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Semistate and Circuits for Driver Neural Network Modules[‡]

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Abstract:
The driver module is a primary one in the biologically realistic neural network program SYNETSIM. Here we introduce semistate equations for the driver module which are suitable for the transistorized electric circuits which we intend to realize from these equations.

I. Introduction

Among the various neural network programs available SYNETSIM [1] stands out as being the most biologically realistic that will run on IBM compatabile PCs. Included in SYNETSIM are a number of modules which the user can insert to mimic various properties of neurons in neural networks. One of these modules is the DRIVER module which models the graded slow responses of some neuron membranes. In essence the driver module incorporates the slow opening and closing behavior of sodium and potassium channels. This modeling is done by allowing the effective conductances of associated channels in the driver module to vary according to laws, given below as Eqs. (1), determined by neurophysiologists. Here we interpret these laws in terms of a basic circuit, given in Fig. 1, with which we associate semistate equations, (2). From these latter we can set up a controlled source model, Fig. 2, which will lead to a transistorized version of the driver module (presently under development). Since the driver module is used in conjunction with other SYNETSIM modules, it can be considered to have an input and an output, and, hence, amenable to a semistate description useful for interconnections. However, because sometimes the membrane voltage is the input with the membrane current as output and sometimes the reverse, we develop two versions of semistate equations. Also we mention that because the nonlinearities are in the form of exponential conductances, these nonlinearities take some effort to properly realize with hardware.

II. The DRIVER Module Equations

In the driver module there are conductances for the main sodium and potassium channels, these two conductances being given by [2, eqs. (8.1, 8.2)]

$$G_i = \frac{\bar{G}_i}{1+\exp[(V_{0i}-v_m(t)/\mu_i]} \qquad i=Na,K \qquad (1a)$$

where v_m is the membrane potential, and $\bar{G}_i$, V_{0i} and μ_i are constants associated with the basic sodium, i=Na, or potassium, i=K, channels [physically $\bar{G}_i$ is a channel conductance for infinite membrane potential, V_{0i} is a half-activation voltage and μ_i is a characteristic voltage for a channel]. Also to go with these conductances are the reversal potentials, $V_{Na}>0$ and $V_K<0$. In addition, in the driver module, there are two other time dependent channels associated with potassium with the conductances for these satisfying the differential equation

$$\tau_{Ki}\dot{G}_{Ki} = \frac{\bar{G}_{Ki}}{1+\exp[(V_{0K}-v_m(t)/\mu_K]} - G_{Ki} \quad i=1,2 \quad (1b)$$

where the dot denotes differentiation with respect to time t and τ_{K1} and τ_{K2} are time constants for these two channels. Interpreting these equations in terms of batteries and resistors we directly obtain the circuit of Fig. 1 where the voltage dependent resistors are given by $R_i=1/G_i$ for i=Na,K,K1,K1.

III. Semistate Equations

Canonical semistate equations are equations of the form [3]

$$E\dot{x} = A(x,t) + Bu \qquad (2a)$$

$$y = Cx \qquad (2b)$$

where x is the semistate vector, u is the input, y is the output, and E, B, C are constant matrices; A is a mapping which includes the nonlinearities and explicit time variations of the system which in the case of a linear time-invariant system is given in terms of a constant matrix A as A·x. Our desire is to place the driver module equations into this form. Since there are many sets of possible semistate equations we wish to choose a set that is favorable to a realization by transistor circuits. For this we first convert Fig. 1 to Fig. 2 where we consider a conductance to be realized by a voltage controlled current source with the controlling (input) voltage being directly across (output) current source. With Fig. 2 in hand we proceed to set the semistate to be the 8-vector (where superscript T denotes the transpose)

$$x^T = [v_{CK1}, v_{CK2}, i_{GNa}, i_{GK}, i_{GK1}, i_{GK2}, v_m, i_m] \quad (3)$$

To continue, we note that the choice of input and output depends upon how the driver module is to be used. In the usual cases either the membrane voltage is the input and the membrane current the output or vice versa. Thus, depending upon the input-output pairing there are two cases of interest while in either case the dynamics is the same. We have

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$$\begin{bmatrix} \tau_{K1} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \tau_{K2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \dot{x} = \begin{bmatrix} -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot \\ \cdot & \cdot & 1 & 1 & 1 & 1 & \cdot & -1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \lambda_v & \lambda_i \end{bmatrix} x + \begin{bmatrix} G_{K1}(x_7) \\ G_{K2}(x_7) \\ G_{Na}(x_7)[x_7-V_{Na}] \\ G_K(x_7)[x_7-V_K] \\ x_1(x_7-V_K) \\ x_2(x_7-V_K) \\ 0 \\ 0 \end{bmatrix} + Bu \qquad (4a)$$

where the $\cdot$'s inside the matrices represent 0 and the λ_v and λ_i depend upon the choice of input, as seen below, and where we write in order to compact the expression

$$G_j(x) = \bar{G}_j(1+\exp[(V_{Oj}-x)/\mu_j])^{-1} \qquad (4b)$$

for $j=Na,K,K1,K2$ with $V_{OK1}=V_{OK2}=V_{OK}$. In obtaining the first two rows of (4a) we have used (1b) while taking numerically $x_1=G_{K1}$, $x_2=G_{K2}$; that is the capacitor voltages in Fig. 2 are taken numerically equal to the potassium subchannel conductances. Rows three through six give i_{Na}, i_K, i_{K1}, and i_{K2} in that order and we use the fact that every branch current is the branch conductance times the voltage across the conductor. The seventh equation is just Kirchhoff's current law giving the membrane current as the sum of membrane branch currents. Finally the eighth equation inserts the input into the semistate equations, there being two cases of interest. In (4a) we have written the linear part of $A(x,t)=A(x)$ via the matrix after the equal sign except for the x_1 and x_2 multipliers of $-V_K$ in the fifth and sixth rows; these are placed in the nonlinear vector since the physical realization via a multiplier is easier when x_7 and $-V_K$ are kept together as the pair x_7-V_K.

Case $u=v_m$, $y=i_m$
Here, by inspection, for equations (2)

$$B = [0,0,0,0,0,0,1,0]^T \qquad (5a)$$
$$C = [0,0,0,0,0,0,0,1] \qquad (5b)$$
$$\lambda_v = -1,\ \lambda_i = 0 \qquad (5c)$$

Case $u=i_m$, $y=v_m$
In this case

$$B = [0,0,0,0,0,0,0,1]^T$$
$$C = [0,0,0,0,0,0,1,0] \qquad (6a)$$
$$\lambda_v = 0,\ \lambda_i = -1 \qquad (6b)$$
$$(6c)$$

In summary, equations (4) with either (5) or (6) give the canonical semistate equations (2) for the driver module.

In order to physically construct a transistorized module we note that the four semistate variables x_3 through x_6 are currents controlled by voltages, that is, they are voltage controlled current sources (VCCS). We obtain the resistors needed for Fig. 1 by using the controlling voltages as those directly across the current source terminals. The best way to make a VCCS is usually to use a differential amplifier with a current mirror [4, pp. 431 - 454]. When BJT transistors are used this leads to an hyperbolic tangent gain function while when MOS transistors are used the result is a square root funtion, neither of which is exactly as needed for the nonlinear terms in (4). What we propose is to use BJT differential amplifiers with inputs V_{Oj} and v_m scaled by V_T/μ_j, V_T=thermal voltage, with outputs one of the collector currents. But rather than being connected as a normal differential amplifier the BJTs should form four quadrant multipliers with the other inputs being v_m-V_{Na} or v_m-V_K for the fifth and sixth rows of (4a). However, at this point the actual transistor implementation still requires details to be worked out.

IV. Discussion

In the above we have presented a set of canonical semistate equations for the driver module. Although a large number of other sets can be given the ones presented are of special interest since they should lead to transistor circuit realization of the driver module. The equations themselves result from a physical situation, that of the slow potential changes that occur across biological neuron membranes. However, they do present a number of difficulties for realization in terms of electronic hardware largely due to the fact that the equations include differential equations for conductance rather than for voltage and current. Consequently, a transfer of the equations into voltage and current ones is needed and that is carried out by numerically converting the conductances into voltages and then realizing the resulting currents via voltage controlled current sources. It is in this spirit that our equations (4) have evolved.

State variable types of equations work with inputs and outputs while driver modules are somewhat like resistors in that sometimes voltage is an input and sometimes current is an input, a property which one might call "non-orientedness." To reflect this non-orientedness of the driver module we have presented above two sets of driver module canonical semistate equations. In both of these the main semistate equations are essentially the same, as is seen in equation (4a), with only two entries in the linear portion of $A(x,t)$ being affected, these being the λ_v and λ_i entries.

References:

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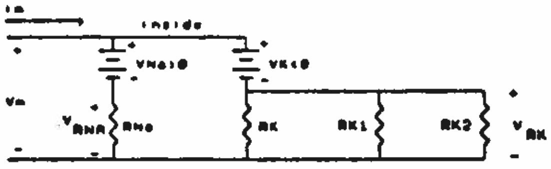

Figure 1
Circuit for Synetsim Equations

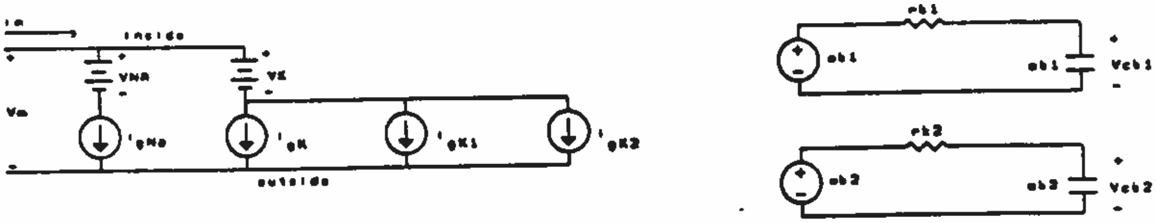

Figure 2
Equivalent for Semistate Equations

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