Examples of Multidimensional Optimal Interpolative Functional Artificial Neural Networks

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Abstract – This paper presents examples of a class of optimal interpolative (OI) functional artificial neural networks (FANNs) which process continuous multidimensional signals. These networks embody for the present case the structure of OI networks, previously derived in the literature, which best approximate a nonlinear dynamical system in a Generalized Fock Space (GFS) under input-output training data constraints. Among other applications, these networks are useful in the modeling and identification of the degradation process of image signals occuring while propagating in nonlinear media.

1 Introduction

This paper presents examples of a class of optimal interpolative (OI) functional artificial neural networks (FANNs) which process continuous multidimensional signals. As shown in the generic case [1], to which we briefly allude below, the structure of these networks results from the best approximation of their input-output map in a Generalized Fock Space under data constraints. Sometimes, in the system theory literature, this type of best approximation has been called system identification [2][3].

FANNs are continuous-time and/or continuous-space versions of conventional artificial neural networks (heretofore referred to as ANNs). Conceptually at least, most results obtained from ANNs easily generalize to FANNs except for some phenomena, such as limit cycles, which genuinely depend on the continuous nature of the system.

A unified approach for the implementation of both FANNs and ANNs, based on a Generalized Fock Space (GFS) framework was presented by de Figueiredo and Dwyer in 1980 [4]. In this framework, the input u to the network is assumed to belong to a real abstract Hilbert space H, and the network's input-output map V is represented as an abstract Volterra series in elements of H, belonging to a Generalized Fock Space $F_s(H)$ over H weighted by a sequence s. The space $F_s(H)$ is a reproducing Kernel Hilbert space with a reproducing kernel $K(u,v)$ (see [1] for details). In the framework just mentioned, the implementation of a neural network map V is specified by a set of interpolative constraints $V(u_i) = y_i$, where (u_i, y_i), $i = 1, \ldots, m$ constitute the training data. This implementation is obtained by projecting V into the span of the representers of the point evaluation functionals $K(u_i, .)$ in $F_s(H)$ corresponding to the training points (in H) $u_1, \ldots, u_m$. For obvious reasons, the implementation V has been called an optimal interpolative (OI) neural network and can be explicitly written in the form

$$V(.) = \sum_{j=1}^{m} c_j K(u_j, .) \qquad (1)$$

where the coefficients c_j are obtained by requiring that (1) satisfy the interpolating training data constraints.

In the case where H is an Euclidean space E^n, the OI net realization V takes the form of a conventional feedforward ANN with two hidden layers. This OI net was presented in 1990 [5] and its theory and applications have been widely discussed in the literature [6]. In the case where H is $L^2(I)$, $I \subset R^1$, the OI net is a FANN which was analyzed by Zyla and de Figueiredo [7] and reconsidered recently by Newcomb and de Figueiredo [1][2]. In the present paper we consider the class of OI FANNs for which H is $L^2(I^n)$, $I^n \subset R^n$.

In what follows we first briefly recall the derivation of the explicit expression for the OI FANN obtained in [1][3]. Then we illustrate this result by some examples from multidimensional signal processing.

2 Multivariable OI FANNs

We assume that m pairs of representative input-output test functions (equivalent to exemplars in artificial neural networks), $u_j(.)$ and $y_j(.)$ for $j = 1, \ldots, m$, are available, with these functions, along with their K derivatives, being square integrable over I^n. We solve for an optimum operator V having the smallest norm in the following sense:

$$\min \|V_x^{(i)}\|_{F_r} \quad \forall x \in I^n \quad and \quad \forall \quad V_x^{(i)} \in F_r \qquad (2)$$

subject to the data constraints

$$V_x^{(i)}(u_j(.)) = y_j^{(i)}(x) \quad i = 0, \ldots, K \quad j = 1, \ldots, m \qquad (3)$$

We note that in order to have sufficient information to perform an identification, we select the m input functions to be linearly independent over I^n. Following [7] and as developed over I^n in [8], the solution to this equivalent problem is outlined below:

1. Form the $m \times m$ Grammian matrix

$$G = [G_{ij}] = \left[\exp\left[\frac{1}{r}\langle u_i(.), u_j(.)\rangle_{L^2(I^n)}\right]\right] \quad (4)$$

   where, for completeness, we recall that

$$\langle u_i(.), u_j(.)\rangle_{L^2(I^n)} = \int_{x \in I^n} u_i(x) u_j(x) dx \quad (5)$$

   Note that G is nonsingular, since the test input functions are linearly independent.

2. Form the column m-vector of test outputs

$$y_{test}(.) = [y_j(.)] \quad (6)$$

3. Obtain a column m-vector of coefficients

$$c(x) = [c_j(x)] = G^{-1} y_{test}(x) \quad (7)$$

4. Determine the optimum estimate $\hat{V}_x(.)$ of $V_x(.)$

$$\hat{V}_x(.) = \sum_{j=1}^{m} c_j(x) \exp\left[\frac{1}{r}\langle u_j(.), .\rangle_{L^2(I^n)}\right] \quad (8)$$

which is the key equation [4] upon which we base our functional artificial neural network discussed next.

The schematic of the resulting functional neural network is depicted in Figure 1, showing a feed-forward two-layer architecture. The first layer consists of m input neurons, each one processing the same input function $u(.)$, presented to the network, and producing a nonlinear response of exponential form, i.e., $\exp[\langle u_i, u\rangle / r]$. The neural network design is carried out in a supervised manner, i.e., using m representative exemplar pairs $u_j(.)$ $y_j(.)$. The entries of $c(x) = G^{-1} y_{test}(x)$, formed with these pairs, correspond to x-varying synaptic weights, whereas the entries of the Grammian matrix G act in a linear manner as neuron nonlinearities, with the weighted neuron outputs added to give the overall output $y(x)$.

When presented with an arbitrary input (of the class allowed by the system), this neural network produces an output that is an approximation to the output of the dynamical system the neural network is modeling. The network uses information acquired during its training on the exemplars to give the desired output in terms of functionals. As a consequence, the network attempts to incorporate with a best fit the nonlinear dynamics of the system being modeled.

3 Examples

Here we illustrate the key theoretical and design ideas with examples from 2D signal processing.

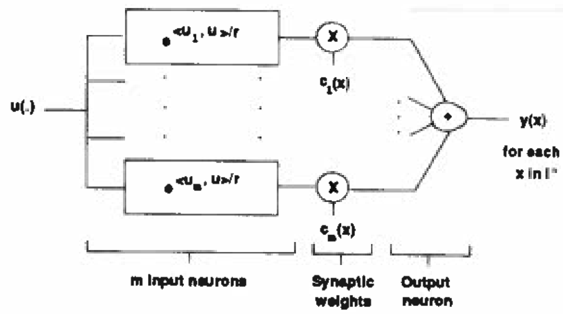

Figure 1: Multidimensional FANN.

3.1 Closed Curve Classification

For this case the exemplar inputs $U_1(x_1, x_2)$ and $U_2(x_1, x_2)$ are surfaces enclosed by closed curves consisting of a circle of radius 1 centered at $(0,0)$ and a square of side 1, also centered at $(0,0)$ (Figure 2). As part of the FANN design, a scheme is set up to detect the inside and the outside of the curves, quantify the result as a 1 or a 0 respectively, and assign the appropriate values to U_1 and U_2.

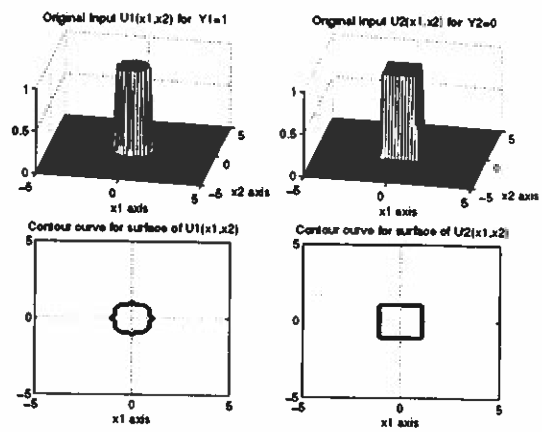

Figure 2: Exemplar inputs $U_1(x_1, x_2)$ and $U_2(x_1, x_2)$

The desired input-output mapping is achieved by forcing the network to associate an output $Y_1(x_1, x_2) = 1$ with the circle and $Y_1(x_1, x_2) = 0$ with the square during the training process, with the aim of enabling the network to generalize the classification to circles and squares of any size, centered anywhere in the plane. The latter is carried out by shifting the center of the curves to $(0,0)$ and normalizing the dimensions to unity.

The generalization ability of the network is tested on a set of circles and squares which we refer to as large and small (see Figures 3 and 4). The associated FANN output values are, as anticipated, 1 for the circles and 0 for the squares.

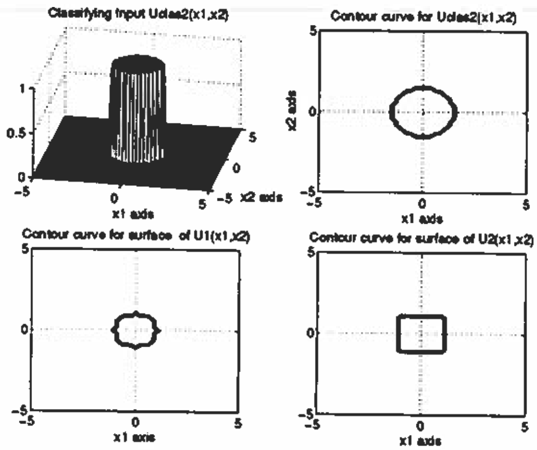

Figure 3: Classification of a large circle

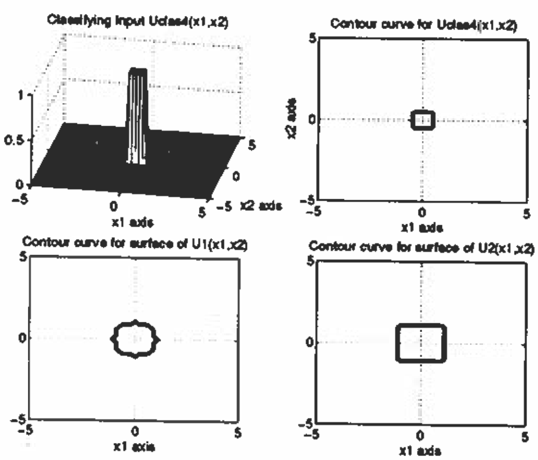

Figure 4: Classification of a small square.

3.2 Pattern Synthesis

Perhaps one of the most powerful and interesting application of the FANN in image processing is pattern synthesis from a database. For example, using either an a priori or a posteriori approach, one can design a FANN for image enhancement and restoration.

As a simple illustration, a FANN is trained to generate a unit circle, centered at $(0,0)$, for an input of 1 over the x_1-x_2 plane and a square of side 1, also centered at $(0,0)$, for an input of 0 over the plane x_1-x_2 (Figures 5 and 6). When the FANN is presented with non-exemplar inputs close to the constants 1 or 0, the FANN produces a circle or a square as expected. However, when presented with a different input, interestingly, as shown in Figures 7, 8, and 9, the FANN combines the circle and the square to create a new pattern.

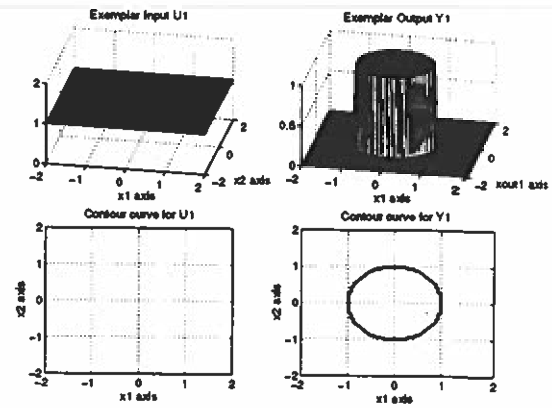

Figure 5: Exemplar input-output pair U_1-Y_1.

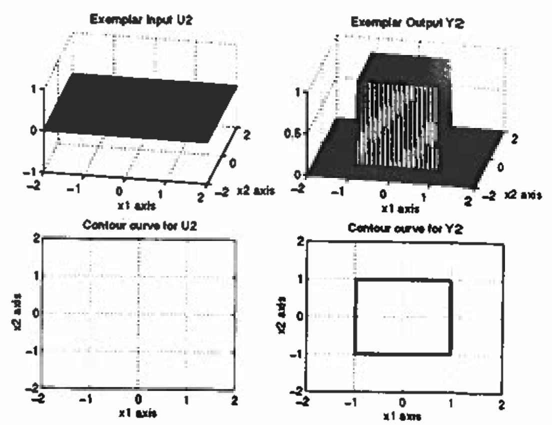

Figure 6: Exemplar input-output pair U_2-Y_2.

4 Discussion

The success of the one-dimensional network introduced in [1, 2] led us to consider the multi-variate case [8]. Thus, in this paper we proposed a neural network approach to the problem of identification of multi-variable nonlinear dynamical systems. The resulting neural network structure, called optimal interpolative multidimensional functional artificial neural network (OI FANN) leads to an optimum characterization of the system via a functional estimation approach.

The proposed approach employs the idea of the reproducing kernel within the mathematical framework of Fock and Hilbert space concepts to approximate nonlinear dynamical systems, specified by representative sets of input-output pairs. In so doing, the approach solves the minimum norm problem in a Bochner space. The use of the reproducing kernel allows the approximation problem to revert back to that of linear systems while still incorporating the nonlinearities for which the Volterra series is tailored. As such it is an attractive alternative to other system modeling techniques [9].

The design of the OI FANN is carried out through a supervised training of the network with exemplar input-output functional pairs and constructs a set of synaptic weights, which are also functionals. When non-exemplar inputs are presented to the network, the latter performs a system identification by associating a Volterra functional input-output map.

The key theoretical and design ideas were exploited in two applications from n-dimensional signal processing, these being closed curve classification and pattern synthesis, the details of which are covered in the full version of the paper.

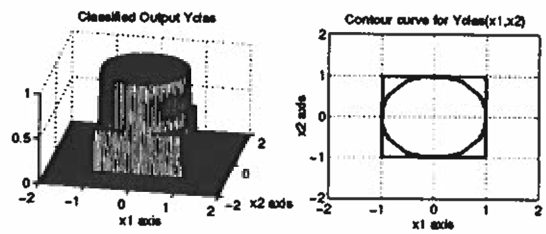

Figure 7: FANN output for $U = 3/4$.

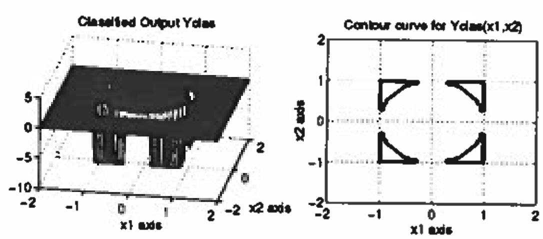

Figure 8: FANN output for $U = 2$

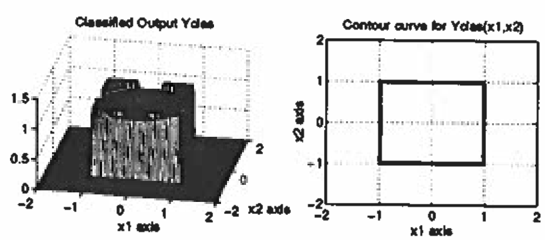

Figure 9: FANN output for $U = -1$.

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