# A BiCMOS Binary Hysteresis Chaos Generator 

S. Ahmadi and R. W. Newcomb<br>Microsystems Laboratory<br>Electrical Engineering Department<br>University of Maryland<br>College Park, MD 20742 USA<br>email: newcomb@eng.umd.edu

Abstract:
A previous op-amp RC circuit which was proven to give chaotic signals is converted to a BiCMOS design more suitable to integrated circuit realization. The structure results from a degree two differential equation which includes binary hysteresis as its nonlinearity. The circuit is realized by differential (voltage to current) pairs feeding two capacitors, which carry the dynamics, with the key component being a (voltage to current) binary hysteresis circuit due to Linares.

## I. Introduction

One of the fascinating application areas of chaos is in coding for secure communication in which chaos is synchronized between sender and receiver [1]. Hardware implementation of this coding rests upon having hardware that consists of accurately to modify oscillas generators [2][3]. And although our experience is that it is rather easy design formulations. Thus thed circuits to obtain chaos it is more difficult to obtain good chaos and this has led to a sequence of spently been an interest in circuits for generating Much of the circuit results rest upon a study of the circuit IEEE devoted to the topic [4]. by Matsumoto [5]. However, there have been other phildue to Chua and first presented which are those for which designs are based upon philosophies, promising ones among among this class is our previous circuit using bipon hysteresis [6][7][8]. A primary one advantages that it is only degree two, readily proven to givis [9][10]. This circuit has the operation conceptually easy to grasp. Previous realizato give chaos, with its principles of could be considered voltage-mode. But for YLSI usually more convenient, so here wire mode, being dependent upon differential a different realization that is essentially currentthe binary hysteresis with a current-mode device; fortutputs. Key to this is the design of has recently been introduced by Linares [11].

In the following we review the princip binary hysteresis chaos generator, review iple of operation of the generic degree two realizations in the form suitable for VLSI construson it yields chaos and then give circuit technology of BiCMOS (Bipolar-Complemstruction using the most recent fabrication first introduce an ideal normalizementary Metal Oxide Silicon). For this latter we denormalize to values that correspod realization of the describing equations and then

ciated with PSpice is to the standand presents a diode, $F$ M and Q denote
,r can be seen by sarated by binary
$\qquad$ $\times 1$
fying $d<a<b<c$
gree two unstable inary hysteresis to cing we can start te points 2 and 3 05 and along an $\ni$, jump up to 10 , 1 forever. A very lesigned to force the start point in m of the $\mathrm{x}_{2}$ axis Rössler [13] and e basic Li-Yorke ical, as well as $n$ can be used to
generate the spirals in the upper and the lower hysteresis half-planes with the binary hysteresis parameter serving to change the origin of the spirals in the upper half-plane from those in the lower. Thus, rather than using degree four, or even three, this system can be realized as a degree two system!

Although there are uncountably many choices of parameters which will yield chaos, in normalized form the following two differential equations have been rigorously proven to give chaos.

$$
\begin{align*}
& \frac{d x_{1}}{d t}=x_{2}+a_{1} h\left(x_{1}\right)  \tag{1a}\\
& \frac{d x_{2}}{d t}=-x_{1}-2 \sigma x_{2}+a_{2} h\left(x_{1}\right) \tag{lb}
\end{align*}
$$

where the binary hysteresis is characterized by

$$
h(x)= \begin{cases}1 & \text { for } x_{L}<x  \tag{1c}\\ 0 & \text { for } x<x_{u}\end{cases}
$$

and the constants are chosen to be

$$
\begin{equation*}
x_{L}=0, \quad x_{U}=0.3, \quad \sigma=-0.2, \quad a_{1}=-1, \quad a_{2}=-1.34996673232527 \ldots \tag{ld}
\end{equation*}
$$

In Eqs. (1) $x_{1}$ and $x_{2}$ are the two dynamicel (state-like) variables and $x_{L}$ and $x_{U}$ the hysteresis jump points (to the lower and to the upper half lines, respectively). And, although Eq. (1c) indicates a double valued function, we consider true hysteresis which is single valued by requiring that $h(x(t+))$ retains the value of $h(x(t))$ unless $x$ hits a jump point (practically in electronic circuits this is usually a consequence of the small parasitic capacitors [15]). The strange value of $a_{2}$ is chosen to give a continuous return map (the map of next crossings of the $\mathrm{x}_{2}>0$ axis) in order that the Li -Yorke Theorem (which requires continuity) is satisfied; however continuity is not really necessary and the system is rather robust and the choice of $\mathrm{a}_{2}=-1.35$ works well.

As was proven in [10] using the Li-Yorke Theorem, given appropriate initial conditions the system of Eqs. (1) will yield chaos.

## III. Ideal DVCCS Realization

In this section we give a realization of Eqs. (1) using ideal components based upon voltage controlled current devices rather than the op-amps previously used. This will be done in such a manner as to conveniently allow denormalizations to practical VLSI circuits. Especially we will use the differential voltage controlled current source (DVCCS), using it to obtain all of the terms on the right hand side of Eqs. (1), including the hysteresis, while realizing the left had sides as currents in unit capacitors. The key

h we use the circuit
the variables $x_{1}$ and of these equations, and by a hysteresis ent, $h\left(\mathbf{x}_{1}\right)$, with two obtain Eqs. (1) by
minary, in Fig. 2(a), mbol G, and in Fig. int, ion whose value

5)
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30th measured with 7ysteresis), in which ent; if this is a linear n. The ideal current $=k_{i}$, where $i_{0}$ is the
nodified) basic ideal ntout of the binary 3(b). The operation utput current, io, of two branches of the ies on positive and - respectively. Then $v<X_{U}$ while when io ave a double valued

$$
i_{0}(v)= \begin{cases}I_{0} & v<X_{v}  \tag{2}\\ -I_{0} & X_{L}<v\end{cases}
$$

In Fig. 3(a) this current is mirrored in the current mirrors, F1 and F2, to give the currents to be fed to the circuit, after a shift of the curve, by current sources I1 and I2, to bring

(a)

(b)

Figure 3
DVCCS Binary Hysteresis Following Linares
$\begin{array}{ll}\text { (a) Circuit Diagram } & \text { (b) DC Characteristics }\end{array}$

sistorized DVCCS we e have chosen $\mathrm{L}_{0}=1 / 2$ ent source Il to give is seen that the binary sproximation to a step icated in Fig. 3(a); to in nodes and replaced , VALUE $=$ gd* $^{*}\left(\left(v_{+}-v_{.}\right)\right.$ needed to guarantee
sble configuration for ttion of Eqs. (1) using state variables, $\mathrm{x}_{1}$ and put current of the unit $-2 \sigma$ via the current All the other terms of 'Spice simulations, the osen via the software. ent sources across the

$\overbrace{0.5}^{11}$
0.675
(4*otan(1))
os Generator


Figure 4 (b) Chaotic Response, Zero Initial Conditions


Figure 4 (c) Longer Time Span of Trajectory of (b)
capacitors with their currents being large but short pulses that drop to zero (to act as impulses); DVCCSs can also be used for realizing these initial condition current sources with these DVCCS inputs being pulses of voltage (which practically are easier to obtain than the pulses of current). However, in our case we have been able to achieve a design that uses zero initial conditions, by proper placement of the spirals in Fig. 1. In parts (b) \& (c) of the figure are given a short time span and a long time span, respectively of a typical trajectory for $x_{1}$, this resulting from our choice of zero initial conditions.

## IV. Practical DVCCS Realization for VLSI

Having the normalized circuit of Fig. 4 it remains to replace the ideal devices indicated there by transistorized components, something which is easily done using the work horses of analog VLSI, the CMOS current mirrors and differential pairs. In the case of the current mirrors, they are realized in VLSI form [16, p. 346] as shown in Fig. 5(a) for an NMOS structure and in Fig. 5(b) for a PMOS structure. The circuits of Fig. 5(al) \& (bl) are the basic current mirrors, but these are improved upon to obtain higher output

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ace the ideal devices easily done using the ntial pairs. In the case as shown in Fig. 5(a) circuits of Fig 5(al) obtain higher output
$[(W 2 / L 2) /(W 1 / L 1)]{ }_{i}$

(03)
mete cascode nMOS mirror
Vervoco Verrbol
ved

$=[(w / L 4) /(w 3 / L 3)]$
(b3)
temode coscode PMOS mirtor $0<\mathrm{Vb}<\mathrm{V} \mathrm{c}$


Figure 5
Basic Transistorized Devices
(c) Basic and (d) Practical Differential Pairs
impedance by use of the standard cascode connection of Fig. 5(a2) \& (b2), this being important for the design of circuits sensitive to parameter changes, as is the case in chaos circuits. Figures 5(a3)(b3) show an alternate cascode connection [17, p. 404] which in our case is more effective since the output drive transistors, M2c \& M4c, are held in saturation for almost all loads via the bias voltage Vb ; although these are better for very sensitive structures, they do require the extra bias source. The current mirrors of Fig. 5 are actually uni-directional, the NMOS one giving a nonzero output current, $i_{o}$, only when the input current, $\dot{i}_{i}$, is positive while the PMOS one yields only a nonzero output current when its input current is negative. But by placing the two types of current mirrors in parallel, a bidirectional current mirror is obtained. Both types of current mirrors work by transferring the voltage of the gate-source, $V_{\text {as, }}$ of the input transistor to the output transistor to force the drain currents of the two transistors to be equal (when $\left|V_{\text {asl }}\right|>\mid V_{\text {aremolal }}$ so the output transistor is tumed on). The DVCCSs are readily constructed from differential pairs, often called differential amplifiers, with both MOS [16, p. 436] and bipolar [16, p. 446] transistor versions being available. For the chaos generator to be constructed in VLSI form, the MOS version is preferable (since pnp transistors and base current loading can be avoided) for which reason we use here MOS differential pairs, as given in Fig. 5(c). The basic differential pair shown in Fig. 5(c) has the source bias current Iss steered between the input transistors M1 \& M2 by the difference, $v+-v-$, between the two input voltages with the difference of the two resulting drain currents taken as the output current formed by using a current mirror, M3-M4, on the M1 drain current. The circuit of Fig. 5(c) is the standard one, but it suffers from some defects which lead to deterioration of the output current especially when operating in a linear region. Consequently, we improve upon it by the circuit of Fig. 5(d) where the input transistors' drain currents are mirrored slightly differently by feeding that of Ml back around though M35-M45-M46 while also using the cascode connections of Fig. 5(a3)(b3). The current source Iss of Fig. 5(c) is realized by applying a fixed voltage to the gate of M5 biased in its saturation region (by a negative enough Vss). The bias voltages for M34-M44, M46, and M5 are obtained by the voltage divider connected N-PMOS pairs, M61-M62, M71-M72, and M51-M52, respectively. The voltages of these dividers being readily set by width to length ratios and the supply voltages Vdd \& Vss as calculated by setting the drain currents of the two transistors in a pair to be equal in magnitude (on assuming the transistors are in the saturation region, which they will be for reasonable supply voltages). In the case of G1 and G2 two output currents are needed; these are simply obtained by mirroring the M1M2 drain currents in another set of output current mirrors (similar to M43-M44-M46M45) for which a different Wout/Lout is chosen. There remains the diodes of the binary hysteresis circuit. These can be realized as diode connected MOS transistors, in which the gates are tied to the drains. However, better diode characteristics result from junction diodes and, since our hysteresis needs sharp corners, we, consequently, construct the diodes from bipolar npn transistors, as are available in the BiCMOS process, by shorting bases to the collectors.

Before designing the fully transistorized circuit it is necessary to carry out a denormalization so that practical values of components can be used, in particular for gm of
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circuit trajectu voltage where
\& (b2), this being $s$ the case in chaos 404] which in our M4c, are held in are better for very iirrors of Fig. 5 are $t$, $i_{0}$, only when the sro output current current mirrors in it mirrors work by stor to the output be equal (when radily constructed IS [16, p. 436] and is generator to be ansistors and base ifferential pairs, as source bias current $v+-v-$, between the rents taken as the drain current. The ects which lead to
a linear region. e input transistors' ack around though i)(b3). The current of M5 biased in its 34-M44, M46, and 1-M62, M71-M72, ily set by width to $g$ the drain currents re transistors are in ). In the case of G1 mirroring the M1o M43-M44-M46siodes of the binary istors, in which the esult from junction ntly, construct the rocess, by shorting
rry to carry out a particular for gm of
the differential pairs, capacitances, and hysteresis parameters. Since our anticipated fabrication is through the MOSIS facility we will use the MOSIS BiCMOS transistor parameters as given in the Appendix. For our designs, to be discussed below, we found by simulations of the circuits outlined above, a base gm=5.3 micro Amp/Volt, an upper hysteresis jump point of $\mathrm{X}_{\mathrm{U}}=0.81$ Volts and an hysteresis upper amplitude of $\mathrm{H}^{+}=15.7$ micro Amp (along with a lower amplitude of 0 ).

To match these practical values obtainable by physical components we perform a denormalization of Eqs. (1) as follows. To adjust for the transconductance value we multiply Eqs. (1a) and (1b) by a constant K1. Then we adjust for the hysteresis amplitude by multiplying and dividing $h(x 1)$ by $K 2$. To adjust the upper jump point of the hysteresis we scale $x_{1}$ to $z_{1}$ by setting $x_{1}=X z_{1}$ and to keep similar labeling set $x_{2}=z_{2}$. Finally to adjust the capacitors, the time scale is next changed by naming $t=t_{\text {odd }}$, replacing $t_{\text {odd }}$ by $t_{\text {odd }}=t_{\text {maw }} / C$ and renaming $\mathrm{t}_{\mathrm{m}}=\mathrm{t}$ to get

$$
\begin{align*}
& C K_{1} X \frac{d z_{1}}{d t}=K_{1} z_{2}+\frac{K_{1}}{K_{2}} K_{2} a_{1} h\left(X_{z_{1}}\right)  \tag{3a}\\
& C K_{1} \frac{d z_{2}}{d t}=-K_{1} X z_{1}-2 \sigma K_{1} z_{2}+\frac{K_{1}}{K_{2}} K_{2} a_{2} h\left(X z_{1}\right) \tag{3b}
\end{align*}
$$

Consequently, we choose $\mathrm{K}_{1}=\mathrm{gm}=5.3$ microA/V, $\mathrm{K}_{2}=\mathrm{H}^{+}=15.7$ microA, $\mathrm{X}=0.3 / 0.81=0.37$ and $\mathrm{C}=0.25 \times 10^{-3}$ to give capacitors $\mathrm{Cl}=0.49$ nanoFd and $\mathrm{C} 2=1.325$ nanoFd. Most width to length ratios are chosen to be unity for convenience, for which we used $\mathrm{L}=10$ micron=W. But in the case of the linear DVCCS's we used a ratio of $\mathrm{W} / \mathrm{L}=2 / 24$ for their input transistors to achieve a larger region of linearity. Also the width to length ratios were appropriately adjusted in the case of the nonunity current ratios of F2 and F3.

Putting everything together Figure 6 results to show the total circuit. Although this looks rather formidable it is formed in a straightforward manner described above from Fig. 4(a). In Fig. 6 we have used a numbering consistent with the ideal circuit of Fig. 4(a) by using the numbers in Fig. 4(a) in the hundreds position of the transistors of Fig. 6. Also in Fig. 6 there are three ideal current sources to compensate for offset currents of the differential pairs; these can be realized in the same manner as Iss in Fig. 5(d) but are kept as current sources to prevent further clutter of the figure. Similarly there are two voltage sources to set the hysteresis jump points, which can be realized by voltage dividers similar to those setting the biases on the cascode current mirrors. In the figure we also include two copies of the supply sources, of values Vdd $=+5$ and Vss $=-5$, in order to avoid further crossing of lines, but only one of each is actually to be used.

The substitutions between the ideal devices of Fig. 4(a) and the transistorized circuit of Fig. 6 are catalogued in Table 1. Figure 7 shows the zero initial condition trajectory obtained from Fig. 6 with part (a) and (b) giving the time traces of the capacitor voltages and part (c) giving the phase plane plot of $z_{2}$ versus $z_{1}$. It is clearly seen when and where $z_{1}$ hits the hysteresis jump point to the upper half plane at 0.81 Volt. By capturing

Figure 6
Practical DVCCS Realization

Table 1
Correspondences of Components of Figure 4(a) with Figure 6
all non listed $\mathrm{L}=10 \mathrm{U}, \mathrm{W}=10 \mathrm{U}$; $\mathrm{x}=0$ through 9; $\mathrm{U}=10 \mathrm{E}-6$

| Fig. 4(a) Device | Fig. 6 <br> Device | Purpose | Parameter | Comments |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{C} 1, \mathrm{C} 2$ | C1, C2 | dynamics | $\begin{aligned} & \mathrm{Cl}=0.49 \mathrm{n} \\ & \mathrm{C} 2=1.325 \mathrm{n} \end{aligned}$ | improvewith capacitor mult |
| D1-V1 | Q1-V1 | diode connected npn, $\mathrm{X}_{\mathrm{U}}$ | $\mathrm{V} 1=-0.6$ | Qladiode connected npn |
| D2 | Q2-V2 | diode connected npı, $\mathrm{X}_{\mathbf{L}}$ | V2x-0.2 | ```Q2=diode connected upn V2 handles Offets``` |
| F1-I] | M165, 15x | for $a_{1}$ erm | W165m3U | II $=0$ due to mirror diode action |
| F2-12 | M175, 15x | for $\boldsymbol{a}_{2}$ term | W175 $=4 \mathrm{U}$ | $12=0$ due to mirror diode action |
| F3 | M38x | for $\sigma$ term | $\begin{aligned} & W 383=4 U \\ & W 385=4 U \\ & \hline \end{aligned}$ |  |
| G1 | M101, 102 | input to differential pair for hysteresis |  |  |
|  | $\begin{aligned} & \text { M103, } 104 \\ & \text { M13x, } 14 \mathrm{x} \end{aligned}$ | current mirrors |  |  |
|  | M16x \& 17x | V | +2\&-2 |  |
|  | M105, 15x | Iss of Diff. Pair | $\begin{aligned} & \text { Iss gate } \\ & \text { vg프 } \end{aligned}$ |  |
| G2 | M201, 202 | input to linear differential pair | $\begin{aligned} & \mathrm{L}=24 \mathrm{U} \\ & \mathrm{~W}=2 \mathrm{U} \end{aligned}$ |  |
|  | M28x | current mirrors for $z_{1}$ term in (3b) | $\begin{aligned} & \mathrm{W} 283=3.7 \mathrm{U} \\ & \mathrm{~W} 285=3.7 \mathrm{U} \\ & \hline \end{aligned}$ |  |
|  | $\begin{aligned} & \text { M203, } 204 \\ & \text { M23x, 24x } \end{aligned}$ | current mirrors |  |  |
|  | M36x \& 37x | Vb | +2 \&-2 |  |
|  | M205, 35x | Iss of Diff. Pair | Iss gate $\mathrm{vg}_{\mathrm{g}}-3$ |  |
|  | 1201 | offset current src |  | 1 Output |
| G3 | M301, 302 | input to linear differential pair | $\begin{aligned} & \mathrm{L}=24 \mathrm{U} \\ & \mathrm{~W}=2 \mathrm{U} \end{aligned}$ |  |
|  | $\begin{aligned} & \text { M303, } 304 \\ & 33 \mathrm{x}, 34 \mathrm{x} \\ & \hline \end{aligned}$ | current mirrors M34x for $z_{2}$ term in (3a) |  |  |
|  | M36x \& 37x | Vb | +2 \& - 2 |  |
|  | M305, 35x | Iss of Diff. Pair | Iss gate $V_{g}=-3$ |  |
|  | 1301, 302 | offset current sres | $\begin{aligned} & 1301=0.4 \mathrm{U} \\ & 1302=0.6 \mathrm{U} \end{aligned}$ | 2 output currents |
| $\begin{aligned} & \text { Vdd } \\ & V_{5 s} \end{aligned}$ | $\begin{aligned} & \text { V3, V4, } \\ & \text { V5, V6 } \end{aligned}$ | $\begin{aligned} & V_{3}=V 5=V d d \\ & V_{4}=V_{6}=V_{s s} \end{aligned}$ | $\begin{aligned} & \text { Vddx+5 } \\ & \text { Vssx-5 } \end{aligned}$ | Power Supply |



Figure 7 (a) Zero Initial Condition Responses of Practical Circuit, $z_{1}$ versus time


Figure 7 (b) Zero Initial Condition Responses of Practical Circuit, $z_{2}$ versus time

it, $\mathbf{z}_{2}$ versus time


Figure 7 (c) Zero Initial Condition Responses of Practical Circuit, $z_{2}$ versus $z_{1}$ Plot
the times when these jumps occur one may be able to obtain some useful properties of this type of chaos.

The nature of responses of the circuit are quite dependent upon initial conditions. Although we have designed the circuit such that it gives a chaotic response with zero initial conditions, it may be desirable to allow for nonzero initial conditions. Such is easily accomplished by feeding the capacitors with a pulse of current which effectively acts as an impulse. Thus, by placing an MOS transistor's drain-source across a capacitor and feeding the gate-source with a short voltage pulse of appropriately large amplitude, desired nonzero initial conditions can be set.

On observation of the circuit of Fig. 6 it is seen that many components are present. Overall, however, the circuit presents no problem for integrated circuit fabrication. But the capacitors are larger than we would like and we feel that some optimization could be done to cut down on the number of transistors. To handle the capacitors, the sizes of which are needed to swamp parasitics, we would like to incorporate capacitor multipliers, with suitable ones possibly being available in the literature [18]. As for the number of
transistors, by optimal choices of Wh ratios, it appears that the current mirrors of Fig. $5(\mathrm{a} 1)(\mathrm{b} 1)$ or (a2)(b2) could be used. In any event the circuit of Fig. 6 is a working circuit suitable for small chip fabrication, though we do feel with time it can be improved upon.

## V. Discussion

For lumped circuit constructions the previous realization of a binary hysteresis chaos generator is convenient since it used readily available operational amplifiers in an analog computer type of circuit. However, for integrated circuit realizations operational amplifiers, which are very high gain voltage controlled voltage sources, are nowhere near as convenient as voltage controlled current sources. Consequently, we have presented here a redesign which is ready to be put into layout for VLSI fabrication.

The binary hysteresis chaos generator is particularly appealing since it is only of degree two and uses hysteresis which is readily made using standard available transistors. It should be noted that we are able to get away with a degree two system since the describing differential equations contain discontinuous functions. Because the electronic realization of the hysteresis relies upon capacitive parasitics it might be fair to believe that really this is a degree three system, the parasitic capacitor giving the third derivative. However, this parasitic can be vanishingly small so in practice we ignore it in the design. Further, if one were to use other means of realizing the binary hysteresis, for example by magnetic amplifier cores, then one need not rely on the parasitics to insure operation; all that is needed is to obtain the operation represented by Fig. 1 and Eqs. (1).

Chaos generating circuits are presenly becoming a dime a dozen since by introducing appropriate return feedback almost any degree two or higher circuit can be made to create chaotic signals. However, obtaining desired characteristics of the chaos is another story since, as yet, there is not available a satisfactory design theory which takes into account the nature of the chaos. But it does seem that the binary hysteresis chaos generator is something like the basic element from which arbitrary chaos can be built by concatenations and other combinations, just as arbitrary vectors are constructed from basis vectors.

Finally we comment that the speed of hardware particularly shows up in working with chaos generators. Whereas it takes hours to obtain results with software the same is obtained in milliseconds with hardware.

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Appendix - SPICE Parameters for MOSIS Analog, BiCMOS, VLSI Fabrication
These are the MOSIS run parameters for run N21H SPICE LEVEL 2 parameters recorded 04/28/93 which were used for the simulations in order to design the circuit for VLSI fabrication.
A. The N Chansel MOS Transistor Model

MODEL MNMOSIS NMOS LEVEL $=2$ LD $=0.250000$ U TOX $=418.000008 \mathrm{E}-10$

+ NSUB $=9.236187 \mathrm{E}+14 \mathrm{VTO}=0.858153 \mathrm{KP}=5.048000 \mathrm{E}-05 \mathrm{GAMMA}=0.198$
$+\mathrm{PH}=0.6$ UO $=596.729$ UEXP $=7.029586 \mathrm{E}-02$ UCRIT $=10266.7$
+ DELTA $=2.7371$ VMAX $=65701.4 \mathrm{XJ}=0.250000 \mathrm{U}$ LAMBDA $=1.843384 \mathrm{E}-02$
+ NFS $=1.086360 \mathrm{E}+12$ NEFF $=1$ NSS $=1.000000 \mathrm{E}+10 \mathrm{TPG}=1.000000$
$+\mathrm{RSH}=28.760000 \mathrm{CGDO}=3.097916 \mathrm{E}-10 \mathrm{CGSO}=3.097916 \mathrm{E}-10 \mathrm{CGBO}=3.838441 \mathrm{E}-10$
$+\mathrm{CJ}=8.997900 \mathrm{E}-05 \mathrm{MJ}=0.783638 \mathrm{CJSW}=5.524800 \mathrm{E}-10 \mathrm{MJSW}=0.285064 \mathrm{~PB}=0.800000$
* Weff = Wdrawn - Delta_W
* The suggested Delta_W is -0.24 um


## B. The P Channel MOS Transistor Model

MODEL MPMOSIS PMOS LEVEL $=2$ LD $=0.250000 \cup$ TOX $=418.000008 \mathrm{E}-10$
$+\mathrm{NSUB}=9.309300 \mathrm{E}+15 \mathrm{VTO}=-0.889271 \mathrm{KP}=1.908000 \mathrm{E}-05 \mathrm{GAMMA}=0.6289$
$+\mathrm{PHI}=0.6$ UO $=216.28$ UEXP $=0.218144$ UCRIT $=62664$

+ DELTA $=0.164572 \mathrm{VMAX}=100000 \mathrm{XJ}=0.250000 \mathrm{U}$ LAMBDA $=5.011626 \mathrm{E}-02$
+ NFS $=9.266623 \mathrm{E}+11 \mathrm{NEFF}=1.001$ NSS $=1.000000 \mathrm{E}+10 \mathrm{TPG}=-1.000000$
$+\mathrm{RSH}=66.820000 \mathrm{CGDO}=3.097916 \mathrm{E}-10 \mathrm{CGSO}=3.097916 \mathrm{E}-10 \mathrm{CGBO}=3.727276 \mathrm{E}-10$
$+\mathrm{CJ}=2.981300 \mathrm{E}-04 \mathrm{MJ}=0.556944 \mathrm{CJSW}=3.002100 \mathrm{E}-10 \mathrm{MJSW}=0.243045 \mathrm{~PB}=0.800000$
* Weff = Wdrawn - Delta_W
* The suggested Delta_W is -0.17 um
C. The NPN Bipolar Transistor Model

MODEL BN2X1 NPN
$+\mathrm{BF}=82 \mathrm{IS}=1.588 \mathrm{E}-16 \mathrm{NF}=9.9563 \mathrm{E}-01 \mathrm{NE}=1.3356 \mathrm{VAF}=57.1$
$+\mathrm{IKF}=2.3067 \mathrm{E}-02 \mathrm{ISE}=1.267 \mathrm{E}-16 \mathrm{RE}=12.7 \mathrm{RC}=420.00 \mathrm{RB}=1.213 \mathrm{E}+03$

+ RBM=7.53 1SC=3.363E-15 NC=1.0202
$+\mathrm{CJE}=0.1952 \mathrm{E}-12 \mathrm{MJE}=0.5050 \mathrm{VJE}=0.85 \mathrm{CJC}=0.18815 \mathrm{E}-12 \mathrm{MJC}=0.4990 \mathrm{VJC}=0.80$
$+\mathrm{CJS}=0.2326 \mathrm{E}-12$ MJS $=0.2033$ VJS $=0.70$
* AREA OF TRANSISTOR
* $\mathrm{AE}=128 \mathrm{um} \wedge 2 \mathrm{PE}=64 \mathrm{um} \mathrm{AB}=644 \mathrm{um}^{\wedge} 2 \mathrm{~PB}=102 \mathrm{um} \mathrm{AC}=2064 \mathrm{um} \wedge 2 \mathrm{PC}=182 \mathrm{um}$


# Nonlinear Dynamics In Circuits 

editors
T. Carroll
L. Pecora

Naval Research Laboratory, USA

Singapore - New Jersey - London • Hong Kong

