving the BVP together with equation (1)-(7), the TE and TM modes of the H-field vector inside the brain (region 3) can be determined by the following equations. Note that by simulation it was found that the magnitude of the TM modes is negligible compared to the TE modes, thus the entire analysis is based only on the TE modes.

$$\vec{H}_3=-i\sqrt{\frac{\varepsilon_3}{\mu_0}}\sum_{l=1}^{\infty}\sum_{m=-l}^{l}F_{lm}\nabla\times\left[j_l(k_3r)\vec{r}\times\nabla Y_{lm}(\theta,\varphi)\right] \qquad (8)$$

Where:

$$F_{lm}=\frac{j\omega\mu_0cJ_{TE}H_l^{(2)}(k_0c)}{\chi_l l(l+1)}\left[\frac{\gamma_lH_l^{(1)}(k_2a)+H_l^{(2)}(k_2a)}{J_l(k_3a)}\right]$$

$$\gamma_l=\frac{H_l^{'(2)}(k_2a)J_l(k_3a)-H_l^{(2)}(k_2a)J_l^{'}(k_3a)}{H_l^{(1)}(k_2a)J_l^{'}(k_3a)-H_l^{'(1)}(k_2a)J_l(k_3a)}$$

$$\chi_l=\tau_l\left[H_l^{'(1)}(k_1c)H_l^{(2)}(k_0c)-H_l^{(1)}(k_1c)H_l^{'(2)}(k_0c)\right]+$$
$$\frac{1}{H_l^{(2)}(k_1b)}\left[\gamma_lH_l^{(1)}(k_2b)+H_l^{(2)}(k_2b)-\tau_lH_l^{(1)}(k_1b)\right]*$$
$$\left[H_l^{'(2)}(k_1c)H_l^{(2)}(k_0c)-H_l^{(2)}(k_1c)H_l^{'(2)}(k_0c)\right]$$

$$\tau_l=\frac{\left\{\begin{array}{l}\left[\gamma_lH_l^{(1)}(k_2b)+H_l^{(2)}(k_2b)\right]H_l^{'(2)}(k_1b)\\ -\left[\gamma_lH_l^{'(1)}(k_2b)+H_l^{'(2)}(k_2b)\right]H_l^{(2)}(k_1b)\end{array}\right\}}{\left[H_l^{(1)}(k_1b)H_l^{'(2)}(k_1b)-H_l^{'(1)}(k_1b)H_l^{(2)}(k_1b)\right]}$$

(9)

$$\vec{r}\times\nabla Y_{lm}(\theta,\varphi)=\hat{\varphi}\frac{\partial Y_{lm}}{\partial\theta}-\hat{\theta}\frac{1}{\sin\theta}\frac{\partial Y_{lm}}{\partial\varphi} \qquad (10)$$

Where $J_l(k_ir)=k_irj_l(k_2r)$ and $j_l(k_ir)$ [2, ch.6-1] is the spherical Bessel function, $J_l^{'}$ is the derivative w.r.t r, $H_l^{(1,2)}(kr)=krh_l^{(1,2)}(kr)$ and $h_l^{(1,2)}(kr)$ [2, ch.6-1] is the spherical Hankel function of the first and second kind, H_l' is the derivative w.r.t r, k_j is the complex wavenumber given by (11), $i=\sqrt{(-1)}$, $\omega=2\pi f$ where $f=1.8GHz$, μ_o is the permeability of free space, F_{lm} is the coefficient for the TE mode (determined by the BVP and given by) and $Y_{lm}(\theta,\varphi)$ is the spherical harmonic.

$$k_j=\omega\sqrt{\mu_0\varepsilon_0(\varepsilon_j+\frac{i\sigma_j}{\omega\varepsilon_0})};\, j=0,1,2\,\&\,3(regions) \qquad (11)$$

Where ε_0 is the permittivity of free space and ε_i and σ_i are the relative permittivity & conductivity of region (j) respectively. J_{TE} is the TE component of the current density $J_i(\theta,\varphi)$ given in (7). The expression for J_{TE} is given in equation (12).

$$J_{TE} = \int_{\theta}\int_{\varphi} [\vec{r} \times \nabla Y_{lm}^{*}(\theta,\varphi)].[\nabla \times \vec{J}_i(\theta,\varphi)] \sin\theta d\theta d\varphi \quad (12)$$

Where $\nabla Y_{lm}^{*}(\theta,\varphi)$ is the gradient of the complex conjugate of Y_{lm} and $\nabla\times$ represents the curl if a vector. Note that (8) gives the magnetic field for one antenna element.

B. Beamforming Technique

Once the magnetic field (H-field) for each antenna element in region (3) is defined, the objective now is to construct a beam with a regular beampattern focused to a specific direction (θ_o,φ_o) for a specific radial distance r_o. In [5] a beamforming technique was used to direct sound to a certain direction of interest using the orthogonality property of spherical harmonics. In this paper a similar approach is followed, but since the aim is to direct the H-field which is a vector (not a scalar like sound pressure) it is necessary to find the right orthogonal basis functions which are vectors to be begin with. The natural choice is to use the orthogonality property of the TE modes.

The desired magnetic field pattern is a delta function in the direction of interest (θ_o,φ_o) for a prefixed point r_o, like the one shown in equation (13). Before beamforming, the delta function in (13) is expanded by using the TE modes of an electromagnetic field.

$$\begin{aligned}\vec{H}_{Desired} &= \hat{\theta}\delta(\theta-\theta_o)\delta(\varphi-\varphi_o) + \hat{\varphi}\delta(\theta-\theta_o)\delta(\varphi-\varphi_o) \\ &= -i\sqrt{\frac{\varepsilon_3}{\mu_0}} \sum_{l=1}^{\infty}\sum_{m=-l}^{l} \Xi_{lm} \nabla\times\left[j_l(k_3 r)\vec{r}\times\nabla Y_{lm}(\theta,\varphi)\right]\Big|_{r=r_o}\end{aligned} \quad (13)$$

One can solve for Ξ_{lm} by applying the orthogonality principle of the spherical harmonics vector $\vec{r}\times\nabla Y_{lm}(\theta,\varphi)$ to get the following equations.

$$\Xi_{lm} = \frac{k_3 r_o \sqrt{\mu_o} / J_l'(k_3 r_o)}{i\sqrt{\varepsilon_3}\, l(l+1)} \left[\sin\theta_o \frac{\partial Y_{lm}^{*}(\theta_o,\varphi_o)}{\partial\theta} - imY_{lm}^{*}(\theta_o,\varphi_o)\right] \quad (14)$$

The expanded H-field given in (13) represents a field that has a delta-like behavior in the direction (θ_o,φ_o).

By comparing (8) and (13) it is clear that the beamforming technique is achieved by mapping F_{lm} (coefficients of the TE modes) to the desired coefficient Ξ_{lm}. Since these coefficients are embedded in the double sum, this mapping cannot be achieved for each antenna element individually but rather by considering the entire array as a system of linear equations.

Recall that F_{lm} is a function of J_{TE} which in turn is a function of the angular position (θ_i,φ_i) of the i-th antenna element where says that $F_{lm}=F_{lm}(\theta_i,\varphi_i)$. From equation (14) it is clear that $\Xi_{lm}=\Xi_{lm}(\theta_o,\varphi_o)$ and so the mapping would be possible by considering the following minimization statement.

$$\begin{aligned}&\arg\min_I \left\|AI - B\right\|^2 \\ &I_{opt} = (A^H A)^{-1} A^H B\end{aligned} \quad (15)$$

Where:

$$A = \left[A_1(\theta_1,\varphi_1) \quad A_2(\theta_2,\varphi_2) \quad A_3(\theta_3,\varphi_3)..... \quad A_N(\theta_N,\varphi_N)\right]$$

$$A_i(\theta_i,\varphi_i) = \left[F_{1,-1}(\theta_i,\varphi_i) \quad F_{1,0}(\theta_i,\varphi_i) \quad F_{1,1}(\theta_i,\varphi_i)...... \right.$$
$$\left. F_{M,-M}(\theta_i,\varphi_i).......F_{M,0}(\theta_i,\varphi_i)....F_{M,M}(\theta_i,\varphi_i)\right]^T$$

$$B = \left[\Xi_{1,-1}(\theta_o,\varphi_o) \quad \Xi_{1,0}(\theta_o,\varphi_o) \quad \Xi_{1,1}(\theta_o,\varphi_o)...... \right.$$
$$\left. \Xi_{M,-M}(\theta_o,\varphi_o).......\Xi_{M,0}(\theta_o,\varphi_o)....\Xi_{M,M}(\theta_o,\varphi_o)\right]^T$$

$$I = \left[I_1 \quad I_2 \quad I_3......I_N\right]^T \quad (16)$$

The matrix A is an MxN matrix that contains the TE coefficients F_{lm} for all N antenna elements. M represents the maximum degree of l desired to be reached (in this paper $M=7$). $M=7$ was chosen because by trial and error it was found that when having 7 orders or higher an acceptable delta-like function is achievable. By solving the minimization problem of equation (15) one can get the optimum current vector I_{opt} that will ensure that the magnetic field will have a delta-like behavior in the desired direction (θ_o,φ_o). Recall that the radial distance r_o of interest is kept fixed and then for that specific r_o the beamforming direction can be solved for.

After determining the optimal set of currents I_{opt}, the total H-field H_{Total} can be expressed as a summation of the H-fields in (8) multiplied by the corresponding current I_i taken from the vector I_{opt} and expressed as follows.

$$\vec{H}_{Total} = -\sum_{i=1}^{N} I_i i\sqrt{\frac{\varepsilon_3}{\mu_0}} \sum_{l=1}^{\infty}\sum_{m=-l}^{l} F_{lmi} \nabla\times\left[j_l(k_3 r)\vec{r}\times\nabla Y_{lm}(\theta,\varphi)\right] \quad (17)$$

III. MATLAB SIMULATION RESULTS

The theory presented in Section II is simulated in MATLAB. As mentioned earlier 42 elements were used in the simulation. Each element has a dimension of $L=2cm$ and since the frequency of operation is taken to be 1.8GHz then the spacing between the centers of the elements is $\lambda/4 \approx 4cm$. For an operating frequency of 1.8GHz the electromagnetic properties of the human head in region 0-3 are as follows (as given in [6]): region (0) has the free space permittivity and permeability; region (1) has $\varepsilon_1=38.87\varepsilon_0$, $\mu_1=\mu_0$ & $\sigma_1=1.88(S/m)$; region (2) has $\varepsilon_2=19.34\varepsilon_0$, $\mu_2=\mu_0$ & $\sigma_2=0.59(S/m)$; and region (3) has $\varepsilon_3=51.8\varepsilon_0$, $\mu_3=\mu_0$ & $\sigma_3=1.5(S/m)$. The simulations in Figures 2-4 show how the magnitude of the H-field in (17) varies with the angular parameters θ and φ. Note that all the simulations made are for a radial distance of $r_o=5cm$ from the center of the coordinate system as shown in Fig. 1.

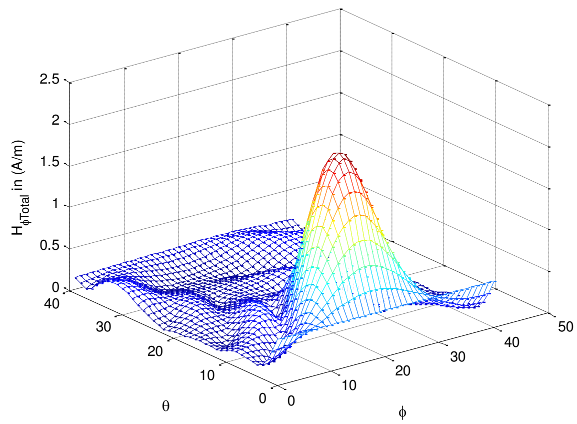

Fig. 2. H-field (*φ-component*) for the Antenna Array for $\theta_o=\pi/4$ & $\varphi_o=\pi$.

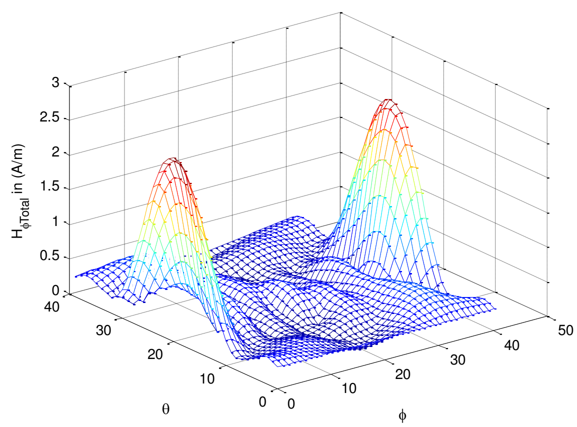

Fig. 3. H-field (*φ-component*) for the Antenna Array for $\theta_o=\pi/2$ & $\varphi_o=0$.

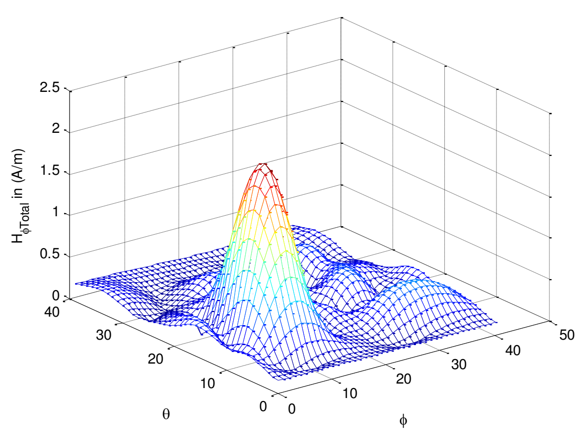

Fig. 4. H-field (*φ-component*) for the Antenna Array for $\theta_o=\pi/4$ & $\varphi_o=\pi/2$.

It is clear from the figures that the beamforming technique is successful in directing the magnitude of the H-field to the desired direction (θ_o,φ_o) indicated by each figure. Note that only the φ-component of (17) is shown on all of the three figures, but also the θ-component takes the same delta-like behavior. Another remark is the scaling used in the figures. The simulation was carried out for values of θ ranging from 0 to π and φ ranged from 0 to 2π. In the plots the two ranges were divided into 40 points, hence the scaling seen. For example, for the φ-axis the point 20 corresponds to $\varphi=20(2\pi/40)=\pi$ and for the θ-axis the same point corresponds to $\theta=20(\pi/40)=\pi/2$.

IV. Conclusions & Future Work

A theoretical hemispherical antenna array is simulated to focus the magnetic field intensity to a certain direction or point in the human head. The antenna elements proposed are to be done in practice. The beamforming technique used is successful in directing all components of the field to a specific point. The electric field and power calculations are yet to be done to examine how much power the human head will be exposed to. By calculating the total electric field on the skin, it will possible to calculate the SAR (specific absorption rate) and after that extend the work to investigate the temperature rise issues of this array to investigate the safety issues ([6] & [7]). It is still early to say how well the patient will cope with the new array or how long is the operation period and what are the long term issues involved. The point that cannot be denied if the device works is its ability to eliminate the need of surgery. A neural network is being designed to carry out the minimization of (15) to determine the optimal current values to realize a robust beamformer that works in real time for all desired points. Also the antenna elements and all the radio-frequency RF front-end circuitry is being designed for simulation.

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