SEMISTATE EQUATIONS FOR TYING A KNOT ON A TORUS USING TWO ROBOT ARMS*

Rassa Rassai, George Syrmos and
Robert W. Newcomb
Microsystems Laboratory
Electrical Engineering Department
University of Maryland
College Park, MD 20742
Phone: (301) 405-3662

ABSTRACT
In this paper we give the semistate equations for tying a knot on a torus using two robot arms. The two robot arms in synchronization with one another follow a trajectory on a torus which is the path generated from the equations of an (m_1,m_2)-torus knot.

I. INTRODUCTION

To tie a knot is a part of our everyday lives. For example we tie our shoe laces, wrap packages, tie ropes while sailing or at the dock. Tying a knot has many industrial applications as well, for example in textile factories. However, under hazardous conditions there will be times that a robot will be desired to do the job, like tying a knot under water or in space. In this paper we give the semistate equations for tying a knot on a torus using two synchronized robot arms. Our available robot arms have four rotation angles, namely waist, shoulder, elbow and wrist, however, each robot arm has only one gripper (end-effector). In [1] we mentioned that a knot which is embedded on a torus is called a torus knot. An (m_1,m_2)-torus knot is a trajectory which travels m_1 times around the meridian circle of a torus with radius R_1 and m_2 times around the axial circle of the torus with radius R_2. In order to tie an (m_1,m_2)-torus knot using two robot arms we need to know the coordinates of the joints and the inverse kinematic equations of each robot arm. That information determines the positioning of the two robots with respect to each other and the torus, which is used as the supporting base for the knot. Here, we place the two robots at a distance "a" from each other, along the positive direction of the Y axis of X-Y-Z coordinate systems. We place the torus supporting the knot at a distance "a/2" from each robot along the Y axis of an X-Y-Z coordinate system to insure that the torus is within reach of both robots, Fig.1a. In part II of this paper we give the coordinates of the joints and the inverse kinematic equations of two robot arms at a distance "a" from each other. The results of part II are used in part III where we give the semistate equations for each robot arm to tie a knot on a torus which is positioned at a distance "a/2" from each robot. We choose a torus which can be disconnected to release the knot once it is tied on the torus. As a result, the two robot arms in synchronization with each other construct an open ended torus knot. Fig. 2a shows a (2,3)-torus knot tied on the torus and Fig. 2b shows the knot after the torus is removed. Part IV contains some discussion and conclusions.

II. THE INVERSE KINEMATIC EQUATIONS OF TWO COORDINATED ROBOT ARMS

In order for humans to tie a knot they normally use both hands. To simulate this process we use two synchronized robot arms. Our available robot arms have four joints, namely waist, shoulder, elbow, wrist and a gripper. In order to tie a knot on a torus using a string, we need to know the coordinates of the joints of each robot and the joints' rotation angles. That information will help us position the torus such that it is well within reach of both robots. Table 1 presents the coordinates of the joints of each robot arm. To understand the entries in Table 1, refer to Fig.1a, where the robots are seen to be positioned at a distance "a" from each other along the positive

* Research supported by NSF Grant MIP 89-21122

direction of Y axis of the X-Y-Z coordinate systems. Continuing to refer to Fig.1a, point O_1 indicates the base or the waist of the robot #1. Ra is the height of its waist, Rb is the length of the upper arm, Rc is the length of the forearm and Rd is the the length of the end-effector, that is, the gripper. θ_{01} is the angle of the waist rotation with positive orientation being counter-clockwise, θ_{11} is the shoulder rotation angle, that is, the angle measured from the positive direction of the Y axis to R_b. θ_{12} is the elbow rotation angle and is equal to the angle measured between R_b and R_c. θ_{31} is the wrist rotation angle and is equal to the angle between R_c and R_d. The coordinates of all of the joints for each robot are given in Table 1. Given the coordinates of the end-effector of robot #1 one can find several possible sets of rotation angles for robot #1, θ_{n1} where n=0,1,2,3. These angles are given in Table 2, a table of direct kinematic equations. In order to find the joint angles of the second robot, θ_{n2}, we use Table 2 and substitute X_{D1}, Y_{D1} and Z_{D1} by X_{D2}, Y_{D2}, Z_{D2}. Depending on the physical limitations and the intended path of each robot only one solution is accepted at a time. Now we proceed to give the semistate equations for these two robot arms tying a knot on a torus.

III. Semistate Equations For Tying A Knot on a Torus Using Two Synchronized Robot Arms

A knot which is embedded on a torus is called a torus knot and, with reference to [1], an (m_1,m_2)-torus knot is a trajectory which travels m_1 times around a meridian circle with radius R_1 and m_2 times around an axial circle with radius R_2. The solution to the set of state Eqs. (1) will result in an (m_1,m_2)-torus knot in four dimensional space of its own coordinates w, x, y, and z.

$$dx/dt=y \quad (1a)$$
$$dy/dt=-m_1^2x \quad (1b)$$
$$dz/dt=w \quad (1c)$$
$$dw/dt=-m_2^2z \quad (1d)$$

with initial conditions $x(0)=0$, $y(0)=R_1$, $z(0)=0$, and $w(0)=R_2$, in which R_1, R_2 are the radii of the two circles used to construct the torus. Now in order to tie an (m_1,m_2)-torus knot we should place the torus supporting the knot well within reach of both robot arms in the robot coordinate system. With referance to Table 1 and examining the maximum and the minimum distances reached by robot #2 on each axis of the coordinate system, we place the torus and its supporting base at a distance "a/2" from each robot on the Y axis. We call the coordinates of the center of the torus X_c, Y_c, and Z_c. These coordinates are defined by the following equations.

$$X_c = 0 \quad (2a)$$
$$Y_c = a/2 \quad (2b)$$
$$Z_c = R_a \quad (2c)$$

With reference to Eqs. (1) and (2), and returning to three dimensional space, we have the semistate equations (3). This solution is the path of the trajectories on a torus shifted a/2 units along the positive direction of Y axis and R_a units up the Z axis, [1].

$$dx/dt=y-Da/2 \quad (3a)$$
$$dy/dt=-m_1^2x \quad (3b)$$
$$dz/dt=w \quad (3c)$$
$$dw/dt=-m_2^2(z-R_aD/m_2) \quad (3d)$$

Where

$$D=d+W \quad (3e)$$

and $d>R_2$ is a free parameter which can be chosen such that

$$d^2=R_1^2+R_2^2 \quad (3f)$$
$$x=X/D,\ y=Y/D, \quad (3g)$$
$$z=(R_1m_2Z)/[D(d^2-R_2^2)] \quad (3h)$$

After some algebraic manipulation and using Eqs. (3), the equation of a torus in X-Y-Z coordinate systems with its center at $(0,a/2,R_a)$, is given by Eq. (4).

$$(Z-R_a)^2+[R-[X^2+(Y-a/2)^2]^2=r^2 \quad (4a)$$

where in Eq. (4a)

$$R=dR_1/(d^2-R_2^2) \quad (4b)$$
$$r=R_1R_2/(d^2-R_2^2). \quad (4c)$$

Now we proceed to give the semistate equations for two synchronized robot arms tying a knot on the torus defined by Eq. (4) which has a very rough surface for holding the string. We assume both robots are at an initial position which for reference we call the HOME position. We call the rotation angles at HOME position θ_{n1H} and θ_{n2H} for each robot where n=0,1,2,3. We assign to robot #1 the task of moving to all points on the torus where $Z>R_a-Z_1$; the task assigned to robot #2 is that of moving to all points on the torus where $Z<R_a+Z_1$, Fig. 1b, where Z_1 is the smallest possible incerement that the end-effector of each robot can take. This increment is one degree or 8.8 steps, [3]. To tie a knot we use a piece of string which is long enough that if the robot holds it in the middle it can still be wrapped around the torus. Robot #1 holds the string near one end by closing its gripper and then it moves the string over the torus to a point where the Z coordinate of the point on the torus is larger than R_a by an increment Z_1. With reference to Table 1 and Table 2 new joint angles are calculated. The above process

continues until $Z=R_a-Z_1$ at which point the gripper throws the string over the torus. When this results, robot #1 opens its gripper and lets go of the string, and goes to HOME position. At this time robot #2 becomes activated and moves its gripper to the point where robot #1 has let go of the string, that is, to where $Z=R_a-Z_1$ and grabs the string by closing its gripper near to the midpoint of the string. At this point since $\theta_{12}<0$ we find θ_{02}, θ_{22}, and θ_{32} using Table 2 and substituting for X_{D2}, Y_{D2}, and Z_{D2} by X_{D1}, Y_{D1}, Z_{D1}, respectively. Taken together these angle constraints mean that robot #2 approaches the point from underneath the torus. For as long as $Z<R_a+Z_1$, robot #2 will follow this new orientation and again, as soon as $Z=R_a+Z_1$, its gripper reaches and attaches the string to the rough surface of the torus and lets go of the string and goes to HOME position with robot #1 being reactivated for $Z>R_a-Z_1$. We should notice that the size of the torus which has the knot embedded on it must be chosen such that it satisfies the workspace constraints of the robot arms. To implement the mentioned idea mathematically, the two robot arms sit in two planes, the upper hysteresis plane and the lower hysteresis plane. Robot #1 moves on points on the upper surface of the torus until it reaches the hysteresis jump plane, which in our case is perpendicular to the Z axis. This jump occurs at $Z=R_a-Z_1$. At this point robot #1 lets go of the string and robot #2 takes over. Robot #2 moves to all the points on the lower half of the torus until it reaches the second hysteresis jump point. This jump occurs at $Z=R_a+Z_1$. At this point robot #2 lets go of the string and robot #1 takes over. This process is shown in Figs. 1a and 1b. With reference to semistate Eqs.(3), the semistate equations for the two robot arms to tie a torus knot on a torus with its center at $(a/2, 0, R_a)$ are given by the following equations along with (3e-h)

$$dx/dt=y-Da/2 \quad (5a)$$
$$dy/dt=-m_1^2x \quad (5b)$$
$$dz/dt=w \quad (5c)$$
$$dw/dt=-m_2^2(z-R_aD/m_2)H(Z) \quad (5d)$$

$$\text{where } H(Z)=\begin{cases}1 & Z>R_a-Z_1 \quad (6a)\\ -1 & Z<R_a+Z_1 \quad (6b)\end{cases}$$

Where with reference to Eqs.(3e), (3f), (3g), and (3h), $Z=R_a-Z_1$ and $Z=R_a+Z_1$ are the hyteresis jump points. For all $Z>R_a-Z_1$ the coordinates of the end-effector of robot #1 should be the same as the coordinates of the point on the knot, while for all $Z<R_a+Z_1$ the coordinates of robot #2 must be the same as the coordinates of a point on the knot. That is:

Case I: $Z>=R_a+Z_1$; $X_{D1}=X$, $Y_{D1}=Y$, $Z_{D1}=Z$ (7a)

Case II: $Z<=R_a-Z_1$; $X_{D2}=X$, $Y_{D2}=Y$, $Z_{D2}=Z$ (7b)

Since each robot after completing its task goes to HOME position there will not be a collision between the two robots. The precautions needed to avoid collision of the robot with the base, the detailed algorithm, and the experimental results are given in [3]. At the completion of the procees mentioned above we have an (m_1,m_2)-torus knot wrapped around the torus, Fig.2a. If the torus can be disconnected and pulled out of the knot, one has an open ended torus knot. Fig.2b shows an open ended (2,3)-torus knot.

IV. CONCLUSION

Here we gave the theory for semistate equations for tying a knot on a torus using two synchronized robot arms. The actual experimental results are fully discussed and given in [3]. Our assumption in this paper was that the knot was wrapped on a torus and after the completion of the task of knot tying the torus is pulled out of the knot. To tie a knot we simulate the motions of human hands. With the robots used here this is a difficult task, since, the robots lack three out of five fingers and, unlike human fingers, which have three joints in each finger, our available robots miss the joints in the fingers. We often tend to overlook the importance of the joints and the flex muscles in our fingers and the complicated coordination among the fingers on each hand while tying a knot. To simulate these movements requires more study.

REFERENCES:

[1]. R. W. Newcomb, "Design of a Torus Knot Oscillator," Proceed- ings of the 27th Midwest Symposium on Circuits and Systems, Morgantown WV, Vol. 1, pp. 344 - 347, June 1984.

[2]. R. Rassai, "Multi-Torus Knot Oscillator," Ph.D. Thesis, 1985, University of Maryland, Electrical Engineering Department.

[3]. R.Raasai, G. Syrmos, and R. W. Newcomb, "An Algorithm For Tying a Knot On A Torus Using Two Synchronized Robot Arms," Sent for publication.

[4]. R. Rassai, G. Syrmos, and R. W. Newcomb, "Solid - Holed Torus Knot Oscillators," Special Issue on "Mathematical Circuit Theory," of the Circuits, Systems and Signal Processing Journal, Vol. 8, No 3, pp. 235 - 260,

1989.

[5]. J. J. Craig, "Introduction to Robotics, Mechanics and Control," pp. 40 - 52, Addison Wesley Publishing Co., Reading, MA, 1986.

TABLE 1

KINEMATIC EQUATIONS OF TWO COORDINATED ROBOT ARMS

Joint	Point	Robot #1	Point	Robot #2
waist	O1	$X_{O1} = 0,$ $Y_{O1} = 0,$ $Z_{O1} = 0.$	O2	$X_{O2} = 0,$ $Y_{O2} = a_o$ $Z_{O2} = 0.$
shoulder	A1	$X_{A1} = 0,$ $Y_{A1} = 0,$ $Z_{A1} = R_a.$	A2	$X_{A2} = 0,$ $Y_{A2} = a_o$ $Z_{A2} = R_a.$
elbow	B1	$X_{B1} = R_b \cos\theta_{11} \cos\theta_{01},$ $Y_{B1} = R_b \cos\theta_{11} \sin\theta_{01},$ $Z_{B1} = R_b \sin\theta_{11} + R_a.$	B2	$X_{B2} = X_{B1},$ $Y_{B2} = -Y_{B1} + a_o$ $Z_{B2} = Z_{B1}.$
wrist	C1	$X_{C1} = [R_b \cos\theta_{11} + R_c \cos(\theta_{11} + \theta_{21})] \cos\theta_{01},$ $Y_{C1} = [R_b \cos\theta_{11} + R_c \cos(\theta_{11} + \theta_{21})] \sin\theta_{01},$ $Z_{C1} = R_b \sin(\theta_{11}) + R_c \cos(\theta_{11} + \theta_{21}).$	C2	$X_{C2} = X_{C1},$ $Y_{C2} = Y_{C1} + a_o$ $Z_{C2} = Z_{C1}.$
end-effector	D1	$X_{D1} = [R_b \cos\theta_{11} + R_c \cos(\theta_{11} + \theta_{21}) + R_d \cos(\theta_{11} + \theta_{21} + \theta_{31})] \cos\theta_{01},$ $Y_{D1} = [R_b \cos\theta_{11} + R_c \cos(\theta_{11} + \theta_{21}) + R_d \cos(\theta_{11} + \theta_{21} + \theta_{31})] \sin\theta_{01},$ $Z_{D1} = [R_b \sin\theta_{11} + R_c \sin(\theta_{11} + \theta_{21}) + R_d \sin(\theta_{11} + \theta_{21} + \theta_{31}) + R_a.$	D2	$X_{D2} = X_{D1},$ $Y_{D2} = -Y_{D1} + a_o$ $Z_{D2} = Z_{D1}.$

TABLE 2

THE ROTATION ANGLES OF TWO COORDINATED ROBOT ARMS

Rotation Angles	Robot #1 (m=1) Robot #2 (m=2)
Waist	$\theta_{0m} = \text{ATN2}\,(X_{Dm}/Y_{Dm})$
Shoulder	θ_{1m} GIVEN
Elbow	$\theta_{2m} = \text{ATN2}\,[(J_1 - J_3)/\text{SQR}\,(1 - (J_1 - J_3)^2)]$ $M_2 < 0\ ,\quad \theta_{1m} > 0$ $\theta_{2m} = \text{ATN2}\,[(J_1 + J_2)/\text{SQR}\,(1 - (J_1 + J_2)^2)]$ $M_2 >= 0\ ,\quad \theta_{1m} < 0$ $R = (X_{Dm} + Y_{Dm})^{1/2}$ $K_1 = R - R_b\ \text{COS}\,(\theta_{1m})$ $K_2 = Z_{Dm} - R_b\ \text{SIN}\,(\theta_{1m}) - R_a$ $M_1 = R_d\ \text{COS}\,(\theta_{1m} + \theta_{2m}) + R_c\ \text{COS}\,(\theta_{1m})$ $M_2 = R_d\ \text{SIN}\,(\theta_{1m} + \theta_{2m}) + R_c\ \text{SIN}\,(\theta_{1m})$ $J_1 = (-M_2K_1)/(M_1^2 + M_2^2)$ $J_2 = \text{SQR}\,[(M_2K_1)^2 - (M_1^2 + M_2^2)(K_1^2 - M_1^2)]/(M_1^2 + M_2^2)$ $J_3 = R_a - R_d + (R_b + R_c)\ \text{SIN}\,(\theta_{1m})$
Wrist	$\theta_{3m} = \text{ATN2}\,(L_2/L_1)$ $L_1 = \text{COS}\,(\theta_{3m}) = (K_1^2 + K_2^2 - R_b^2 - R_c^2)/(2R_bR_c)$ $L_2 = \text{SIN}\,(\theta_{3m}) = \pm(1 - L_1^2)^{1/2}$

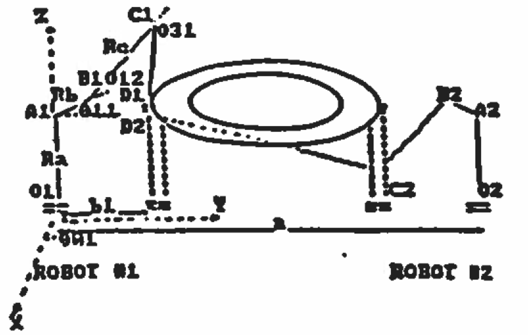


Fig.1a Two Synchronized Robot Arms

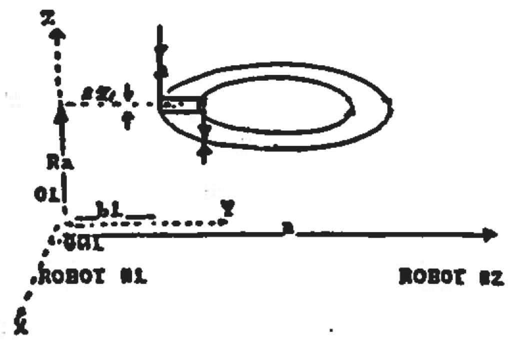


Fig.1b The Path Taken by Each Robot Arm On The Torus

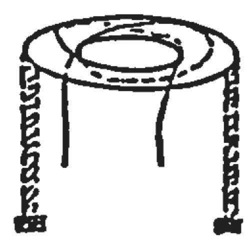
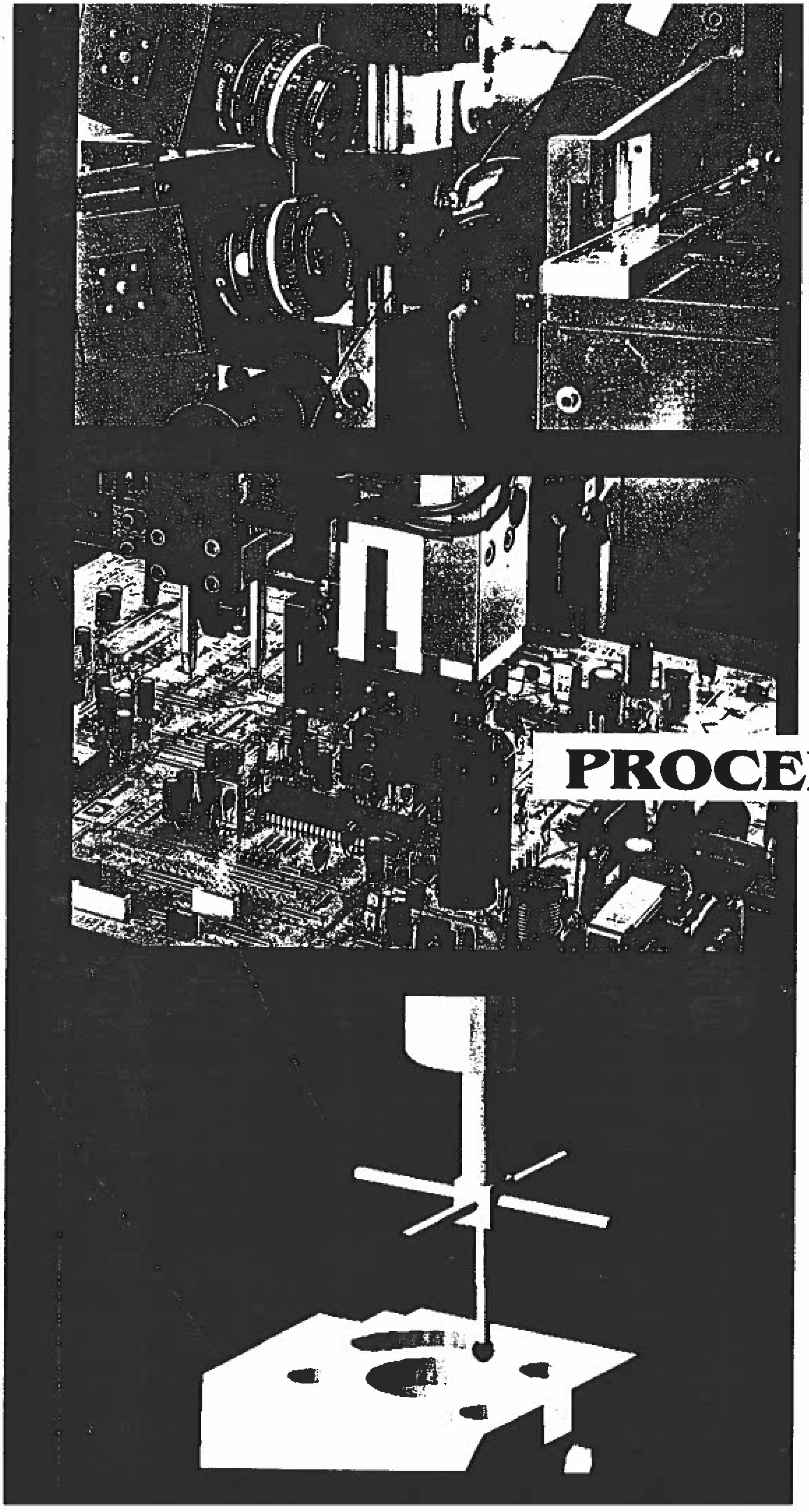


Fig.2a A (2,3)-Torus knot Formed On A Torus

Fig.2b An Open Ended (2,3)-Torus Knot



ICARCV '92

SECOND INTERNATIONAL CONFERENCE ON AUTOMATION

ROBOTICS AND

COMPUTER VISION

PROCEEDINGS

VOLUME 3 OF 3

ORGANISED BY

School of Electrical and Electronic Engineering
Nanyang Technological University
Singapore

The Institution of Engineers, Singapore

16-18 September 1992