# Linear Bilateral CMOS Resistor for Neural-Type Circuits

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Abstract—A previous CMOS bilateral linear resistor is analyzed and shown to be reducible from four to two transistors with improved linearity. This is developed for neural-type circuits to allow its use in emulating both excitatory and inhibitory voltage variable synapses. Simulation results using parameters of MOSIS transistors are presented to verify the theory.

#### I. INTRODUCTION

In VLSI realization of Artificial Neural Networks (ANNs) [1] weights can be set by resistor values [2], in which case it is important to have good VLSI bilateral resistors. Besides that, if the weights are to be adapted it is advantageous to be able to adjust the resistances by controlling voltages. Consequently we present here a suitable class of simple VLSI bilateral, voltage adjustable resistors which are essentially linear. Previously in most standard MOS processes monolithic resistors were implemented as polysilicon or diffusion strips acting as passive devices but recently many authors have presented unidirectional linear active resistors containing MOS devices [3]-[13]. However, the nonlinearity of the MOSFET prohibits the use of large signals and limits its signal swing. To circumvent these limitations different circuit structures are introduced [4], [8], [10], [11]. Since these active resistor implementations usually take less die area as compared to passive resistors we present a generalized active linear bilateral enhancement-mode CMOS device resistor. This resistor is based on Youssef's [10], [11] circuit, which is a bilateral version of that of Moon [8]. This is improved in terms of elimination of two bias batteries which were inserted to insure that the diode-connected transistors operate in the linear region, while her theory is also improved. This improvement allows us to simplify the circuit from four transistors to two.

The circuit is presented in section II with resulting equations given in section III including substrate bias effects. In section IV design criteria for the two- and

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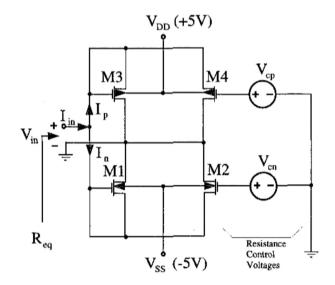


Fig. 1. The linear bilateral four-transistor CMOS resistor.

four-transistor circuits are derived. In section V, SPICE simulation results using MOSIS parameters for the two-transistor circuit are given.

### II. THE BILATERAL CMOS RESISTOR

The basic configuration of the linear bilateral CMOS resistor of Youssef [10], without the substrate offset bias voltage sources, is shown in Figure 1. It consists of four MOS transistors and two control voltage sources which could be voltage controlled sources (VCVS) for use in ANN weight adaptation. The pair of NMOS transistors  $M_1$  and  $M_2$  is connected as in the unilateral resistor of [8] to cover positive  $V_{in}$  while the other pair of PMOS transistors  $M_3$  and  $M_4$  is similarly connected to cover negative range of input voltage  $V_{in}$ . The resistance of the circuit is controlled by  $V_{cp}$  for the PMOS pair and by  $V_{cn}$  for the NMOS pair as illustrated in Figure 1. The substrates of the PMOS transistors are connected to  $V_{DD}(+5V)$  and that of the NMOS transistors are connected to  $V_{SS}(-5V)$ .

The expression for the total input current  $I_{in}$  of the transistor circuit of Figure 1 is obtained by taking the sum of two currents through NMOS and PMOS pairs and

is given by

$$I_{in} = I_n + I_p \tag{1}$$

where  $I_n$  is the current of the NMOS transistor pair and  $I_p$  the current of the PMOS transistor pair. The applied input voltage  $V_{in}$  can be either positive or negative in the range

$$V_{SS} \le V_{in} \le V_{DD} \tag{2}$$

and determines which of the NMOS or the PMOS transistors draw current. Note that when  $V_{in}$  is positive  $M_3$  is turned off while the other transistors draw currents with  $M_1$  in saturation. The presence of  $M_4$  compensates for the substrate bias batteries used by Youssef and Moon. As in their cases, the nonlinearity of  $M_2$  cancels that of  $M_1$  but now also that of  $M_4$  over an input voltage range determined by the control voltage  $V_{cn}$ . By symmetry the same effects occur for negative  $V_{in}$ .

### III. CIRCUIT ANALYSIS

Here we analyze the circuit of Figure 1 for the case of positive input voltage  $V_{in}$ ; the other case results by symmetry. From the start we will assume  $V_{cp} < 2V_{th4}$  and  $2V_{th2} < V_{cn}$  in order to guarantee the possibility of linearization. As this forces  $V_{cp} < V_{th4}$  then  $M_4$  is operating in the Ohmic region (see Appendix 1 for the proof). The drain of  $M_4$  is at ground and its current, designated by  $I_{d4p}$ , for the Ohmic region of interest is

$$I_{d4p} = -\frac{KP_p}{2} \frac{W_4}{L_4} \left[ V_{in}^2 - 2(V_{cp} - V_{th4}) V_{in} \right] (1 - \lambda_p V_{in})$$
(3)

where  $W_4$  is the transistor width,  $L_4$  the length and  $\lambda_p$  the channel length modulation parameter of  $M_4$ . We also note the dependence of the threshold voltage  $V_{th4}$  of  $M_4$  on the input voltage  $V_{in}$ 

$$V_{th4}(V_{in}) = VTO_p - \gamma_p \left[ \sqrt{\phi + V_{dd} - V_{in}} - \sqrt{\phi} \right]$$
 (4)

Now Equation (1) becomes

$$I_{in} = I_{d1} + I_{d2} - I_{d4p} \tag{5}$$

where  $I_{d1}$  and  $I_{d2}$  are the drain currents of  $M_1$  and  $M_2$ . There are three different regions based on the value of the input voltage  $V_{in}$  which determines the regions of operation for the NMOS transistors  $M_1$  and  $M_2$ . The expressions for the total input current  $I_{in}$  for all three regions are rather complicated and space precludes to include the full equations for all the regions. Thus, we derive the expression for region I only in Appendix 2. Here we give the simplified forms suitable for design purposes by setting  $\lambda_n = \lambda_p = 0$ .

A. Region I:  $0 \le V_{in} \le V_{th1}$ 

Here  $V_{th1} = V_{th2}$  is the threshold voltage for the NMOS transistors  $M_1$  and  $M_2$  given as

$$V_{th1} = V_{th2} = VTO_n + \gamma_n \left[ \sqrt{\phi - V_{ss}} - \sqrt{\phi} \right]$$
 (6)

In this range of  $V_{in}$   $M_1$  remains in the cut-off region and  $M_2$  is forced to the Ohmic region by assuming  $V_{cn} > 2V_{th2}$ . Their drain currents  $I_{d1}$  and  $I_{d2}$  are

$$I_{d1} = 0$$

$$I_{d2} = \frac{KP_n}{2} \frac{W_2}{L_2} \left[ 2(V_{cn} - V_{th2})V_{in} - V_{in}^2 \right] (1 + \lambda_n V_{in})$$
(7)

where  $W_2$  and  $L_2$  are the channel width and length,  $KP_n$  is the transconductance parameter,  $\lambda_n$  the channel length modulation parameter and  $V_{th2} = V_{th1}$  the threshold voltage for  $M_2$  given by Equation (6). Dropping the  $\lambda_n$  dependence we get

$$I_{in} = \left[\frac{KP_p}{2} \frac{W_4}{L_4} - \frac{KP_n}{2} \frac{W_2}{L_2}\right] V_{in}^2$$

$$+ \left[2 \frac{KP_n}{2} \frac{W_2}{L_2} (V_{cn} - V_{th2}) - 2 \frac{KP_p}{2} \frac{W_4}{L_4} (V_{cp} - V_{th4}(0))\right] V_{in}$$
(8)

# B. Region II: $V_{th1} \leq V_{in} \leq V_{cn} - V_{th2}$

For this region  $M_1$  will be in saturation because the gate is connected to the drain and  $V_{in} \geq V_{th1}$ . Transistor  $M_2$  will be in the Ohmic region for the assumed range of  $V_{cn} > 2V_{th2}$ . Thus, its drain current  $I_{d2}$  stays the same as in (7). Setting  $\lambda_n = 0$  the drain current  $I_{d1}$  is written as

$$I_{d1} = \frac{KP_n}{2} \frac{W_1}{L_1} \left[ (V_{in} - V_{th1})^2 \right]$$
 (9)

where  $W_1$  and  $L_1$  are the channel width and length. The total input current  $I_{in}$  is

$$I_{in} = \left[ \frac{KP_n}{2} \left( \frac{W_1}{L_1} - \frac{W_2}{L_2} \right) + \frac{KP_p}{2} \frac{W_4}{L_4} \right] V_{in}^2$$

$$+ \frac{KP_n}{2} \frac{W_1}{L_1} V_{th1}^2$$

$$+ \left[ -2 \frac{KP_n}{2} \frac{W_1}{L_1} V_{th1} + 2 \frac{KP_p}{2} \frac{W_2}{L_2} (V_{cn} - V_{th2}) \right] V_{in}$$

$$- \left[ 2 \frac{KP_p}{2} \frac{W_4}{L_4} (V_{cp} - V_{th4}) \right] V_{in}$$

$$(10)$$

# C. Region III: $V_{cn} - V_{th2} < V_{in}$

Transistor  $M_1$  remains in saturation, therefore its drain current  $I_{d1}$  is the same as in (9). However  $M_2$  goes into the saturation region and its drain current becomes

$$I_{d2} = \frac{KP_n}{2} \frac{W_2}{L_2} \left[ \left( V_{cn} - V_{th2} \right)^2 \right]$$
 (11)

TABLE I

Various SPICE simulated  $R_{eq}$  resistance values obtained with  $V_{cn}$  and  $V_{cp}$   $= -V_{cn} + V_{thn} + V_{thp} \text{ as parameters, } L_2 = W_2 = L_4 = 10\mu, W_4 = W_2 * 2.646 = W_2 * \frac{KP_n}{KP_p}, V_{thn} = 1.173V, V_{thp} = -1.89V = V_{th4}(0).$ 

Resistance	Control Voltage	Control Voltage	
Value	$V_{cn}$	$V_{cp}$	
$R_{eq}$ in $\Omega$	Parameter	Parameter	
12.5K	12V	-2.16V	
6.5K	3V	-3.16V	
4.4K	4V	-4.16V	
3.3K	5V	-5.16V	

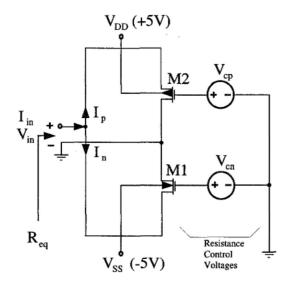


Fig. 2. The linear bilateral two-transistor CMOS resistor

Therefore the total input current can be written as

$$I_{in} = \left[ \frac{KP_p}{2} \frac{W_4}{L_4} - \frac{KP_n}{2} \frac{W_2}{L_2} \right] V_{in}^2$$

$$+ \frac{KP_n}{2} \left[ \frac{W_1}{L_1} V_{th1}^2 + \frac{W_2}{L_2} (V_{cn} - V_{th2})^2 \right] V_{in}$$

$$+ \left[ -2 \frac{KP_n}{2} \frac{W_1}{L_1} V_{th1} - 2 \frac{KP_p}{2} \frac{W_4}{L_4} (V_{cp} - V_{th4}) \right] V_{in}$$

$$(12)$$

## IV. DESIGN CRITERIA

By a proper choice of transistor parameters we show that almost linear relationships between the input voltage,  $V_{in}$ , and the input current,  $I_{in}$ , can be obtained in all regions. In order to reduce nonlinear effects, we eliminate the  $V_{in}^2$  term in region II by forcing the following condition to hold

$$\frac{W_2}{L_2} = \frac{W_1}{L_1} + \frac{KP_p}{KP_n} \frac{W_4}{L_4} \tag{13}$$

At this point we note that if we choose  $W_1 = 0$  we obtain linearity in all three regions and we have the advantage

TABLE II

Various SPICE determined  $R_{eq}$  values obtained with  $(L_2, L_4)$  and  $(W_2, W_4)$  as parameters and with  $V_{cn} = 5$ V and  $V_{cp} = -V_{cn} + V_{thn} + V_{thp} = -5.16$ V

$R_{eq} \Omega$	Parameter $L_2$	Parameter $W_2$	Parameter L <sub>4</sub>	Parameter W <sub>4</sub>
30K	$90\mu$	$10\mu$	$90\mu$	$26.46\mu$
25K	$70\mu$	$10\mu$	$70\mu$	$26.46\mu$
15K	$50\mu$	$10\mu$	$50\mu$	$26.46\mu$
10K	$30\mu$	$10\mu$	$30\mu$	$26.46\mu$
3.3K	$10\mu$	$10\mu$	$10\mu$	$26.46\mu$

that  $M_1$  is deleted (and with it, by symmetry,  $M_3$ ). Thus, the optimal design has

$$\frac{W_4}{L_4} = \frac{KP_n}{KP_p} \frac{W_2}{L_2} \quad and \quad W_1 = W_3 = 0 \tag{14}$$

The conductance for the two-transistor case in all three regions is

$$G_{+} = K P_{n} \frac{W_{2}}{L_{2}} \left[ (V_{cn} - V_{thn}) - (V_{cp} - V_{thp}) \right]$$
 (15)

Since  $V_{cn}$  and  $V_{cp}$  control the resistance in the first and third quadrant, respectively, we choose  $V_{cp} = -V_{cn} + V_{thn} + V_{thp}(0)$  to balance the effect of differences in NMOS and PMOS threshold voltages over positive and negative ranges of  $V_{in}$ .

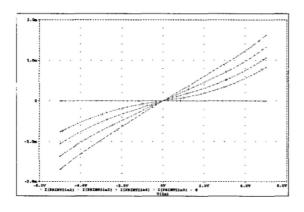


Fig. 3. PSPICE simulated curves for the two-transistor linear bilateral CMOS resistor of Figure 2 with the top right curve for  $V_{cn} = 5$ V and decreasing in 1V step to the bottom curve with  $V_{cn} = 2$ V.

## V. SIMULATION RESULTS

To verify the analysis, the two-transistor circuit of Figure 2 was simulated with PSpice. The input voltage  $V_{in}$  was swept from -5V to +5V and the total input current  $I_{in}$  is plotted in Figure 3. SPICE MOS level 2 parameters are used following MOSIS from run N21H of 04/28/93 for which the key parameters are

 $KP_n=5.048E-5$ ,  $KP_p=1.908E-5$ ,  $VT0_n=0.858$ ,  $VT0_p=-$ 0.889,  $\lambda_n = 1.844 \text{E}-2$ ,  $\lambda_p = 5.012 \text{E}-2$ ,  $\phi = 0.6$ ,  $\gamma_n = 0.198$ ,  $\gamma_p = 0.6289$ . The key design parameters used to get various resistor values are transistor length L and width W, as tabulated in Table II, and the controlling voltage  $V_{cn}$ which is varied from 3V to 5V in 1V steps as a global parameter (see Table I). As shown in Table II, resistance values from  $R_{eq} = 3.3K\Omega$  to  $30K\Omega$  are attained. The calculated value of resistance from equation (15) for  $W_2 = 10\mu$ ,  $L_2 = 10\mu$ ,  $W_4 = 26.46\mu$  and  $L_4 = 10\mu$  comes out to be  $2.6K\Omega$  whereas the SPICE simulated value (from Table II) is  $3.3K\Omega$ . This discrepancy is due to the fact that the Early effect was neglected in the derivation of equation (15) for the purpose of simplification. Note also that for  $V_{cn} = 2V$  and 3V, at least one of the constraints  $V_{cn} > 2V_{th2}$  and  $V_{cp} < 2V_{th4}$  is violated, but the curves are very linear in region I.

### VI. Conclusions

In this paper, we have presented a linear bilateral CMOS resistor for neural-type circuits. This resistor is an improved version of Youssef's resistor, which with the number of transistors now reduced to two (instead of four) has improved linearity. It offers an efficient usage of die area since area consumed by it is much smaller compared to similarly sized directly implemented polysilicon or diffusion strip based passive resistors. Also, it is voltage controllable over a suitable dynamic range. Although it is targeted for neural-type circuits to enable its use in both excitatory and inhibitory synapses, it can be used in other VLSI applications. Even though Equation (14) leads in theory to a linear circuit, the simulations show some nonlinear behavior. This nonlinear behavior is due to  $V_{th4}$  being a function of  $V_{in}$  and to the term  $1 + \lambda_n V_{in}$ . The linear range of region II varies proportionally to  $V_{cn} - V_{th2}$ , as can be derived from Equation (10) by substituting  $V_{cn} - V_{th2}$  for the maximum value of  $V_{in}$ , and, thus, region II linearity decreases with decreasing  $V_{cn}$ .

### VII. APPENDICES

1): Here we show that  $M_4$  is in saturation for  $V_{in} > 0$  and  $V_{cp} < V_{th4}(0)$ . Depending upon the value of  $V_{cp}$ ,  $M_4$  can be either in the Ohmic or in the saturation region. The gate to source voltage and drain to source voltage are given by

$$V_{gs} = V_{cp} - V_{in} \quad \text{and} \quad V_{ds} = -V_{in} \tag{16}$$

The condition for  $M_4$  to operate in the Ohmic region is

$$0 \le |V_{ds}| \le |V_{qs}| - |V_{th4}| \tag{17}$$

which leads to

$$V_{cp} \le V_{th4} < 0 \tag{18}$$

The condition for  $M_4$  to operate in the saturation region is

$$0 \le |V_{qs}| - |V_{th4}| \le |V_{ds}| \tag{19}$$

which gives

$$V_{th4} \le V_{cp} < 0 \tag{20}$$

But  $V_{cp} < V_{th4}(0) \le V_{th4}(V_{in})$  is needed to turn on the device for  $V_{in} \ge 0$  (by Equation (4)), therefore  $M_4$  cannot be in saturation.

2) : In terms of the full dependence upon  $V_{in}$ , the equation for the total input current  $I_{in}$  in region I is

$$\begin{split} I_{in} &= \left[\frac{KP_p}{2} \frac{W_4}{L_4} - \frac{KP_n}{2} \frac{W_2}{L_2}\right] V_{in}^2 + \left[2 \frac{KP_n}{2} \frac{W_2}{L_2} (V_{cn} - V_{th2})\right] V_{in} \\ &- \left[2 \frac{KP_p}{2} \frac{W_4}{L_4} \left(V_{cp} - (V_{th0} - \gamma_p \left[\sqrt{\phi + V_{dd} - V_{in}} - \sqrt{\phi}\right])\right)\right] V_{in} \\ &- \left[\lambda_p \frac{KP_p}{2} \frac{W_4}{L_4} + \lambda_n \frac{KP_n}{2} \frac{W_2}{L_2}\right] V_{in}^3 + \left[2\lambda_n \frac{KP_n}{2} \frac{W_2}{L_2} (V_{cn} - V_{th2})\right] V_{in}^2 \\ &- \left[2\lambda_p \frac{KP_p}{2} \frac{W_4}{L_4} \left(V_{cp} - (V_{th0} - \gamma_p \left[\sqrt{\phi + V_{dd} - V_{in}} - \sqrt{\phi}\right])\right)\right] V_{in}^2 \end{split}$$

### REFERENCES

- J. Zurada, Introduction to Artificial Neural Systems, West Publishing Co., St. Paul, MN, 1992.
- [2] S. K. Singh, R. W. Newcomb, P. Gomez and V. Rodellar, "A Means of VLSI Current Controlled Weight Setting in ANNs," Proceedings of the IEEE International Conference on Neural Networks, vol. 4, Perth, pp. 1919-1922, Dec. 1995.
- [3] M. Banu and M. Tsividis, "Floating voltage-controlled resistor in CMOS technology," *Electronics Letters*, Vol. 18, no. 15, pp. 678-679, Jul. 1982.
- [4] K. Nay and A. Budak, "A voltage-controlled resistance with wide dynamic range and low distortion," *IEEE Transactions* on Circuits and Systems, Vol. CAS-30, no. 10, pp. 770-772, Oct. 1983.
- [5] M. Banu and M. Tsividis, "Fully integrated active RC-filter in MOS technology," *IEEE Journal of Solid-State Circuits*, Vol. sc-18, no. 6, pp. 644-651, Dec. 1983.
- [6] I. S. Han and S. B. Park, "Voltage-controlled linear resistor by two MOS transistors and it's application to active RC-filter mos integration," *Proceedings of IEEE*, Vol. 72, no. 11, pp. 1655-1657, Nov. 1984.
- [7] J. N. Babanezhad and G. C. Temes, "A linear NMOS depletion resistor and its application in an integrated amplifier," *IEEE Journal of Solid-State Circuits*, Vol. sc-19, no. 6, pp. 932-938, Dec. 1984.
- [8] G. Moon, M. E. Zaghloul, and R. W. Newcomb, "An enhancement-mode MOS voltage-controlled linear resistor with large dynamical range," *IEEE Transactions on Circuits* and Systems, Vol. 37, no. 10, pp. 365-366, Mar. 1986.
- [9] P. M. Vanpeteghem and G. L. Rice, "New CMOS resistor implementation for linear RC applications," *Electronics Letters*, Vol. 24, no. 5, pp. 288-290, Mar. 1988.
- [10] H. Youssef, R. W. Newcomb, and M. E. Zaghloul, "A CMOS voltage-controlled linear resistor with wide dynamic range," in Proceedings 21st South-eastern Symposium on System Theory, Talahasee, Florida, pp. 680-683, Mar. 1989.
- [11] H. Youssef, "An enhancement-mode CMOS voltage-controlled linear resistor with wide dynamic range," M.S. thesis, Dept. Elect. Eng., Univ. of Maryland, May 1989.
- [12] G. Wilson and P. K. Chan, "Novel voltage-controlled grounded resistor," *Electronics Letters*, Vol. 25, no. 25, pp. 1725–1726, Dec. 1989.
- [13] G. Wilson and P. K. Chan, "CMOS serial/parallel quad resistor," *Electronics Letters*, Vol. 28, no. 3, pp. 335-336, Jan. 1992.