semiconductors. The crystal was mounted as shown in Fig. 1. The electrodes were platinum and the insulator was a sheet of 0.001-inch Mylar. The device was mounted in a jig which could be inserted into a Perkin-Elmer 221 spectrophotometer. The 221 uses a prism dispersing system and a blackened gold leaf thermocouple detector. A wavelength range was selected which gave sufficient transmission through the TlBr-TlI crystal with zero modulation voltage. The modulation voltage was applied to the crystal and transmission was observed on the output recorder of the spectrometer. The time response was not measurable in this experiment since the recording-thermocouple response was extremely slow.

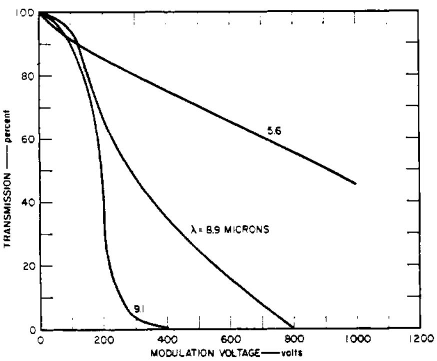

Fig. 2. Transmission vs. voltage.

Figure 2 shows the transmission characteristics as a function of dc modulation voltage. It can be seen that 100 percent modulation is possible with this device in the 8.0 to 9.0 micron region. The modulation, at a fixed voltage, can be seen to be less effective as the wavelength is decreased, as predicted in (1). The necessary surface charge density to obtain zero transmission may be estimated from the experimental data of Fig. 2, using the fact that the induced charge is equal to the electric field displacement ϵE_m where ϵ is the dielectric constant of the insulator and E_m is the average electric field or V_m/g. Dividing the displacement by the charge of an electron e gives the surface charge density σ which is

$$\sigma = \frac{\epsilon V_m}{eg}. \tag{2}$$

Substituting $V_m = 400$ (at 9.1 microns for cutoff), $g = 2\times10^{-6}$ meters, $\epsilon = 2.7\times10^{-11}$ (Mylar) and $e = 1.6\times10^{-19}$ coulombs gives $\sigma = 3.4\times10^{11}/\text{cm}^2$. This surface density agrees with the sensitivity obtained in surface study experiments on silicon [11]. The modulation effect is limited fundamentally by transitions within the crystal at the shorter wavelengths (edge absorption), surface state absorption at discrete wavelengths, and trapping of injected carriers in surface states for all wavelengths. Surface states could affect the modulator performance by: a) trapping the injected carriers, which would severely modify their absorption capabilities, and b) decreasing the drift mobility of the carriers, which would decrease the high frequency response. The TlBr-TlI crystal is an insulator which probably has a small drift mobility and poor crystalline perfection and is far from being an optimum material for this application. However, the experimental evidence does demonstrate the basic feasibility of the device. Since the device is dependent on a surface effect, the insulating layer could be made very thin. This would allow small voltages to control the transmission.

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On the Terminal Description of Nonlinear Networks

Although there recently have been discussed general nternal descriptions of nonlinear networks [1], [2], there is very little available on general external descriptions of nonlinear networks. Here we show that the general description $AV = BI$, of use for investigating the terminal behavior of linear n ports [3], [4], [5], can be extended to nonlinear n ports. It appears that the general description was first introduced by Bayard [3] but Tellegen [6] used related ideas to introduce the nullor [7] which is most conveniently described by the general description. Nevertheless, although the paper of Tellegen considered some nonlinear network elements, the general description has been considered only for linear networks.

The envisaged description is simply stated. Consider an n-port $\boldsymbol{N}$ of port voltages $\boldsymbol{v}$ and currents $\boldsymbol{i}$, these latter being real-valued n-vector functions of time, which are zero until a finite time and infinitely differentiable, written $\boldsymbol{v}, \boldsymbol{i} \in \boldsymbol{D}_+$, [5], [8]. Then in many cases there exist operators $\mathcal{A}[\ ]$ and $\mathcal{B}[\ ]$, perhaps nonlinear and time-dependent such that the defining relation for the network $\boldsymbol{N}$ can be expressed as

$$\mathcal{A}[v] = \mathcal{B}[i] \tag{1}$$

for all allowed pairs $[\boldsymbol{v}, \boldsymbol{i}] \in \boldsymbol{N}$. It is natural to call (1) the *general description* for $\boldsymbol{N}$.

Having defined the general description it is useful to exhibit a large, and meaningful class of networks for which it can be found. For this we recall that a network is called *solvable* if for every $\boldsymbol{e} \in \boldsymbol{D}_+$ the equation

$$e = v + i \tag{2}$$

has a unique solution $[\boldsymbol{v}, \boldsymbol{i}] \in \boldsymbol{N}$, [9]. This corresponds to stating that a unique current vector $\boldsymbol{i}$ results in Fig. 1 when $\boldsymbol{e} \in \boldsymbol{D}_+$ is applied; 1_n denotes n unit resistors in series with the ports.

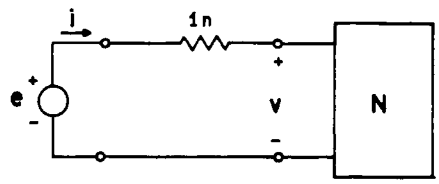

Fig. 1. Augmented network for (2).

Theorem: Every solvable network possesses a general description.

Proof: If $\boldsymbol{N}$ is solvable (2) defines a unique mapping $\mathcal{Y}_a[\ ]$ of $\boldsymbol{e} \in \boldsymbol{D}_+$ into $\boldsymbol{i} \in \boldsymbol{D}_+$. This with (2) gives

$$i = \mathcal{Y}_a[e] \tag{3a}$$

$$v = \mathcal{I}[e] - \mathcal{Y}_a[e] \tag{3b}$$

where $\mathcal{I}[\ ]$ is the identity mapping of $\boldsymbol{D}_+$ into $\boldsymbol{D}_+$. Since the range and domain of $\mathcal{I}[\ ] - \mathcal{Y}_a[\ ]$ and $\mathcal{Y}_a[\ ]$ are $\boldsymbol{D}_+$ we can apply the former to (3a) and the latter to (3b) to get the desired result

$$\mathcal{Y}_a[v] = \mathcal{I}[i] - \mathcal{Y}_a[i]. \tag{4}$$

If further $\mathcal{C}[\ ]$ is any "nonsingular" map of $\boldsymbol{D}_+$ into $\boldsymbol{D}_+$ it can be applied to both sides of (4) to obtain

$$\mathcal{A}[\ ] = \mathcal{C}[\mathcal{Y}_a[\ ]], \tag{5a}$$

$$\mathcal{B}[\ ] = \mathcal{C}[\mathcal{I}[\ ] - \mathcal{Y}_a[\ ]]. \tag{5b}$$

Therefore, many equivalent forms of the general description exist, (4) being a particular one in the solvable case. Q.E.D.

Observing Fig. 1, it seems clear that every physical circuit in the laboratory should be solvable and, consequently, that the general description is applicable to physical constructs. Still (1) applies to other than solvable networks, as a square-law device $v^2 = i$, or the norator, the 1 port defined by $0.v = 0.i$. Therefore, it appears that the general description is one of universal importance for any fundamental studies of networks, or more generally of nonoriented objects [10]. For linear networks the $\boldsymbol{D}_+$ constraint can often be relaxed and $\boldsymbol{D}'$, the set of n-vector distributions, used. The situation is different for general $\boldsymbol{N}$, since for instance $[du/dt]^2$ is undefined for u the unit step function.

In summary, we have introduced the general description as a means of expressing

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the terminal constraints on port variables for nonlinear networks. Although the most general conditions for the existence of (1) are unknown, in the solvable case $\mathfrak{A}[\ ]$ and $\mathfrak{B}[\ ]$ can be chosen as the single-valued maps $\mathfrak{A}[\ ]=\mathcal{Y}_a[\ ]$, $\mathfrak{B}[\ ]=\mathcal{I}[\ ]-\mathcal{Y}_a[\ ]$, of $\boldsymbol{D}_+$ into $\boldsymbol{D}_+$. It appears that classification of the $\mathfrak{A}[\ ]$ and $\mathfrak{B}[\ ]$ may lead to useful ways of cataloging nonlinear networks.

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Bandwidth Requirement for Frequency-Modulated Signals

The purpose of this communication is to supplement a previous report [1] on the spectral method of determining bandwidth requirement for FM signals based on the amount of tolerable distortion at the discriminator output.

Let the channel through which the signal is to be transmitted be characterized by the idealized transfer function

$$G(\omega) = |G(\omega)|\, e^{-j\tau\omega} \qquad (1)$$

with

$$\left.\begin{aligned} |G(\omega)| &= 1,\ \omega_{-\kappa} < \omega < \omega_{\kappa} \\ |G(\omega)| &= 0,\ \text{otherwise} \end{aligned}\right\}. \qquad (2)$$

Here ω is the angular frequency of a steady-state signal, τ is a constant independent of ω,

$$\omega_{\pm\kappa} = \omega_c \pm \kappa\omega_m \qquad (3)$$

are the frequencies of the κth sidebands of the FM signal being considered, and ω_c and ω_m are the carrier- and modulating-signal frequencies, respectively. Using a rigorous spectral analysis, the same author [1] has shown that the distortion-to-signal power ratio can be expressed as

$$D/S = \frac{1}{\pi\beta^2}\int_{-\pi}^{\pi}\left\{\sum_{-\infty,\kappa}^{-\kappa,\infty} J_k(\beta)(\beta\cos\theta - k)\cos(\beta\sin\theta - k\theta)\right\}^2 d\theta, \qquad (4)$$

where β is the index of modulation, $J_k(\beta)$ denotes the Bessel function of the first kind and kth order, and the summation over the index k is meant to be from $-\infty$ to $-\kappa$ and from κ to ∞. We have computed D/S as functions of the index of modulation β for different channel bandwidths ($\kappa=2, 3, \cdots, 16$) on a Honeywell H1800 digital computer. The series on the right side of (4) was first summed up to the 19th- and 27th-order Bessel functions for $\beta \le 5$ and $5 < \beta \le 10$, respectively. The integrals were then evaluated by Simpson's rule. The results are converted into decibels and plotted in Fig. 1 with the curves labeled by the dimensionless channel bandwidth

$$BW/f_m = 2(\kappa - 1). \qquad (5)$$

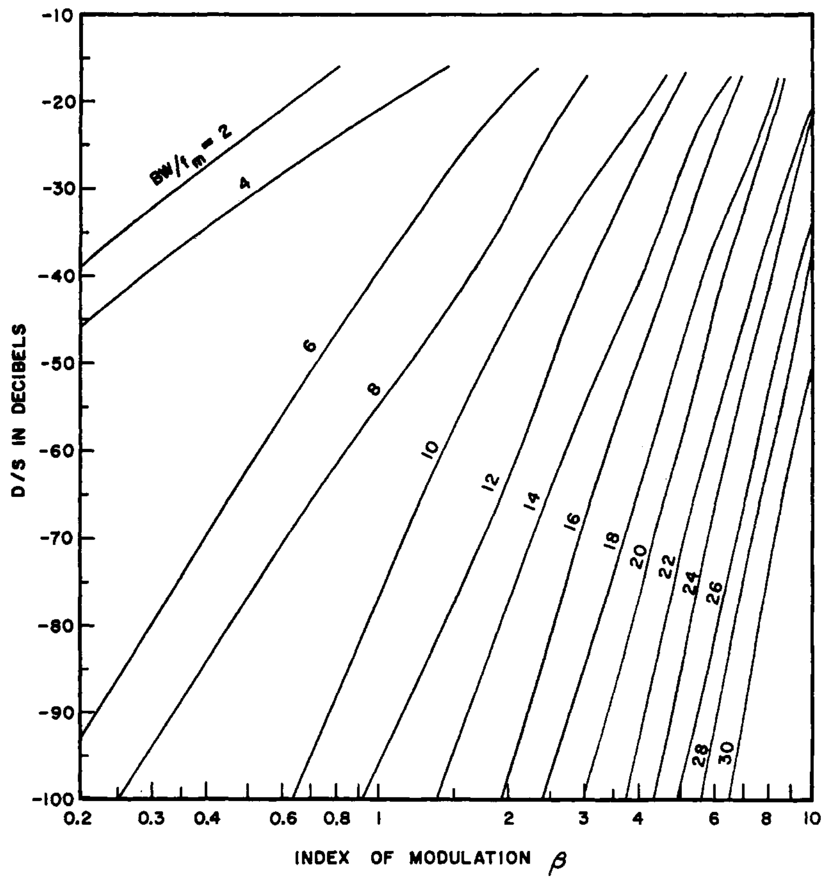

Fig. 1. Distortion-to-signal ratio vs. index of modulation in FM systems of finite transmission channel bandwidth.

The uneven spacings between the family of curves is due to certain combinations of Bessel functions of positive and negative integral orders. As the index of modulation is increased, more sidebands must be included; the effect of the combination of a particular pair of sidebands (or Bessel functions) becomes less significant and the distortion curves tend to be more evenly spaced for higher indices of modulation. The family of distortion curves provides a more rational and flexible basis for the determination of channel bandwidth requirement for the transmission of FM signals. For example, if the allowable distortion is 30 dB below the signal and the index of modulation is 2.0, a bandwidth of 8 times the modulating frequency would be adequate. Distortion decreases rapidly with the index of modulation; for small values of β (say <0.5), bandwidths of $2f_m$ to $4f_m$ obviously would suffice under most circumstances, a fact well known for "narrow-band" FM systems. As in the conventional method of bandwidth estimation by percentage sideband amplitude [2], [3], the highest frequency component in the information signal usually determines the channel bandwidth for a fixed modulating system and tolerable distortion level.

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