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A Pipelined Synthesis of Cochlea DSP Lattice Filters

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Abstract

A pipelined lattice realization of a digital scattering model of the cochlea is presented, based on real, lossless lattice synthesis of ARMA filters. The structure is recursively designed and each lattice is precisely implemented by a pair of complex conjugate transmission zeros via Richard's function extractions. In addition to being suitable for VLSI realization, the structure leads to a systematic cochlea parameter estimation, owing to the scattering nature of the model.

Keywords: Lattice Filters, Lossless Filter Synthesis, Cochlea Filters, VLSI Implementations.

1 Introduction

The lattice filter structure, as a realization of a digital transfer function, presents several advantages. These include ease of VLSI implementation, suitability for adaptive filtering, and superiority in round-off noise performance. Moreover, for particular classes of signals (such as speech, acoustic, seismic, etc.), the lattice structure and the scattering parameters (the transmission and reflection coefficients) have physical interpretations which enable the understanding of the properties of the processes [1].

The most extensive use of the lattice filter has been for speech processing applications, including speech compression, speech synthesis, and modeling of the human vocal tract. With the discovery of Kemp echoes and their experimentally well established properties, the development of a new category of models for non-invasive cochlea assessment and hearing loss correction becomes possible [2]. Since Kemp echoes are based on incident and reflected waves from the ear, a scattering type of model, amenable to a cascade lattice structure, is the most relevant. The cochlea model developed in [3] reproduces Kemp echoes in their impulse response and is very well suited for cochlea assessment through estimation of basilar membrane parameters. Thus, in this paper we focus on the synthesis technique for this model in a pipeline canonical form that leads not only to a systematic ear characterization, but to possible VLSI implementations as well.

The paper is organized as follows. In section 2, a summary of the ARMA synthesis technique is given. In section 3, an overview of the scattering model of the cochlea is presented. In section 4, the scattering cochlea model is realized as a cascade of 16 real lossless lattice filters. In section 5, possible VLSI implementations are discussed.

2 Cascade Synthesis of Real Lossless Lattice Filters

In a previous paper [4], we developed a technique designed to synthesize the transfer function of a stable, single-input, single-output ARMA(n,m) filter as a cascade of real lossless degree-one or degree-two lattice filters with a minimum number of delay elements. The method relies on a four-step Richard's function extraction, where two steps are used for reducing the degree of the transfer function and two for obtaining real lattices. Compared to other methods available in the literature [1, 5], this technique offers the advantage of realizing real degree-two lattices from complex degree-one lattices, a result that cannot be achieved by simply cascading two complex lattices. In addition, it is computationally very efficient since the section extraction proceeds from one entry of the scattering matrix (the input reflection coefficient) rather than from the entire matrix.

During the extraction step, a canonic section is removed that realizes a chosen transmission zero, followed by the computation of the load reflection coefficient. This process generates a new input reflection coefficient that corresponds to a lower order network. Synthesis is completed by iterating the basic extraction step.

The degree reduction induced during the extraction steps insures that, after all the factors have been exhausted, the remainder load reflection coefficient corresponds to a zero-order section. Below we outline the basic theoretical formalism and synthesis algorithm for the case of degree-two real lattices and transmission zeros inside the unit circle.

Let $H(z)$ denote the overall transfer function of the ARMA filter to be synthesized, $S(z)$ the overall scattering matrix of the corresponding cascade two-port lattice filter representation, having the four S_{ij} as entries, and $S_I(z)$ and $S_L(z)$ the input and load reflection coefficients [4].

- **Step 1:** Calculate $S_I(z)$ by factorization using the para-unitary property of the lossless $S(z)$ as follows:

$$1 - S_{21}(z)S_{21*}(z) = S_I(z)S_{I*}(z) \qquad (1)$$

  where $_*$ means transposing and replacing z by $1/z^*$, with * being the complex conjugate, and S being para-unitary means $S^{-1} = S_*$. Here $S_{21}(z)$ is calculated by normalizing the given transfer function H according to

$$S_{21}(z) = \frac{H(z)}{M}, \quad M \geq \max|H(z)|, \quad |z| = 1 \qquad (2)$$

- **Step 2:** Calculate the zeros of transmission which are the zeros of $S_{21}(z)$.

- **Step 3:** Extract a lattice section characterized by a transfer scattering matrix $\theta(z)$, realizing a chosen transmission zero a.

$$\theta(z) = I_2 + \frac{(z-1)}{(1-a^*)(z-a)} x x_* J \qquad (3)$$

  where $x = [x_1, x_2]^T$ is a complex vector such that

$$|x_1|^2 = \frac{|a|^2 - 1}{1 - |S_I(a)|^2}, \quad x_2 = S_I(a)x_1 \qquad (4)$$

  I_2 the identity matrix, and J the para-unitary matrix given by

$$J = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad (5)$$

- **Step 4:** Evaluate the load reflection coefficient as follows

$$\begin{aligned} S_L(z) &= (\theta_{22}(z) - S_I(z)\theta_{12}(z))_*^{-1} \\ &\quad (S_I(z)\theta_{11}(z) - \theta_{21}(z)) \end{aligned} \qquad (6)$$

- **Step 5:** $S_L(z)$ represents the input reflection coefficient of the remainder network. Thus, to repeat the extraction, set $S_I(z) = S_L(z)$ and select a^* as the corresponding transmission zero to be realized. Repeat steps 3, 4, and 5 until the degree of $S_L(z)$ is reduced to zero.

- **Step 6:** For each section, multiply two degree-one transfer scattering matrices realizing complex conjugate transmission zeros to obtain a real degree-two transfer scattering matrix.

3 The Cochlea Model

The cochlea model proposed in [3, 6] is of a digital scattering nature, based on a nonuniform lossy unidimensional transmission line structure. The fundamental analog modeling concepts embody the properties of the cochlea fluid and basilar membrane mechanics. The digital scattering model is obtained by rephrasing the analog description in terms of incident and reflected waves and digitizing in space and time. The resulting structure takes the form of a pipeline of degree-two real lattice filters, with each lattice filter representing one section of the cochlea of the structure shown in Figure 1, propagating the incident and reflected traveling pressure waves p_k^i and p_k^r.

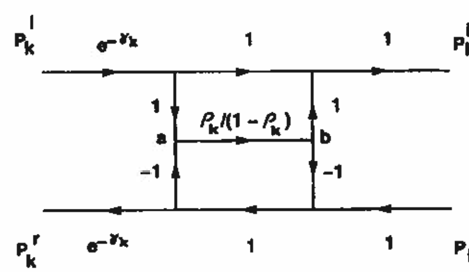

Figure 1: Signal-flow graph of a cochlea section.

Here the functions $\rho_k(z)$ and $\gamma_k(z)$ are similar to a "reflection coefficient" and a "propagation function" yielding a delay through the k^{th} section. These functions are reasonably approximated in the z-transform domain by [3]

$$\rho_k(z) = \frac{A_{k2}z^2 + A_{k1}z + A_{k0}}{B_{k2}z^2 + B_{k1}z + B_{k0}} \quad (7)$$

$$\gamma_k(z) = \sqrt{\frac{C_{k2}z^2 + C_{k1}z + C_{k0}}{D_{k2}z^2 + D_{k1}z + D_{k0}}} \quad (8)$$

with the A_k, B_k, C_k, and D_k coefficients being functions of the geometrical, fluid, and mechanical parameters of the cochlea. The lattice filters are described by transfer scattering matrices whose entries are functions of these parameters, thus allowing their systematic extraction [3]. For the k^{th} section, the expression of the transfer scattering matrix is

$$\theta_k(z) = \frac{1}{1+\rho_k(z)} \begin{bmatrix} e^{\gamma_k(z)}\rho_k(z) & e^{\gamma_k(z)} \\ e^{-\gamma_k(z)} & e^{-\gamma_k(z)}\rho_k(z) \end{bmatrix} \quad (9)$$

4 Cochlea Lattice Filters

Since in the 0 - 30 kHz range, which contains the Kemp echo dominant frequency range (between 0.5 and 4 kHz), $\gamma_k(z)$ is negligible compared to $\rho_k(z)$ [3], the lattice structure described by the transfer scattering matrix of (9) [and whose flow-graph is given in Figure 1] is equivalent to the one generated by the synthesis algorithm and given by (3). Thus, we use the approach outlined in section 2 to approximate the cochlea lattices.

In previous research the cochlea model was identified as an ARMA(32,16) and its transfer function estimated [7]. This transfer function is the actual input reflection coefficient of the system, since in the case of the ear the response of the system and the input are measured at the same point. In this section we synthesize the cochlea model as a cascade of 16 lossless real lattice filters of degree two, closed on a constant terminating section representing the helicotrema end of the cochlea.

4.1 Transfer Function

The digital cochlea model is assumed to be a stable ARMA filter of unknown order. Kemp echo signals, recorded from human ears and provided to us by Dr. H. P. Wit and Dr. P. V. Dick from the Institute of Audiology (the Netherlands), were used as the output signal in an ARMA system identification technique, developed by Youla, Pillai and Shim [8], to estimate the transfer function of the cochlea [7]. The model is estimated as an ARMA(32,16) with the resulting transfer function $H(z) = N(z)/D(z)$ such that

$$\begin{aligned} N(z) &= 0.57 - 1.18z + 0.33z^2 + 0.43z^3 + 0.74z^4 \\ &\quad -1.35z^5 - 1.81z^6 + 0.97z^7 + 0.92z^8 + \\ &\quad 1.42z^9 + 0.09z^{10} - 0.29z^{11} + 0.31z^{12} - \\ &\quad 0.63z^{13} - 1.44z^{14} - 1.87z^{15} + 4.46z^{16} \end{aligned} \quad (10)$$

$$\begin{aligned} D(z) &= 0.14 - 0.28z + 0.17z^2 - 0.13z^3 + \\ &\quad 0.01z^4 + 0.16z^5 - 0.15z^6 + 0.09z^7 + \\ &\quad 0.03z^8 - 0.05z^9 - 0.15z^{10} - 0.12z^{11} - \\ &\quad 0.06z^{12} + 0.09z^{13} + 0.22z^{14} - 1.13z^{15} \\ &\quad +2.77z^{16} - 1.6z^{17} - 0.58z^{18} - 1.66z^{19} \\ &\quad +2.27z^{20} + 1.98z^{21} - 2.65z^{22} + 0.39z^{23} \\ &\quad -1.00z^{24} + 1.17z^{25} + 0.33z^{26} - 1.45z^{27} \\ &\quad +0.44z^{28} + 1.68z^{29} + 3.25z^{30} - 9.09z^{31} \\ &\quad +5.00z^{32} \end{aligned} \quad (11)$$

4.2 Zeros of Transmission

The transmission zeros of the cochlea model are calculated by factorization using the para-unitary property of S. This factorization does not provide any information on how to choose the locations (inside or outside the unit circle) of these zeros or on the order in which they should be synthesized. These two pieces of information are crucial to the synthesis of the cochlea lattices because these zeros depend on the displacement along the basilar membrane via the geometrical and mechanical parameters of the ear. Thus, a preliminary calculation is done through the use of experimental parameter values for a typical ear. The results show that all the transmission zeros of the cochlea occur in complex conjugate pairs, located inside the unit circle, and whose magnitudes decrease from the first section to the last section.

Consequently, we select the transmission zeros resulting from factoring $1 - S_{21}S_{*21} = S_I S_{*I}$ inside the unit circle. This factorization yields 15 pairs of complex conjugate zeros, comparable to the ones obtained from experimental data, and two real zeros which do not appear in the experimental case. For this reason, we choose to realize the complex conjugate pairs first, in the order of decreasing magnitude, then realize the real pair of zeros. All of these zeros are listed below

$$
\begin{array}{ll}
z_1 = 0.38 - 0.92j & z_2 = 0.91 - 0.39j \\
z_3 = 0.93 - 0.25j & z_4 = -0.73 - 0.61j \\
z_5 = 0.72 - 0.61j & z_6 = -0.17 - 0.92j \\
z_7 = -0.89 - 0.15j & z_8 = -0.52 - 0.74j \\
z_9 = -0.85 - 0.19j & z_{10} = -0.72 - 0.48j \\
z_{11} = 0.79 - 0.36j & z_{12} = -0.34 - 0.77j \\
z_{13} = 0.46 - 0.69j & z_{14} = 0.20 - 0.79j \\
z_{15} = -0.07 - 0.80j & z_{16} = 0.65 \\
z_{17} = 0.99 &
\end{array} \tag{12}
$$

4.3 The Lattice Realization

The extraction proceeds from the input reflection coefficient $S_I(z)$ obtained from $H(z)$ through normalization (equation (2), where $M = 180$). Since it has degree 32, the cochlea model is realized as a cascade of 16 degree-two real sections (15 complex conjugate pairs and 2 unpaired reals). To illustrate their nature in the limited space available, below we give the transfer scattering matrices for the first, the eighth, the fifteenth, and the sixteenth sections, realizing the pairs (z_1, z_1^*), (z_8, z_8^*), (z_{15}, z_{15}^*), and (z_{16}, z_{17}) respectively.

For the first section, we have

$$
\begin{bmatrix}
\frac{0.84+1.89z+1.09z^2}{0.83+1.79z+z^2} & \frac{10^{-5}(-8.3-0.8z+9.1z^2)}{0.83+1.79z+z^2} \\
\frac{10^{-5}(-9.1-0.8z+8.7z^2)}{0.83+1.79z+z^2} & \frac{0.83+1.73z+1.10z^2}{0.83+1.79z+z^2}
\end{bmatrix} \tag{13}
$$

For the eighth section, we have

$$
\begin{bmatrix}
\frac{0.66+0.15z+1.02z^2}{0.64+0.14z+z^2} & \frac{10^{-4}(0.78-2.84z+2.06z^2)}{0.64+0.14z+z^2} \\
\frac{10^{-4}(1.06-2.85z+2.78z^2)}{0.64+0.14z+z^2} & \frac{0.64+0.14z+1.01z^2}{0.64+0.14z+z^2}
\end{bmatrix} \tag{14}
$$

For the fifteenth section, we have

$$
\begin{bmatrix}
\frac{0.95-1.86z+1.00z^2}{0.93-1.86z+z^2} & \frac{10^{-2}(0.83-1.95z+1.12z^2)}{0.93-1.86z+z^2} \\
\frac{10^{-2}(1.12-1.95z+0.93z^2)}{0.93-1.86z+z^2} & \frac{0.93-1.87z+1.02z^2}{0.93-1.86z+z^2}
\end{bmatrix} \tag{15}
$$

The sixteenth section has

$$
\begin{bmatrix}
\frac{1.66-2.91z+1.25z^2}{0.65-1.65z+z^2} & \frac{1.08-2.39z+1.32z^2}{0.65-1.65z+z^2} \\
\frac{1.32-2.39z+1.08z^2}{0.65-1.65z+z^2} & \frac{1.66-2.91z+1.25z^2}{0.65-1.65z+z^2}
\end{bmatrix} \tag{16}
$$

and a degree-zero load reflection coefficient $S_{16} = 0.98$, an indication that no more lattice extractions are possible and representing section losses and the helicotrema.

5 VLSI Realization of Cochlea Lattices

Our cochlea lattice structure is in a very appropriate form for VLSI construction, essentially being a cascade of degree-two real sections. By obtaining the semistate equations for each section and then realizing these equations in canonical VLSI circuits (using two delays via integrators) a pipelined form of VLSI layout can be obtained. These circuits can be realized by switched current mode circuits for low bias voltages. Moreover, generalizations of translinear circuits and the development of a theory of synthesis based upon them and the semistate representation should lead to good design methods for obtaining practical cochlea-like lattices in VLSI form.

6 Discussion

In this paper, we presented a pipelined synthesis of real lossless cochlea lattices suitable for cochlea assessment and VLSI implementation. In the same context, we point out that this particular structure may help in advancing and improving the methods that are currently used in many related fields, such as language learning, hearing aid design, and in exploring signal processing techniques and coding models for the auditory system that could be supported by VLSI circuits. Note that the cochlea is inherently lossy and its loss term is represented here by the terminating load section. This loss term could be moved into the sections using the loss distribution technique given in [9]. Finally, we mention that the synthesis method could be nicely extented to extract the delay term $e^{-\gamma_k(z)}$ and, thus, preserve the cochlea structure of (9). Towards this, a generalization of the Youla-Pillai method seems to be ideal for handling such a case.

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