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alization for multipath environments
ogy Conference, Countdown to the

e equalization for co-channel inter-
mun vol. 43, 1441–1453, February–

ement receiver structures for sSpa-
mation Theory, vol. IT-32, 195–219,

Communications, Palaiseau, France,

orks, John Wiley & Sons, New York

or GSM cellular systems, 46th IEEE

Trans. Commun., vol. COM-34, 89–

istics of DFE transversal-combining
netric-combining schemes, Electron-
–98, 1994.

Computable Real Lattice Structures for Cochlea-Like Digital Filters*

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Abstract. A synthesis algorithm for a pipelined lattice implementation of cochlea-like digital filters is presented, based upon the properties of real, lossless lattice synthesis of ARMA filters. The algorithm operates on a simplified characterization of elementary lattice sections of degree one or degree two. This leads to a structure that is recursively designed and for which each lattice is precisely implemented by a pair of complex conjugate transmission zeros via Richard's function extractions. Except for zeros of transmission on the unit circle, all other types and multiplicities are allowed. Necessary and sufficient conditions are derived for the degree-two lattices to guarantee computability, i.e., realizability with no delay-free loops. In addition to being suitable for VLSI realization, the structure enables a systematic cochlea assessment from the scattering ear parameters.

1. Introduction

The problem of cascade synthesis of lattice filters using scattering variables has received a great deal of attention during the last several years [2], [8], [9], [17] because of its many applications. Both its theoretical and practical aspects have been widely exploited in various fields of signal processing. In particular, the lattice filter structure as a realization of a digital transfer function presents several advantages. These include modular structure, which makes it an attractive candidate for VLSI implementation, its nice stability properties, suitability for adaptive filtering, superiority in finite word length performance, and the flexibility it offers in choosing the order. Moreover, for particular classes of signals (speech, acoustic, seismic, etc.), the lattice structure and the scattering parameters (which include the

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transmission and reflection coefficients) have physical interpretations that enable the understanding of the properties of the processes.

The most extensive use of the lattice filter has been for speech processing applications, including speech compression, speech analysis and synthesis [1], and modeling of the human vocal tract [22]. With the discovery of Kemp echoes and their experimentally well-established properties [7], the development of a new category of cochlea models suitable for hearing impairment diagnosis via Kemp echoes becomes possible. The implementation of these models in lattice form can provide important information about many kinds of cochlea damages, thus allowing for the development of potential clinical tools for noninvasive screening of the cochlea and hearing loss correction [3], [18], [21].

Kemp echoes are oto-acoustic signals emitted by the ear in response to an acoustic stimulation (a click); they are exhibited by the majority of healthy ears of both humans and animals. Although the echoes are of small magnitude in the case of damaged ears, they can still be measured with appropriate filtering techniques. Because the echoes are based on incident and reflected waves from the ear, a scattering type of model, amenable to a cascade lattice structure that mimics cochlea behavior, is the most relevant for diagnosis purposes via Kemp echoes. The cochlea model developed in [18] reproduces Kemp echoes in their impulse response and is very well suited for cochlea assessment through estimation of basilar membrane parameters. Thus, in this paper we focus on the synthesis technique for this model in a pipeline canonical lattice form that leads not only to a systematic ear characterization, but to possible VLSI implementations of cochlea like ARMA filters as well.

The remainder of the paper is structured as follows. In Section 2, a summary of the technique for synthesis of ARMA filters by real lossless lattice filters is given. In Section 3, an overview of the digital scattering model of the cochlea is presented, emphasizing the cascade lattice structure and its description in terms of transfer scattering matrices and reflection coefficients. Section 4 is devoted to the implementation of the cochlea model in question as a pipeline of 16 real lossless lattice filters terminated on a lossy nondynamical section representing the helicotrema and the losses of the cochlea structure. Necessary and sufficient conditions for obtaining a computable lattice realization are derived in Section 5, and comments on possible VLSI implementations are given in Section 6. Section 7 presents the concluding discussion.

2. Synthesis algorithm for lossless lattice filters

In a previous paper [17], we developed a technique designed to synthesize the transfer function of a stable, single-input, single-output ARMA(n, m) filter as a cascade of lossless degree-one or degree-two lattice filters with a minimum number of delay elements. The method relies on a four-step Richard's function extraction, where two steps are used for reducing the degree of the transfer function and two

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for obtaining real lattices. Compared to other methods available in the literature [2], [9], this technique offers the advantage of realizing real degree-two lattices from complex degree-one lattices, a result that cannot be achieved by simply cascading two complex lattices. In addition, it is computationally more efficient because the section extraction proceeds from one entry of the scattering matrix (the input reflection coefficient) rather than from the entire matrix.

The synthesis procedure involves repeated basic extraction steps. It starts with a given transfer function (or an input reflection coefficient) and at each step extracts a first- or second-order canonic section that realizes a chosen transmission zero, followed by the computation of the load reflection coefficient. This process generates a new input reflection coefficient that corresponds to a lower-order network. Synthesis is completed by iterating the basic extraction step.

The degree reduction induced during the extraction steps ensures that, after all the dynamical factors have been exhausted, the remainder load reflection coefficient corresponds to a zero-order terminating section containing the losses of the structure. It also ensures that the overall cascade connection, with its termination, has the prescribed transfer function, thus generating a structure with a minimum number of delay elements. In the following paragraphs we outline the basic theoretical formalism and synthesis algorithm for the case of degree-two real lattices with transmission zeros inside the unit circle. We concentrate on this particular case because it represents the physical model of the cochlea considered in Section 3; a treatment of the case of degree-one sections and transmission zeros inside and outside the unit circle can be found in [17].

Let $H(z)$ denote the overall digital transfer function of the ARMA filter to be synthesized, let $S(z)$ be the overall scattering matrix of the corresponding cascade two-port lattice filter representation, having the four S_{ij} as entries, and let $S_I(z)$ and $S_L(z)$ be the input and load reflection coefficients, respectively. Each extracted lattice will be described by a (2×2) transfer scattering matrix $\theta(z)$. Because we work with lossless lattices, $S(z)$ is para-unitary and $\theta(z)$ is J-para-unitary and J-expansive, that is,

$$S(z)S_*(z) = I_2, \quad \theta_*(z)J\theta(z) = J \ , \tag{1}$$

where for any square matrix $A_*(z) = A^{*T}(1/z^*)$, I_2 is the identity matrix, and J is given in equation (2) below. The superscripts * and ^{T} denote complex conjugation and matrix transposition, respectively.

$$J = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} . \tag{2}$$

Because the ARMA filter is assumed finite and, hence, described by a rational transfer function with real coefficients, $S(z)$ and $\theta(z)$ are both rational matrices with real coefficients. Furthermore, because the lattice structure of interest is passive, $S(z)$ is bounded real in $|z| > 1$. The extraction steps are as follows:

- **Step 1.** Calculate $S_I(z)$ by normalizing the given transfer function $H(z)$ according to

$$S_I(z) = \frac{H(z)}{M}, \quad M \geq \max|H(z)|, \quad |z| = 1 . \tag{3}$$

- **Step 2.** Determine the zeros of transmission, these being the zeros of $S_{21}(z)$, by factorization using the para-unitary property of the lossless $S(z)$ as follows:

$$S_{21}(z)S_{21*}(z) = 1 - S_I(z)S_{I*}(z) . \tag{4}$$

  Choose $S_{21}(z)$ to have the same denominator as $S_I(z)$ and assign half of the zeros of (4) (the ones inside the unit circle, for minimum phase) to the numerator of $S_{21}(z)$.

- **Step 3.** Using a Richard's function extraction of factors of the form $(z-a)$ and $(z-1/a^*)$, extract a degree-one lossless passive lattice section characterized by a transfer scattering matrix $\theta_x(z)$, realizing a chosen transmission zero a.

$$\theta_x(z) = I_2 + \frac{(z-1)}{(1-a^*)(z-a)} x x_* J , \tag{5}$$

  where $x = [x_1, x_2]^T$ is a complex vector calculated such that the degree of the load reflection coefficient $S_{Lx}(z)$ (calculated in Step 4) is reduced by one, while satisfying the J-para-unitary property of $\theta_x(z)$:

$$|x_1|^2 = \frac{|a|^2 - 1}{1 - |S_I(a)|^2}, \quad x_2 = S_I(a)x_1, \quad x_* J x = |a^2| - 1 . \tag{6}$$

- **Step 4.** Evaluate the load reflection coefficient as follows:

$$S_{Lx}(z) = \frac{S_I(z)\left[(1-a^*)(z-a) + (z-1)|x_1|^2\right] - (z-1)x_1^* x_2}{(1-a^*)(z-a) + (z-1)\left[S_I(z)x_1 x_2^* - |x_2|^2\right]} . \tag{7}$$

  At first glance, $S_{Lx}(z)$ appears to be one degree higher than $S_I(z)$; therefore, to reduce its degree by one with respect to $S_I(z)$, the complex parameters x_1 and x_2 of Step 3 are chosen so that the numerator and the denominator of $S_{Lx}(z)$ have $(z-a)$ and $(z-1/a^*)$ factors in common.

- **Step 5.** $S_{Lx}(z)$ represents the input reflection coefficient of the remainder network and has the same properties as $S_I(z)$; that is, $S_{Lx}(z)$ is rational in z and is analytic and bounded where $S_I(z)$ is analytic and bounded (i.e., in $|z| > 1$). Thus, the extraction can be repeated as follows: set $S_I(z) = S_{Lx}(z)$ and select a^* as the corresponding transmission zero to be realized. Extract a second degree-one lossless passive section $\theta_y(z)$:

$$\theta_y(z) = I_2 + \frac{(z-1)}{(1-a)(z-a^*)} y y_* J , \tag{8}$$

  where the complex vector y is calculated so that the degree of the output reflection coefficient $S_{Ly}(z)$ is reduced by one with respect to $S_{Lx}(z)$, on the one hand, and the cascade of $\theta_x(z)$ and $\theta_y(z)$ is a degree-two transfer scattering matrix with real coefficients, on the other hand. In this step, it is the factors $(z - a^*)$ and $(z - 1/a)$ that are extracted via a Richard's function extraction.

$$y = \frac{x^*}{K^*} - x\frac{\gamma^*}{K^*(r + 2jd)} \tag{9}$$

$$\gamma = x^T J x \tag{10}$$

$$r = |a|^2 - 1 \tag{11}$$

$$d = \frac{a - a^*}{2j} = \text{Im}(a), \quad j = \sqrt{-1} \tag{12}$$

$$|K|^2 = 1 - |\gamma|^2 \left(r^2 + 4d^2\right)^{-1} . \tag{13}$$

- **Step 6.** In equation (7), replace $S_I(z)$ by $S_{Lx}(z)$, $S_{Lx}(z)$ by $S_{Ly}(z)$, a by a^*, and evaluate S_{Ly}. Then repeat Steps 3, 4, and 5 until all the zeros of transmission are extracted.
- **Step 7.** For each section, multiply the two degree-one transfer scattering matrices $\theta_x(z)$ and $\theta_y(z)$ to obtain a real degree-two transfer scattering matrix.

3. The digital scattering cochlea model

The cochlea model proposed in [18] is of a digital scattering nature, based on a nonuniform lossy unidimensional transmission line structure. The fundamental analog modeling concepts embody the properties of the cochlea fluid and basilar membrane mechanics. The two cochlea chambers, scala tympani and scala vestibuli, which are filled with fluid and are separated by the basilar membrane, except at the helicotrema end, receive individually incident and reflected pressure waves analogous to signals in standard lattice structures. Therefore, to obtain the corresponding digital scattering model, the analog description is rephrased in terms of incident and reflected waves and digitized in space and time. The resulting structure takes the form of a pipeline of degree-two real lattice filters, with each lattice filter representing one section of the cochlea of the structure shown in Figure 1, propagating the incident and reflected traveling pressure waves p_k^i and p_k^r. Here the functions $\rho_k(z)$ and $\gamma_k(z)$ are similar to a "reflection coefficient" and a "propagation function," yielding a delay through the kth section. These functions are reasonably approximated in the z-transform domain by [3], [18]

$$\rho_k(z) = \frac{A_{k2}z^2 + A_{k1}z + A_{k0}}{B_{k2}z^2 + B_{k1}z + B_{k0}} \tag{14}$$

$$\gamma_k(z) = \sqrt{\frac{C_{k2}z^2 + C_{k1}z + C_{k0}}{D_{k2}z^2 + D_{k1}z + D_{k0}}} , \tag{15}$$

where the A_k, B_k, C_k, and D_k coefficients are explicit functions of the geometrical, fluid, and mechanical parameters of the cochlea. The lattice filters are described by

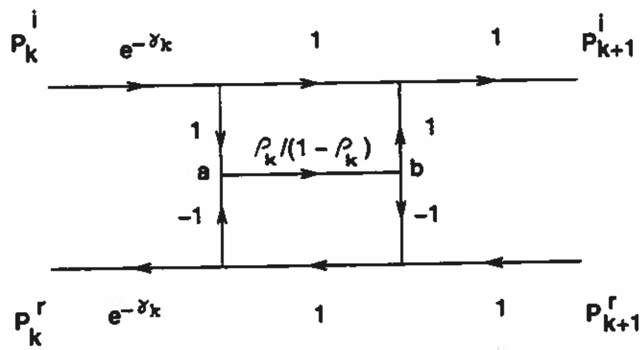

Figure 1. Signal-flow graph of a cochlea section.

transfer scattering matrices whose entries are functions of these parameters, thus allowing their systematic estimation through lattice extractions [18]. For the kth section, the expression of the transfer scattering matrix is

$$\theta_k(z) = \frac{1}{1+\rho_k(z)}\begin{bmatrix} e^{\gamma_k(z)}\rho_k(z) & e^{\gamma_k(z)} \\ e^{-\gamma_k(z)} & e^{-\gamma_k(z)}\rho_k(z) \end{bmatrix}. \tag{16}$$

4. Cochlea-like lattice filters

In the 0 – 30-kHz range, which contains the Kemp echo dominant frequency range (between 0.5 and 4 kHz), $\gamma_k(z)$ is negligible compared to $\rho_k(z)$ [18], thus the lattice structure described by the transfer scattering matrix of (16) (whose flow graph is given in Figure 1) is equivalent to the one generated by the synthesis algorithm, whose transfer scattering matrix is the product of $\theta_x(z)$ and $\theta_y(z)$ given in (5) and (8), respectively. Consequently, we use the algorithm outlined in Section 2 to approximate the cochlea lattices.

In previous research the cochlea model was identified as an ARMA(32,16) and its transfer function estimated [20]. After normalization, this transfer function represents the actual input reflection coefficient of the system, because in the case of the ear the response of the system and the input are measured at the same point. Using this transfer function, we synthesize the cochlea model as a cascade of 16 lossless real lattice filters of degree two, closed on a constant terminating section representing the helicotrema end of the cochlea.

4.1 Transfer function.

The digital cochlea model is assumed to be a stable ARMA filter of unknown order. Kemp echo signals [16], recorded from human ears and provided to us by Dr. Hugo

P. Wit and Dr. Pim V. Dijk from the Institute of Audiology, the Netherlands, were used as the output signal in the ARMA filter identification method developed by Pillai et al. [13] to estimate the transfer function of the cochlea without a priori knowledge of its degree [20]. The model is estimated as an ARMA(32,16) with the resulting transfer function $H(z) = N(z)/D(z)$ such that

$$\begin{aligned} N(z) = 0.57 &- 1.18z + 0.33z^2 + 0.43z^3 + 0.74z^4 - 1.35z^5 \\ &- 1.81z^6 + 0.97z^7 + 0.92z^8 + 1.42z^9 + 0.09z^{10} - 0.29z^{11} \\ &+ 0.31z^{12} - 0.63z^{13} - 1.44z^{14} - 1.87z^{15} + 4.46z^{16} \end{aligned} \tag{17}$$

$$\begin{aligned} D(z) = 0.14 &- 0.28z + 0.17z^2 - 0.13z^3 + 0.01z^4 + 0.16z^5 \\ &- 0.15z^6 + 0.09z^7 + 0.03z^8 - 0.05z^9 - 0.15z^{10} \\ &- 0.12z^{11} - 0.06z^{12} + 0.09z^{13} + 0.22z^{14} \\ &- 1.13z^{15} + 2.77z^{16} - 1.6z^{17} - 0.58z^{18} - 1.66z^{19} + 2.27z^{20} + 1.98z^{21} \\ &- 2.65z^{22} + 0.39z^{23} - 1.00z^{24} + 1.17z^{25} + 0.33z^{26} \\ &- 1.45z^{27} + 0.44z^{28} + 1.68z^{29} + 3.25z^{30} - 9.09z^{31} + 5.00z^{32}. \end{aligned} \tag{18}$$

4.2 Zeros of transmission.

The transmission zeros of the cochlea model are calculated by factorization of $S_l(z)$ using the para-unitary property of the (2×2) scattering matrix S. This factorization does not provide any information on how to choose the locations (inside or outside the unit circle) of these zeros or on the order in which they should be synthesized. These two pieces of information are crucial to the synthesis of the cochlea lattices because these zeros depend on the displacement along the basilar membrane via the geometrical and mechanical parameters of the ear. Thus, a preliminary calculation is done through the use of measured and curve fitted geometrical and mechanical parameters from the literature for a typical ear [15]. The results given in equation (19) for one of each conjugate pair show that all the transmission zeros of the cochlea occur in complex conjugate pairs, located inside the unit circle, whose magnitudes decrease from the first section to the last section:

$$\begin{array}{lll} z_1 = -0.99 + 0.045j & z_2 = -0.99 + 0.066j & z_3 = -0.99 + 0.096j \\ z_4 = -0.98 + 0.14j & z_5 = -0.98 + 0.19j & z_6 = -0.95 + 0.28j \\ z_7 = 0.93 + 0.32j & z_8 = -0.91 + 0.40j & z_9 = 0.88 + 0.45j \\ z_{10} = -0.81 + 0.55j & z_{11} = 0.77 + 0.61j & z_{12} = -0.65 + 0.73j \\ z_{13} = 0.58 + 0.79j & z_{14} = -0.40 + 0.89j & z_{15} = 0.29 + 0.93j \\ z_{16} = -0.064 + 0.97j\,. & & \end{array} \tag{19}$$

Consequently, we select the transmission zeros inside the unit circle resulting from factoring $1 - S_{21}S_{*21} = S_l S_{*l}$. This factorization yields 15 pairs of complex conjugate zeros and two real zeros which closely approximate the ones obtained from the preliminary calculation. Because our cochlea lattice sections are real and of degree two, we extract the complex conjugate pairs first, in order of decreasing magnitude, then extract the real pair of zeros. Omitting the complex conjugates, these zeros are listed as follows:

$$\begin{array}{lll} z_1 = 0.38 - 0.92j & z_2 = 0.91 - 0.39j & z_3 = 0.93 - 0.25j \\ z_4 = -0.73 - 0.61j & z_5 = 0.72 - 0.61j & z_6 = -0.17 - 0.92j \\ z_7 = -0.89 - 0.15j & z_8 = -0.52 - 0.74j & z_9 = -0.85 - 0.19j \\ z_{10} = -0.72 - 0.48j & z_{11} = 0.79 - 0.36j & z_{12} = -0.34 - 0.77j \\ z_{13} = 0.46 - 0.69j & z_{14} = 0.20 - 0.79j & z_{15} = -0.07 - 0.80j \\ z_{16} = 0.99 & z_{17} = 0.65\,. & \end{array} \tag{20}$$

4.3 The lattice realization.

The extraction proceeds from the input reflection coefficient $S_I(z)$ obtained from $H(z)$ through normalization (equation (3), where $M = 180$). Because it has degree 32, the cochlea model is realized as a cascade of 16 degree-two real sections (15 complex conjugate pairs and 2 unpaired reals). To illustrate their nature, we now give the transfer scattering matrices for the first, the eighth, the fifteenth, and the sixteenth sections, realizing the pairs (z_1, z_1^*), (z_8, z_8^*), (z_{15}, z_{15}^*), and (z_{16}, z_{17}) respectively. The transfer scattering matrices of the remaining sections are given in the Appendix.

For the first section, we have

$$\theta_1(z) = \begin{bmatrix} \dfrac{0.83+1.79z+1.00z^2}{0.83+1.79z+z^2} & \dfrac{10^{-5}(-8.33-0.87z+9.19z^2)}{0.83+1.79z+z^2} \\ \dfrac{10^{-5}(-9.19-0.87z+8.33z^2)}{0.83+1.79z+z^2} & \dfrac{0.83+1.79z+1.00z^2}{0.83+1.79z+z^2} \end{bmatrix}. \tag{21}$$

For the eighth section, we have

$$\theta_8(z) = \begin{bmatrix} \dfrac{0.66+0.15z+1.02z^2}{0.64+0.14z+z^2} & \dfrac{10^{-4}(0.78-2.84z+2.06z^2)}{0.64+0.14z+z^2} \\ \dfrac{10^{-4}(1.06-2.85z+2.78z^2)}{0.64+0.14z+z^2} & \dfrac{0.64+0.14z+1.01z^2}{0.64+0.14z+z^2} \end{bmatrix}. \tag{22}$$

For the fifteenth section, we have

$$\theta_{15}(z) = \begin{bmatrix} \dfrac{0.95-1.86z+1.00z^2}{0.93-1.86z+z^2} & \dfrac{10^{-2}(0.83-1.95z+1.12z^2)}{0.93-1.86z+z^2} \\ \dfrac{10^{-2}(1.12-1.95z+0.93z^2)}{0.93-1.86z+z^2} & \dfrac{0.93-1.87z+1.02z^2}{0.93-1.86z+z^2} \end{bmatrix}. \tag{23}$$

The sixteenth section, combining z_{16} and z_{17} has

$$\theta_{16}(z) = \begin{bmatrix} \dfrac{1.66-2.91z+1.25z^2}{0.65-1.65z+z^2} & \dfrac{1.08-2.39z+1.32z^2}{0.65-1.65z+z^2} \\ \dfrac{1.32-2.39z+1.08z^2}{0.65-1.65z+z^2} & \dfrac{1.66-2.91z+1.25z^2}{0.65-1.65z+z^2} \end{bmatrix}$$

and a degree-zero load reflection coefficient $S_{16} = 0.98$, an indication that no more lattice extractions are possible, and representing cochlea section losses and the helicotrema.

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tting the complex conjugates,

$$z_3 = 0.93 - 0.25j$$
$$z_6 = -0.17 - 0.92j$$
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, z_8^*), (z_{15}, z_{15}^*), and (z_{16}, z_{17})
remaining sections are given

$$\frac{8.33-0.87z+9.19z^2)}{.83+1.79z-z^2} \quad \frac{79z+1.00z^2}{-1.79z+z^2} \quad . \quad (21)$$

$$\frac{.78-2.84z-2.06z^2)}{.64+0.14z+z^2} \quad \frac{14z+1.01z^2}{-0.14z+z^2} \quad . \quad (22)$$

$$\frac{0.83-1.95z+1.12z^2)}{0.93-1.86z-z^2} \quad \frac{1.87z+1.02z^2}{-1.86z+z^2} \quad . \quad (23)$$

$$\frac{-2.39z+1.32z^2}{5-1.65z+z^2} \quad \frac{-2.91z+1.25z^2}{5-1.65z+z^2}$$

= 0.98, an indication that no
ting cochlea section losses and

5. Delay-free loop lattices

One of the major problems in the cascade realization of digital filters is the occurrence of delay-free loops between the sections. A delay-free loop is an internal loop without a delay element. Thus, in an implementation with delay-free loops, it would be impossible to carry out the arithmetic operations required by these loops because a variable's new value would depend on that value itself. To eliminate delay-free loops from the cochlea structure (i.e, make the structure computable), we perform a local normalization on the lattices by extracting right-matched (or left-matched) lossless passive sections following the schemes presented in [2], [19] for the case of complex degree-one sections.

The normalization begins with the multiplication on the right (or on the left) of the overall transfer scattering matrix by a constant J-unitary, J-expansive transfer scattering matrix θ_c, which is then factored and distributed among the sections. Extending the results obtained in [2], [19] to degree-two real sections, it can be easily established that, after distribution, the losslessness, the passivity, and the realness properties, as well as the structure of the sections, are preserved. Thus, after distribution, the sections are characterized by J-unitary, J-expansive transfer scattering matrices of the form $\theta_x(z)\theta_y(z)\theta_c$ given in (5),(8), and (25), where the new vectors x and y are calculated in terms of the old x and y and the entries of θ_c. Consequently, we take $\theta_x(z)\theta_y(z)\theta_c$ as the basic matrix from which we derive conditions to guarantee computability.

We proceed from the fact that any constant lossless passive lattice section can be described by a J-unitary transfer scattering matrix of the form given in (25) and that it can be factored as a product of n J-unitary, J-expansive transfer scattering matrices.

$$\theta_c = \begin{bmatrix} 1 & C \\ C^* & 1 \end{bmatrix} \begin{bmatrix} \theta_c^1 & 0 \\ 0 & \theta_c^2 \end{bmatrix} . \quad (25)$$

where $1 - CC^* > 0$ to guarantee that θ_c is J-expansive and θ_c^1 and θ_c^2 must satisfy (26) for θ_c to be J-unitary. The corresponding signal-flow diagram is shown in Figure 2.

$$\theta_c^1 = \theta_c^2 = \frac{1}{\sqrt{1 - CC^*}} . \quad (26)$$

For computability using right-matched normalization, it is necessary and sufficient to choose C so that the (1, 2) entry of $\theta_x(z)\theta_y(z)\theta_c$ is zero at $z = \infty$ for finite transmission zeros [2], [19]. Although here we only treat this case, we point out that similar results can be obtained for left-matched normalization by forcing the (2, 1) entry to be zero at $z = 0$ for nonzero transmission zeros. Let

$$\theta_{xy}(z) = \theta_x(z)\theta_y(z) = \begin{bmatrix} \theta_{11}(z) & \theta_{12}(z) \\ \theta_{21}(z) & \theta_{22}(z) \end{bmatrix} . \quad (27)$$

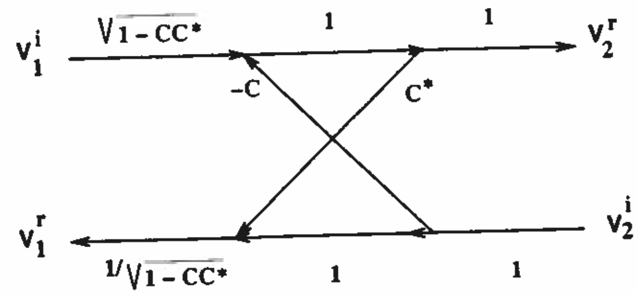


Figure 2. Signal-flow graph of the constant lossless passive section of equation (25).

It is easily seen that the (1, 2) term of $\theta_{xy}(z)\theta_c$ is forced to zero at infinity by the choice

$$C = -\left[\frac{\theta_{12}(z)}{\theta_{11}(z)}\right]_{z=\infty}, \tag{28}$$

which is real and satisfies $1 - CC^* = 1 - C^2 > 0$ because

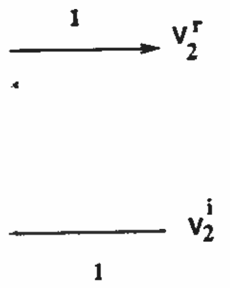
$$1 - C^2 = 1 - \frac{\theta_{12}^2}{\theta_{11}^2} > 0 \tag{29}$$

by virtue of the J-expansive property of $\theta_{xy}(z)$. Using the expressions of $\theta_{11}(z)$ and $\theta_{12}(z)$ at $z = \infty$ in terms of the vectors x and y and the zeros of transmission a and a^*, C is calculated as follows:

$$C = \frac{\left[(1-a^*) + |x_1|^2\right] y_1 y_2^* + \left[(1-a) - |y_2|^2\right] x_1 x_2^*}{\left[(1-a^*) + |x_1|^2\right]\left[(1-a) + |y_1|^2\right] - x_1 x_2^* y_1^* y_2}. \tag{30}$$

6. VLSI realization of cochlea-like lattices

Our cochlea lattice structure is in a very appropriate form for VLSI construction of cochlea like filters, essentially being a cascade of degree-two real sections. By obtaining the semistate equations for each section and then realizing these equations in canonical VLSI circuits (using two delays via integrators), a pipelined form of VLSI layout can be obtained. These circuits can be realized with switched current mode circuits for low bias voltages. Although scarcely mentioned in the literature, these circuits operate on principles familiar from the theory of dynamic-RAM [4], where charge is transferred to the parasitic gate-source capacitance of an MOS transistor to control the drain current. By appropriately transferring the charge, one can achieve various classes of (discretized) integrators [6] for continuous-time systems and delay elements for discrete-time systems [5] leading to excellent filter designs.



sive section of equation (25).

orced to zero at infinity by the

, (28)

because

0 (29)

Jsing the expressions of $\theta_{11}(z)$
y and the zeros of transmission

$$\frac{-a) - |y_2|^2] x_1 x_2^*}{y_1|^2] - x_1 x_2^* y_1^* y_2} \quad (30)$$

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The switched current mode systems studied to date are primarily linear ones, whereas we would like to eventually capitalize upon nonlinearities. Toward this end we find that translinear filter circuits offer a promising alternative for filter design, as they are based upon current mode operation to yield a large class of nonlinearities, including products and absolute values. A number of valuable synthesis methods have been reported in the literature, with each differing in the basic building functions used to design the filter. One synthesis approach is to use an integrator as the basic function block and to design the filter by choosing the right number of such integrators and making the appropriate network connections [14]. Another synthesis approach is to use incomplete translinear loops as the basic function blocks [12]. These are generated by applying a transformation to the voltage mode state space description of the filter to be implemented and mapping the resulting equations on the basic function blocks. Drawing upon these synthesis techniques, generalizations of translinear circuits and the development of a theory of synthesis based upon them and the semistate representation should lead to good design methods for obtaining practical cochlea-like lattices in VLSI form.

7. Discussion

In this paper, based upon Richard's function extractions, we presented a pipelined synthesis of real lossless cochlea-like lattices suitable for cochlea assessment and VLSI implementation. In the same context, we point out that this particular structure could be useful in advancing and improving the methods that are currently used in many related fields, such as language learning, hearing aid design, and in the exploration of signal processing techniques and coding models for the auditory system that could be supported by VLSI circuits.

Note that the cochlea is inherently lossy, and its loss term is represented here by the terminating load section. This loss term could be distributed among the sections using the lossy synthesis technique derived in [19]. This method generates lossy sections while still preserving the realness and the passivity properties of the structure and, at the same time, allows for its realization with no delay-free loops. For completeness, we mention that the synthesis method could be nicely extended to extract the delay term $e^{-n(z)}$ and, thus, preserve the cochlea structure given in (16). Toward this, a generalization of the Youla-Pillai-Shim method seems to be ideal for handling such a case, therefore allowing for an estimation of the transfer function of the cochlea as an irrational function.

Additionally, as far as the mechanical behavior of the cochlea is concerned, although most of the cochlear mechanics measurements reported by researchers point to a linear or quasi-linear behavior, Kemp echoes have been shown to possess nonlinear properties contributed in part by the feedback from the hair-cells. Thus, it is felt that the linear synthesis treated here could be improved upon by adapting nonlinear treatments such as the Parker-Perry method presented in [11] to lead to the development of nonlinear lattice synthesis algorithms. Finally, we point out that upgrading the lattice structure to two dimensions using the lattice parameter model of Parker and Kayran [10] or to three dimensions might bring noticeable improvements in the characterization of the cochlea.

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This paper is dedicated to the memory of Sydney R. Parker.

Appendix

Here we give the transfer scattering matrices for the rest of the cochlea lattice sections.

$$\theta_2(z) = \begin{bmatrix} \frac{0.77+1.72z+1.00z^2}{0.77-1.72z+z^2} & \frac{10^{-4}(1.35-0.26z-1.09z^2)}{0.77-1.72z+z^2} \\ \frac{10^{-4}(-1.09-0.26z+1.35z^2)}{0.77-1.72z+z^2} & \frac{0.77+1.72z+1.00z^2}{0.77-1.72z+z^2} \end{bmatrix} \tag{31}$$

$$\theta_3(z) = \begin{bmatrix} \frac{0.90+1.46z+1.00z^2}{0.90+1.46z+z^2} & \frac{10^{-5}(6.38+4.98z-11.37z^2)}{0.90+1.46z+z^2} \\ \frac{10^{-5}(-11.37+4.98z-6.38z^2)}{0.90+1.46z+z^2} & \frac{0.90+1.46z+1.00z^2}{0.90+1.46z+z^2} \end{bmatrix} \tag{32}$$

$$\theta_4(z) = \begin{bmatrix} \frac{0.76+1.45z+1.00z^2}{0.76+1.45z+z^2} & \frac{10^{-5}(-12.88+0.23z+12.64z^2)}{0.76+1.45z+z^2} \\ \frac{10^{-5}(12.64+0.23z-12.88z^2)}{0.76+1.45z+z^2} & \frac{0.76+1.45z+z^2}{0.76+1.45z+z^2} \end{bmatrix} \tag{33}$$

$$\theta_5(z) = \begin{bmatrix} \frac{0.82+1.04z+z^2}{0.82+1.04z+z^2} & \frac{10^{-4}(-1.00+1.33z-0.32z^2)}{0.82+1.04z+z^2} \\ \frac{10^{-4}(-0.32+1.33z-1.00z^2)}{0.82+1.04z+z^2} & \frac{0.82+1.04z+z^2}{0.82+1.04z+z^2} \end{bmatrix} \tag{34}$$

$$\theta_6(z) = \begin{bmatrix} \frac{0.72+0.69z+1.00z^2}{0.72+0.69z+z^2} & \frac{10^{-4}(0.62-2.91z+2.29z^2)}{0.72+0.69z+z^2} \\ \frac{10^{-4}(2.29-2.91z+0.62z^2)}{0.72+0.69z+z^2} & \frac{0.72+0.69z+1.00z^2}{0.72+0.69z+z^2} \end{bmatrix} \tag{35}$$

$$\theta_7(z) = \begin{bmatrix} \frac{0.88+0.33z+1.00z^2}{0.88+0.33z+z^2} & \frac{10^{-4}(0.48+1.74z-2.22z^2)}{0.88+0.33z+z^2} \\ \frac{10^{-4}(-2.22+1.74z+0.48z^2)}{0.88+0.33z+z^2} & \frac{0.88+0.33z+1.00z^2}{0.88+0.33z+z^2} \end{bmatrix} \tag{36}$$

$$\theta_9(z) = \begin{bmatrix} \frac{0.67-0.41z+1.00z^2}{0.67-0.41z+z^2} & \frac{10^{-4}(0.15+3.89z-4.04z^2)}{0.67-0.41z+z^2} \\ \frac{10^{-4}(-4.04+3.89z+0.15z^2)}{0.67-0.41z+z^2} & \frac{0.67-0.41z+1.00z^2}{0.67-0.41z+z^2} \end{bmatrix} \tag{37}$$

$$\theta_{10}(z) = \begin{bmatrix} \frac{0.99-0.75z+1.00z^2}{0.99-0.75z+z^2} & \frac{10^{-5}(0.21+0.67z-0.88z^2)}{0.99-0.75z+z^2} \\ \frac{10^{-5}(-0.88+0.67z+0.21z^2)}{0.99-0.75z+z^2} & \frac{0.99-0.75z+1.00z^2}{0.99-0.75z+z^2} \end{bmatrix} \tag{38}$$

$$\theta_{11}(z) = \begin{bmatrix} \frac{0.68-0.91z+1.00z^2}{0.68-0.91z+0.99z^2} & \frac{10^{-4}(-6.01+15.19z-9.17z^2)}{0.68-0.91z+0.99z^2} \\ \frac{10^{-4}(-9.17+15.19z-6.01z^2)}{0.68-0.91z+0.99z^2} & \frac{0.68-0.91z+1.00z^2}{0.68-0.91z+0.99z^2} \end{bmatrix} \tag{39}$$

$$\theta_{12}(z) = \begin{bmatrix} \frac{0.90-1.45z+1.00z^2}{0.90-1.45z+z^2} & \frac{10^{-4}(-2.69+5.65z-2.95z^2)}{0.90-1.45z+z^2} \\ \frac{10^{-4}(-2.95+5.65z-2.69z^2)}{0.90-1.45z+z^2} & \frac{0.90-1.45z+1.00z^2}{0.90-1.45z+z^2} \end{bmatrix} \tag{40}$$

$$\theta_{13}(z) = \begin{bmatrix} \frac{0.76-1.58z+1.00z^2}{0.76-1.58z+z^2} & \frac{10^{-4}(-2.15+5.59z-3.43z^2)}{0.76-1.58z+z^2} \\ \frac{10^{-4}(-3.43+5.59z-2.15z^2)}{0.76-1.58z+z^2} & \frac{0.76-1.58z+1.00z^2}{0.76-1.58z+z^2} \end{bmatrix} \tag{41}$$

$$\theta_{14}(z) = \begin{bmatrix} \frac{0.97-1.81z+1.00z^2}{0.97-1.81z+z^2} & \frac{10^{-5}(-2.29-0.52z+2.81z^2)}{0.97-1.81z+z^2} \\ \frac{10^{-5}(2.81-0.52z-2.29z^2)}{0.97-1.81z+z^2} & \frac{0.97-1.81z+1.00z^2}{0.97-1.81z+z^2} \end{bmatrix} \tag{42}$$

References

[1] H. M. Ahmed, M. Morf, D. T. Lee, and P. H. Ang, A VLSI speech analysis chip set based on square-root normalized ladder forms, *Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing*, Atlanta, vol. 1, pp. 648–653, March 1981.

[2] E. Deprettere and P. Dewilde, Orthogonal cascade realization of real multiport digital filters, *Circuit Theory and Applications*, vol. 8, No. 3, pp. 245–272, July 1980.

[3] P. Gomez, V. Rodellar, and R. W. Newcomb, Kemp echo digital filters, *Proceedings of the 1st International Conference on Advances in Communications and Control Systems*, pp. 38–46, Washington, D.C., June 1987.

[4] H. Haznedar, *Digital Microelectronics*, Benjamin/Cummings, Redwood City, CA, 1991.

[5] J. B. Hughes, I. C. Macbeth, and D. M. Pattullo, Switched current filters, *IEE Proceedings-G*, vol. 137, no. 2, pp. 156–162, April 1990.

[6] J. B. Hughes, I. C. Macbeth, and D. M. Pattullo, Second generation switched-current signal processing, *Proceedings of the IEEE International Symposium on Circuits and Systems*, pp. 2805–2808, New Orleans, May 1990.

[7] D. T. Kemp, Stimulated acoustic emissions from within the human auditory system, *Journal of the Acoustical Society of America*, vol. 64, no. 5, pp. 911–913, September 1978.

[8] Y. C. Lim and S. R. Parker, On the synthesis of lattice parameter digital filters, *IEEE Transactions on Circuits and Systems*, vol. CAS-31, no. 7, pp. 593–601, July 1984.

[9] R. W. Newcomb, N. El Leithy, V. Rodellar, P. Gomez, and M. Cordoba, Computable minimum lattice-like ARMA synthesis, *IEEE Transactions on Circuits and Systems.*, vol. 35, no. 5, pp. 577–583, May 1988.

[10] S. R. Parker and A. H. Kayran, Lattice parameter autoregressive modeling of 2-D fields, Part I: The quarter plane case, *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 32, pp. 872–885, August 1984.

[11] S. R. Parker and F. A. Perry, A discrete ARMA model for nonlinear system identification, *IEEE Transactions on Circuits and Systems*, vol. CAS-28, no. 3, pp. 224–233, March 1981.

[12] D. Perry and G. W. Roberts. Log-domain filters based on LC ladder synthesis, *Proceedings of the IEEE International Symposium on Circuits and Systems*, vol. 1, pp. 311–314, 1995.

[13] S. U. Pillai and T. I. Shim, *Spectrum Estimation and System Identification*, Springer-Verlag, Berlin and New York, 1993.

[14] M. Punzenberger and C. Enz, Low-voltage companding current-mode integrators, *Proceedings of the IEEE International Symposium on Circuits and Systems*, vol. 3, pp. 2112–2115, 1995.

[15] L. Sellami, *Kemp echo lattices incorporating hair-cell nonlinearities*, Doctoral dissertation, Electrical Engineering Department, University of Maryland, College Park, MD, 1992.

[16] L. Sellami and R. W. Newcomb, Ear-type analog and digital systems, *Recent Research Developments in Circuits and Systems*, vol. 1, pp. 59–83, 1996.

[17] L. Sellami and R. W. Newcomb, Synthesis of ARMA filters by real lossless digital lattices, *IEEE Transactions on Circuits and Systems II*, vol. 43, no. 5, pp. 379–386, May 1996.

[18] L. Sellami and R. W. Newcomb, A digital scattering model of the cochlea, *IEEE Transactions on Circuits and Systems I*, vol. 44, no. 2, pp. 174–180, February 1997.

[19] L. Sellami and R. W. Newcomb, Lossy synthesis of digital lattice filters, *IEEE Transactions on Circuits and Systems II*, to appear.

[20] L. Sellami and R. W. Newcomb, ARMA system identification: The case of a digital scattering cochlea model, manuscript prepared.

[21] L. Sellami, R. W. Newcomb, V. Rodellar, and P. Gomez, A digital model for cochlea characterization, *Proceedings of the 5th International Conference on Biomedical Engineering*, pp. 15–16, Spain, September 1994.

[22] H. Wakita, Direct estimation of the vocal tract shape by inverse filtering of acoustic speech waveforms, *IEEE Transactions on Audio and Electrostatics*, vol. 21, pp. 417–427, October 1973.

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