would require the use of switches 1, 3, 5, 10, 7, 9, 8. By now the procedure pattern should have become evident. Repeat the process till all α's, a_{kn_k}'s, and n_k's ($k=1, 2, \cdots, r$) are obtained. The end of the experiment would be indicated by zero output at D. To find the rest of a's, measure $h(t)$ at some $t_1, t_2, \cdots, t_{n_1+n_2+\cdots n_r}$ (no large t restriction); use these and other determined data in (9) to form a set of simultaneous equations from which get a's through solving. Now $h(t)$ of (8) is completely known; Laplace transform it and get $N(s)$, relevant to (1), after suitable manipulation. This completes the parameter estimation of the given system which is known to belong to (1).

Concluding Remarks

When the transfer function $H(s)$, with reference to (1), has a simple pole at $s=0$, the technique of the present note is still applicable except for a trivial modification; namely, a constant function, relevant to $s=0$, should be subtracted from $h(t)$ before proceeding with the estimation process. In a situation where any of the n_k's, with reference to (1), are too large and, therefore, the number of times of differentiation required is too high to be practically handled, it may be more appropriate to carry out the differentiation of the output at D in Fig. 1 graphically. In addition to this situation of high n_k if "large t" is too large for practical consideration, then the entire estimation procedure is to be carried out graphically though the process becomes laborious. The whole procedure is, of course, approximate. Nevertheless, with a number of runs of the experiment, reasonable results can conceivably be obtained.

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The Time-Variable Transformer

The definition of a network [1] allows many ideal devices to be considered as networks, some of which have proven useful for theoretical studies as well as for modeling physical constructs; such an example is the gyrator [2]. Here we wish to draw attention to the definition and possible physical realizations and uses of another network little considered hitherto, namely the time-variable turns-ratio transformer.

Belevitch [3] has given a definition of a multiport transformer which is readily extendable to the time-varying case. Let $\boldsymbol{T}(t) = [t_{ij}(t)]$, the turns-ratio matrix, be an $m \times l$ matrix whose elements are real-valued functions of time t. Further, let v_1 and i_1 denote primary voltage and current l-vectors, and v_2 and i_2 denote secondary voltage and current m-vectors; again all entries are assumed to be real-valued functions of time t. Then an ideal time-variable transformer $(l+m)$-port is defined by the constraint equations

$$v_1(t) = \tilde{T}(t)v_2(t) \quad (1a)$$

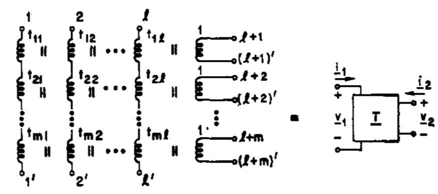
$$i_2(t) = -T(t)i_1(t). \quad (1b)$$

Here a superscript tilde ~ denotes matrix transposition. This transformer is conveniently illustrated as in Fig. 1 where Belevitch's notation is given in (a) and a compact symbolism is used in (b). Of course one is quite at liberty to set arbitrary restrictions on the elements of the turns-ratio matrix $\boldsymbol{T}$ requiring for example time invariance, or periodicity, or infinite differentiability.

Since, from (1), $\tilde{v}_1 i_1 + \tilde{v}_2 i_2 = 0$ for all values of time, it follows that the transformer is a passive and lossless network [4].

As with all ideal and lossless elements the time-variable transformer can only be approximated by physical devices. Nevertheless, just as a potentiometer with a tap setting varied in time by some agency can be considered as a crude time-variable resistor, so the variable auto-transformer can serve as an example of a familiar device approximating the ideal time-varying transformer. A more exact realization of a two-port transformer results from cascading [5] two gyrators of time-variable gyration resistance. Such gyrators can be in turn realized by simply varying the transconductance of the pentodes (through screen grid voltage variations) in the pentode gyrator of Sharpe [6]. Although other electrical realizations can undoubtedly be given, it is worthwhile noting that the concept of a time-variable transformer is a familiar one in mechanical engineering where continuously variable ratio transmissions exist in a number of forms using gears, veebelt, or fluid drives. For example if one changes, as a function of time, the radius of contact $r(t)$ of the friction gears of Fig. 2, one obtains a time-variable transformer, where angular velocity and torque are analogous to voltage and current.

Just as the gyrator is the natural element to adjoin to the common circuit elements (R's, L's, C's, and transformers) for studies of linear, passive, time-invariant, nonreciprocal networks [2], the time-variable transformer is the natural element to adjoin for a study of linear, passive, time-varying networks. Alternatively one can look on the time-variable transformer as giving an a priori extension of time-invariant transformers in the sense that time-varying resistors, inductors, and capacitors are an extension of the corresponding time-invariant elements. Thus theoretical analysis of time-variable networks demonstrates the utility of the time-variable transformer, if only by virtue of the fact that any passive time-variable resistor, capacitor, inductor, or gyrator can be replaced by time-invariant elements and time-varying transformers [7], as partially illustrated in Fig. 3. Because of these equivalences synthesis of passive time-variable networks can now proceed in a systematic form [7], [8]. Although many time-variable networks are not passive, passive ones are of use in modeling many processes as, for example, the transmission characteristics of a time-variable medium (as the ionosphere). Further, active time-variable devices can often be modeled with the assistance of the



(a) (b)

Fig. 1. $(l+m)$-Port transformer.

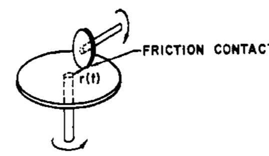
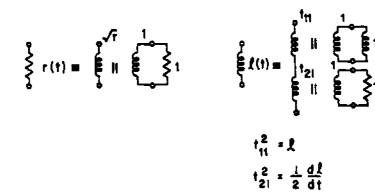


Fig. 2. Variable gear ratio transformer.



(a) (b)

Fig. 3. Passive equivalents using time-variable transformers.

transformer by considering active time-invariant elements in the equivalences illustrated in Fig. 3. Likewise it is clear that in the modeling of mechanical systems by electrical networks the time-variable transformer is a needed element. Conceivably many other modeling problems could require, or at least be simplified by, its inclusion.

In summary, we have defined and pointed out uses and realizations of the ideal time-variable transformer. Although physical realizations must of necessity only be approximate, the uses in modeling and synthesis of time-variable systems make it an object for consideration in its own right. From the observations of this correspondence, the time-variable transformer appears as a useful element to adjoin to the common circuit elements.

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