induction law" in the form:

$$mmf = \frac{d\psi_e}{dt} \tag{11a}$$

$$mmf = \oint_{C(t)} \bar{H}\cdot d\bar{l} = \iint_{S(t)} \nabla\times\bar{H}\cdot\bar{n}dS \tag{11b}$$

$$\frac{d\psi_e}{dt} = \frac{d}{dt}\iint_{S(t)} \bar{D}\cdot\bar{n}dS$$

$$= \iint_{S(t)} \frac{\delta\bar{D}}{\delta t}\cdot\bar{n}dS. \tag{11c}$$

From (11) one may find the "Maxwell induction law" in its differential form:

$$\nabla\times\bar{H} = \frac{\delta\bar{D}}{\delta t}\cdot \tag{12}$$

Taking

$$\nabla\cdot\bar{D} = \rho \tag{13}$$

and substituting (3) in (12) one obtains:

$$\nabla\times\bar{H} = \frac{\partial\bar{D}}{\partial t} + \rho\bar{v} - \nabla\times(\bar{v}\times\bar{D}). \tag{14}$$

The term $\rho\bar{v}$ in (14) represents *both* convection and conduction current densities. However, because of practical considerations, we should distinguish between those two types of current densities, denoting the latter by $\bar{J}$; (14) then becomes:

$$\nabla\times\bar{H} = \frac{\partial\bar{D}}{\partial t} + \bar{J} + \rho\bar{v} - \nabla\times(\bar{v}\times\bar{D}) \tag{15}$$

where (15) is the second Maxwell equation for moving media. One should realize that both (10) and (15) have been derived for an arbitrarily given nonuniform velocity field.

It is of interest to show how we may derive the results of the special theory of relativity for the case of small uniform velocity $|\bar{v}|\ll c$ directly from (10) and (15). Let us assume that the X' frame of reference is moving with a constant velocity $\bar{v}$ with respect to the X frame of reference, which is at rest. From (10) one has:

$$\nabla\times\bar{E} = -\frac{\partial\bar{B}}{\partial t} \tag{16a}$$

$$\nabla\times\bar{E}' = -\frac{\partial\bar{B}'}{\partial t} + \nabla\times(\bar{v}\times\bar{B}'). \tag{16b}$$

In addition one has:

$$\nabla\cdot\bar{B} = 0 \tag{17a}$$

$$\nabla\cdot\bar{B}' = 0. \tag{17b}$$

From (15) one has:

$$\nabla\times\bar{H} = \frac{\partial\bar{D}}{\partial t} + \bar{J} \tag{18a}$$

$$\nabla\times\bar{H}' = \frac{\partial\bar{D}'}{\partial t} + \bar{J}' + \rho'\bar{v} - \nabla\times(\bar{v}\times\bar{D}'). \tag{18b}$$

In addition one has:

$$\nabla\cdot\bar{D} = \rho \tag{19a}$$

$$\nabla\cdot\bar{D}' = \rho'. \tag{19b}$$

In (16) to (19), both space and time variables are with respect to the X frame of reference, which is at rest. From (17) one has:

$$\bar{B}' = \bar{B}. \tag{20a}$$

Since the charge density ρ is invariant in both frames of reference:

$$\rho' = \rho. \tag{20b}$$

One obtains from (19)

$$\bar{D}' = \bar{D}. \tag{20c}$$

Using (20a) in (16) one obtains:

$$\bar{E}' = \bar{E} + \bar{v}\times\bar{B}. \tag{20d}$$

Using (20b) and (20c) in (18) one obtains:

$$\bar{J} = \bar{J}' + \rho\bar{v} \tag{20e}$$

$$\bar{H}' = \bar{H} - \bar{v}\times\bar{D}. \tag{20f}$$

Equation (20) is identical with the results of the special theory of relativity for small velocities [4].

It has been shown in the present correspondence that the first two Maxwell equations in their general form may be derived directly from the Faraday induction law (7), or (8), and from the corresponding "Maxwell induction law" (11), or (12). Since those induction laws are given in a very simple form, they could constitute the most basic laws in electromagnetic theory, together with Gauss' laws.

During the past decade, numerous texts have been published in electromagnetic field theory, and most of them do not include the derivation of the Faraday induction law for moving media in its rigorous form [1], [2], [3]. In the light of the present correspondence, it is hoped that this will be corrected in the future.

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On the Definition of a Network

In the relatively recent literature one can find quite general definitions of a network as an operator taking currents in the operator's domain into voltages in its range [1], [2]. Philosophically one would then consider currents as inputs and voltages as outputs for such networks. However, one most often considers a network as a "black box" which is used in various contexts; sometimes currents are considered as inputs, sometimes voltages are so considered, etc., all for the same network. This latter fact led McMillan to define a network in terms of pairs of voltages and currents allowed at the terminals [3]. Although it seems that McMillan's formulation is the most physically meaningful of those available, it is limited to a very special class of networks (those described by a finite number of ordinary differential equations with constant coefficients). Consequently, here we give a definition of a network using the framework of McMillan but allowing the generality of the more modern approaches.

We first consider some preliminaries. Given a physical n-port network $\mathfrak{N}$ one associates with it pairs of terminals (T_j, T_j'), $j=1, \cdots, n$, called the ports. With each port can be further associated two variables v_j and i_j which, respectively, represent the voltage of T_j with respect to T_j' and the current entering T_j and leaving T_j'. Associated with $\mathfrak{N}$ are then two sets of n-vector real-valued functions of time t, the port voltages $\boldsymbol{v}=[v_j]$ and port currents $\boldsymbol{i}=[i_j]$. Letting a tilde, $\sim$, denote the transpose, we see that $p(t)=\tilde{\boldsymbol{v}}\boldsymbol{i}$ denotes the total instantaneous power into $\mathfrak{N}$.

Since we are unable to comprehend the beginning of time, and since we believe engineering constructions to be of recent origin in the history of the universe, we assume all voltage and current variables to be zero until some finite time (which may depend upon the particular $\boldsymbol{v}$ or $\boldsymbol{i}$ under consideration). This allows us to use the same time origin $t=-\infty$ for all networks and also builds in a type of causality, a priori [4]. Further, since no physical measurement can prove otherwise, we assume all $\boldsymbol{v}$ and $\boldsymbol{i}$ to be infinitely differentiable. For compactnesss we write $\mathfrak{D}_+$ to denote the set of infinitely differentiable n vectors which are zero until a finite time; thus $\boldsymbol{v}\in\mathfrak{D}_+$, $\boldsymbol{i}\in\mathfrak{D}_+$.

What distinguishes one network from another are the constraints C_N placed upon the components of $\boldsymbol{v}$ and $\boldsymbol{i}$. Among all the possible pairings $[\boldsymbol{v}, \boldsymbol{i}]$ only certain ones are allowed by the network constraints to be port variables; for conciseness we write $\boldsymbol{v}C_N\boldsymbol{i}$ if $[\boldsymbol{v}, \boldsymbol{i}]$ is an allowed pair of port variables for the network. In fact we are are free to read into $\boldsymbol{v}C_N\boldsymbol{i}$ the fact that $\boldsymbol{v}, \boldsymbol{i}\in\mathfrak{D}_+$ and the choice of $p=\tilde{\boldsymbol{v}}\boldsymbol{i}$. The constraints C_N then have the precise mathematical meaning of being a binary relation [5].

Given a physical n-port $\mathfrak{N}$, a binary relation C_N is defined between $\boldsymbol{v}\in\mathfrak{D}_+$ and $\boldsymbol{i}\in\mathfrak{D}_+$ which in turn defines a mathematical model $\boldsymbol{N}$ of $\mathfrak{N}$ through $[\boldsymbol{v}, \boldsymbol{i}]\in\mathfrak{N}$ if and only if $\boldsymbol{v}C_N\boldsymbol{i}$. For many studies it is convenient to identify $\boldsymbol{N}$ and $\mathfrak{N}$, calling both the n-port. We are then led to the following.

Definition: An *n-port network* $\boldsymbol{N}$ is defined as

$$\boldsymbol{N} = \{[\boldsymbol{v}, \boldsymbol{i}] \mid \boldsymbol{v}C_N\boldsymbol{i}\} \tag{1}$$

where C_N, the *network constraints*, is a binary relation with $\boldsymbol{v}\in\mathfrak{D}_+$, $\boldsymbol{i}\in\mathfrak{D}_+$.

Rephrasing: A network $\boldsymbol{N}$ is a set of allowed pairs of port variables $\boldsymbol{v}$ and $\boldsymbol{i}$, these being allowed by virtue of their satisfying

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the network constraints C_N. It is convenient to note that a network in this formulation is a special case of a nonoriented object [6]. The constraint $p=\tilde{v}i$ is a physical one, as is the interpretation of v as voltage and i as current. The interpretation previously used leads to electrical networks, but the mathematics allow other possibilities of physical significance when p is considered as power [7].

Since nothing more can be said about the constraints without considering specific examples we turn to two such for illustration. The 1-port resistor N_r of resistance r can be defined in the notation of (1) as $N_r=\{[v,\ i]\,|\,v=ri,\ v,\ i\in\mathfrak{D}_+\}$; that is, the allowed pairs are of the form of $[ri,\ i]$, this being for all $i\in\mathfrak{D}_+$ if r is independent of i and infinitely differentiable. The $(l+m)$-port transformer N_T can be defined through an $m\times l$ turns ratio matrix T by partitioning v and i into l and m-vector subvectors, $\tilde{v}=[\tilde{v}_1,\ \tilde{v}_2]$, $\tilde{\imath}=[\tilde{\imath}_1,\ \tilde{\imath}_2]$, and writing $N_T=\{[v,\ i]\,|\,v_1=\tilde{T}v_2,\ i_2=-Ti_1;\ v,\ i\in\mathfrak{D}_+\}$. Note that the resistor could easily be thought of as a map of currents into voltages, but that voltages are independent of currents for the transformer.

The ideal battery $v=V=$constant (for all time) is not a network in this formulation since $V\notin\mathfrak{D}_+$. But this is reasonable, since all batteries must be constructed; at the time of construction a battery's voltage can be considered as rising to V in an infinitely differentiable manner, in which case a battery becomes an N. However, since all distributions (in the sense of Schwartz [8]) are limits of infinitely differentiable functions zero until a finite time, any ideal network N_I (as the ideal battery) can be considered through

$$N_I=\{\lim_{j\to\infty}[v_j,\ i_j]\,|\,[v_j,\ i_j]\in N\},$$

where the limit is in the sense of distributions. The $\mathfrak{D}_+$ constraint is then seen to be a physical one which often can be relaxed; relaxing it sometimes leads to paradoxical results, however, such as violation of "causality" [4] or an inductor appearing nonreciprocal [9]. It is also worth observing that a unit step function of current can not be considered for the network whose voltage is the square of the derivative of the current. Sometimes it is useful to work with k-terminal networks where voltages are taken with respect to references other than port terminals. By the equivalence of k-terminal networks with $(k-1)$-ports, given by Cauer [10], the definition of (1) applies to $(n+1)$-terminal networks.

From the preceding one can see that the definition is cumbersome to work with directly, but so must be any definition at the envisaged level of generality. Consequently one investigates various classes of networks, such as the linear, passive, etc., in the standard manner [9]. However the difference between (1) and an operator definition is well worth reiterating. Equation (1) defines a network as a *set* of variables; operator definitions define a network as a (perhaps multivalued) *function* mapping a domain into a range. Besides placing equal importance to v and i, the set theoretical definition has certain notational advantages since the contained in notion can conveniently be used for defining related concepts. For instance the dual N_d of N is readily defined by $[v_d,\ i_d]\in N_d$ if and only if $[i_d,\ v_d]\in N$.

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Traveling-Wave Tube with 2.7 dB Noise Figure at 12 Gc/s

During the past year, work was carried out in the laboratories of the Watkins-Johnson Company that led to the development of an ultra-low-noise traveling-wave tube that operates over the 8.0 to 12.0 Gc/s frequency range. Terminal noise figure less than 5.0 dB across the full frequency band, with spot noise figures approaching 4.0 dB at the low-frequency end of the band, have been measured with the tube focused in a permanent magnet. The unique features of the tube have been described in a paper presented at the 1964 Electron Devices Meeting in Washington, D. C. The purpose of this communication is to report the most significant result of this program, i.e., the achievement of a noise figure of 2.7 dB at 12 Gc/s as measured at the input terminal. This result was accomplished by focusing the tube in a high-field solenoid with the helix cooled to liquid-nitrogen temperature, as has been previously described by Israelsen and Peter [1] (1.7 dB in L band) and Hammer and Thomas [2] (1.0 dB in S band).

The tube used in this experiment, the WJ-276-6, employs a multielectrode gun enabling optimum ajustment of the potential profile. The electron beam is extremely hollow, being drawn from a very thin-rim annular cathode. The low-loss helix is copper-plated tungsten wire supported by three quartz rods. Principal operating and design parameters are summarized below:

beam current	100 μA
helix voltage	400 V
helix TPI	175
helix ID	0.042 inch
wire diameter	0.003 inch
γa (at 12 G/cs)	3.7
cathode OD	0.024 inch
cathode ID	0.021 inch
gain	25 dB
power output	1 mW

The variation of terminal noise figure at 12 Gc/s as a function of axial magnetic field is shown in Fig. 1. The top curve was measured with the tube and solenoid at room temperature; the bottom curve with the tube and solenoid at liquid nitrogen temperature. All electrode voltages were reoptimized for best noise figure at each value of magnetic field. The difference in noise figure between the two curves reflects the reduced helix loss and reduced helix thermal noise as the tube is refrigerated [3], [4].

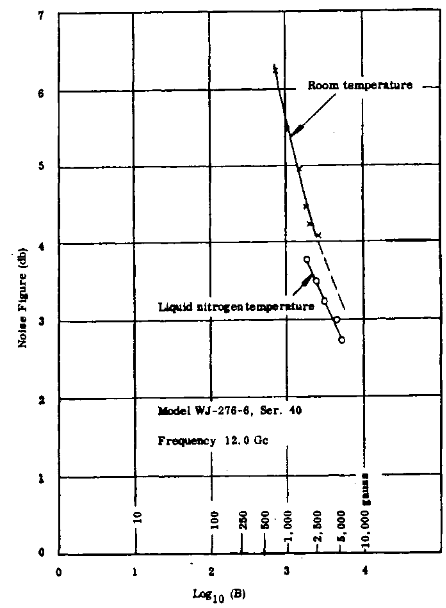

Fig. 1. Terminal noise figure of the WJ-276-6 vs the log of magnetic field with the tube focused in a high-field solenoid. Data shown in the top curve were taken with the tube and solenoid at room temperature, while the bottom curve represents conditions at liquid-nitrogen temperature. The dashed line is an extrapolation.

The reduction in noise figure obtained by the increase in magnetic focusing field is ascribed principally to the suppression of electrostatic lens effects in the electron gun [5] and to the reduction of beam noise in the region of the potential minimum where the beam voltage is less than a few tenths of a volt [6]. It is also of interest to note the absence of a lower limit on noise figure as the magnetic field is raised, indicating that further reduction in beam noise at higher values of magnetic field might be possible.

The noise figure was measured using a Hewlett-Packard model 340B noise figure meter and a Hewlett-Packard model X347A noise source with an excess noise of 15.2 $\pm$0.5 dB, including insertion loss.

The equation for the minimum noise

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