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Equations Depicting a Revolute Robot Tracing Torus Knots*

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In this paper we consider an application of semistate theory in the area of robotics. In particular a robot arm is depicted tracing a three-dimensional curve and more specifically a torus knot. In our development we desire the end-effector of a revolute robot arm (with three degrees of freedom) to trace a knot on a torus. Therefore the equation describing a torus knot can be added in the semistate equations as a constraint. In [1] a treatment of robots and in particular constrained robots shows that indeed the use of semistate theory is ideal.

Addressing a problem such as the tracing of a two- or three-dimensional curve can be considered as the first step towards completing a much more involved task, that is, having more than one robot arms employed in unison in order to tie various kinds of knots in the timber or fishing industry, for example, or sketching various types of automobile designs in the automotive industry. The depiction of a robot arm along with the curve that it is to trace is very useful if computer aided design is the intermediate step in realizing the actual task at hand. In particular such a depiction may provide significant insight as to the proper location of the robot relative to the curve or the need for employing more than one robot arms to successfully perform the required task.

Recall that a torus can be considered as a meridian circle revolved around an axial circle to form the doughnut shaped torus and that an (m_1, m_2)-torus knot is a trajectory on a torus which goes m_1 times around the meridian circle and m_2 times around the axial circle of the torus [2].

Based on the development in [2] the equation describing an (m_1, m_2)-torus knot in $\mathbb{R}^3$ is available along with its coordinates $(x(t), y(t), z(t))$ on the torus in $\mathbb{R}^3$ parameterized by t. We use a three-degree of freedom revolute robot [3, 4] whose inverse kinematic equations can be easily provided [4, pp. 117-126]. By inverse kinematic equations we refer to determining a particular set of link values, namely a waist rotation $\psi_1(t)$, a shoulder rotation $\psi_2(t)$ and an elbow rotation $\psi_3(t)$, parameterized by t, which we take to be robot time, that will produce a desired end-effector configuration, namely in our case, the tracing of a given point by the end-effector on the knot, that is, $(x(t), y(t), z(t))$ for any $t_0 \le t \le t_f$.

Let us now develop the equations that depict a revolute robot tracing a given point $(x(t), y(t), z(t))$ on a torus knot. Let us assume that the base of the robot is located at (x_b, y_b, z_b) in the $x-y-z$ plane. Here the assumption is made that given any point $(x(t), y(t), z(t))$, for $t_0 \le t \le t_f$, on the knot, (x_b, y_b, z_b) has been chosen so that the robot can trace the knot without conflict to its physical limitations.

Let

$$\begin{bmatrix} \bar{x}(t) \\ \bar{y}(t) \\ \bar{z}(t) \end{bmatrix} = \begin{bmatrix} x(t) - x_b \\ y(t) - y_b \\ z(t) - z_b \end{bmatrix}, \qquad (1)$$

for any $t_0 \le t \le t_f$.

Then using (1) and [4, pp. 117-126], with positive e, f, and h the known elbow length, forearm length and height of the robot, respectively, for any $t_0 \le t \le t_f$, a physically attainable triplet $(\psi_1(t), \psi_2(t), \psi_3(t))$ is assumed found. To keep the presentation of the remaining equations simple we refer to the triplet $(\psi_1(t), \psi_2(t), \psi_3(t))$ as (ψ_1, ψ_2, ψ_3) where the dependence on time t is implicitly understood. Also, since (1) is used in determining (ψ_1, ψ_2, ψ_3) it is apparent that this triplet will de-

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pend on the radii R_1 (radius of meridian circle) and R_2 (radius of axial circle) through which the knot is created [1]. Using this triplet solution and the coordinates indicated in Figure 1, after some work, the equations for drawing the robot tracing a particular point $(x(t), y(t), z(t))$, for a given $t_0 \leq t \leq t_f$ are as follows. In the robot base frame, points on the trunk are parameterized by a variable l,

$$x_0(l) = x_b,\ y_0(l) = y_b,\ z_0(l) = l + z_b, \qquad (2a, b, c)$$

where

$$0 \leq l \leq h. \qquad (3)$$

In a frame centered at the shoulder, parameterized by a variable m, points on the upper arm are given by

$$x_1(m) = \cos(\psi_1)\cos(\psi_2)\cdot m + x_b, \qquad (4a)$$

$$y_1(m) = z_2(m) = \sin(\psi_1)\cos(\psi_2)\cdot m + y_b, \qquad (4b)$$

$$z_1(m) = \sin(\psi_2)\cdot m + h + z_b, \qquad (4c)$$

where

$$0 \leq m \leq e. \qquad (5)$$

In a frame centered at the elbow, parameterized by a variable k, points on the forearm are given by

$$\begin{aligned} x_3(k) &= \cos(\psi_1)\cos(\psi_2 + \psi_3)\cdot k \\ &+ \cos(\psi_1)\cos(\psi_2)\cdot e + x_b, \qquad (6a) \end{aligned}$$

$$\begin{aligned} y_3(k) &= x_2(k) = \sin(\psi_1)\cos(\psi_2 + \psi_3)\cdot k \\ &+ \sin(\psi_1)\cos(\psi_2)\cdot e + y_b, \qquad (6b) \end{aligned}$$

$$z_3(k) = \sin(\psi_2 + \psi_3)\cdot k + \sin(\psi_2)\cdot e + h + z_b, \quad (6c)$$

where

$$0 \leq k \leq f. \qquad (7)$$

For $k = f$ equations (6) are (1), with $x(t) = x_3(k)$, $y(t) = y_3(k)$, and $z(t) = z_3(k)$ where the ψ_i's vary in t.

Figure 2 shows a projection of an (m_1, m_2)-torus knot in the specific case of $m_1 = 2$, $m_2 = 3$, $R_1 = 6$, $R_2 = 2$. Here at $t = t_1 = 11$, we also draw a revolute robot employing eqs. (2)-(7) with $e = 4$, $f = 5$, $h = 3$, $x_b = 1$, $y_b = 1$, $z_b = 1$, and using inverse kinematics we find $(\psi_1(t_1) = 204.777°$, $\psi_2(t_1) = 10.858°$, $\psi_3(t_1) = -118.459°)$ as the solution corresponding to the end-effector desired configuration $(x(t_1) = -1.2,\ y(t_1) = -0.011,\ z(t_1) = -0.014)$. The base of the robot is indicated by an oversized dot, and the arrows on the knot indicate the sense of travel as time t increases.

Finally, a couple of important points need to be brought forward. Firstly, in our development here the orientation of the end-effector (tool) is ignored. The reason behind this is that we are interested in depicting a robotic manipulator tracing the desired knot and not otherwise manipulating it, therefore keeping the development as simple as possible. In distinction from tracing the knot, when the knot tying task is to be performed, the orientation of the end-effector is important and should be taken into account. In that case a six-degree of freedom robotic manipulator could be required which in turn would make the development much more involved. Secondly, the appropriate location of the robot relative to the given knot that is to be traced and the need for employing possibly more than one robot arms are provided in more detail in [5].

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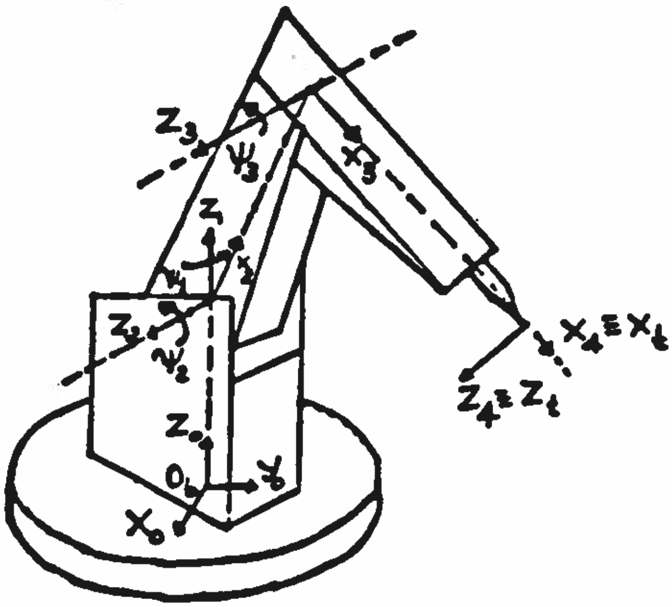

Figure 1. A Revolute Robot

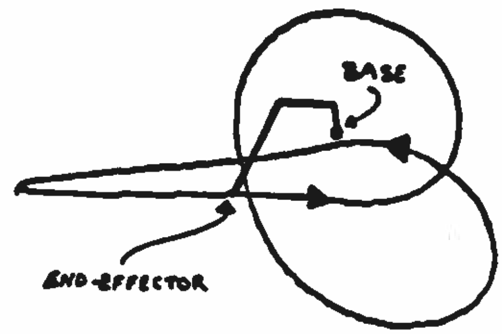

Figure 2. A (2,3)-Torus Knot and A Revolute Robot with End-Effector on It.

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