REALIZATION OF TRANSMISSION LATTICE FILTERS

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A double channel ARMA lattice filter is proposed. This filter is realized as a cascade of fundamental filter sections. The filter can be realized with the minimal number of delay elements and with no delay free loops, with the realization based on the given scattering function.

Introduction

There has been considerable success in modeling AR and MA processes in the past decade. The ARMA modeling can be used for speech analysis and synthesis [1]. Among them, the lattice structure has received an increasing amount of interest, because of its superiority in round off performance [2] and its ease of VLSI implementation [3].

The ARMA transmission lattice filter introduced here uses the lattice structure to channel signals that imitate those related to the movement of the fluid in the inner ear. Both channels of the cochlea [4] have individual incoming and reflecting forces as represented by signals in a standard lattice structure. In addition to the individual fluid force in each channel, motion in the upper channel of the cochlea affects the basilar membrane and this in turn affects the lower channel and vice versa. This phenomenon leads us to a lattice structure coupling between channels. Because this filter cascades into many sections, there are transmission signals passing between every two tandem sections. The transmission properties from one section to another section are important in the implementation of our filter design. Therefore, the term "transmission lattice filter" provides a meaning from the overall structural point of view.

Our transmission lattice filter is basically a cascaded type digital filter which divides the filter into several sections. The flexibility of the cascade structure allows us to arbitrarily add new sections of either AR or MA type to the end of the digital filter. The purpose of adding a new section is to increase the degree of the overall transmission lattice filter for obtaining better filter characteristics. In the issue of degree, the degree of each of our sections is equal to the number of delay elements in the section. Along with the cascade structure, we obtain the advantage that the overall degree is equal to the degree of the transfer function and as such is the number of all the delay elements present in the filter. This yields a minimal delay element realization of the transfer function.

Basic Structure and Properties

Scattering matrix concepts are well developed [5]. Our technique is closely related to 4-port synthesis through the use of the scattering matrix. Our ARMA digital filter can be looked upon as a cascade structure constructed from several elementary sections. The transfer scattering matrix is defined as the transformation taking the left-hand-side signals into the right-hand-side signals in each section.

For the properties of the transfer scattering matrix, let T_i be the transfer scattering matrix of the i th section. We regard the overall n cascaded sections as a box, with T_b as its transfer scattering matrix. Then T_b is the multiplication in opposite order of n individual transfer scattering matrices,

$$T_b = T_n T_{n-1} \cdots T_2 T_1 \tag{1}$$

where T_b is the 4x4 transfer scattering matrix for the structure of figure 1 without the input and load sections.

The basic structure of our filter are the integral sections shown in figures 2 for the AR and MA types, respectively, with their transfer scattering matrices being

$$T_{AR} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -ez^{-1} \\ 0 & 0 & 1 & -gz^{-1} \\ h & 0 & 0 & z^{-1} \end{bmatrix} \tag{2}$$

$$T_{MA} = \begin{bmatrix} z^{-1} & 0 & 0 & k \\ -dz^{-1} & 1 & 0 & 0 \\ -fz^{-1} & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

The integral section can constructed with four fundamental sections. Fundamental sections are the most simple components that can be used in forming a transmission lattice filter. A fundamental section is either a single delay 4-port section or a 4-port with a simple lattice signal flow from one port to the another port.

AR and MA integral sections have more meaningful properties than a single fundamental section, because they represent the degree one basic unit. Besides the integral sections, we introduce input and load sections to form the entire lattice filter structure.

The input and load sections are defined in figure 3 with their transfer scattering descriptions defined, respectively, by

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_x(0) \\ v_y(0) \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} v_x(1) \\ v_y(1) \\ v_r(1) \\ v_s(1) \end{bmatrix} \tag{4}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ a & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_x(n+1) \\ v_y(n+1) \\ v_r(n+1) \\ v_s(n+1) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{5}$$

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In the following theorems, let T_{AR} denote the transfer scattering matrix of an AR integral section. For n such sections cascaded, with different parameters in each section, we will denote $T^{(n)}_{AR}$ as their transfer scattering matrix. Let $\delta[A]$ represent the degree of A.

Theorem 1 (Scattering function in terms of T_b matrix):
Given an arbitrary rational scattering function $S(z^{-1})$

$$S(z^{-1}) = \frac{a_0+a_1z^{-1}+\cdots+a_nz^{-n}}{1+b_1z^{-1}+\cdots+b_mz^{-m}} \tag{6}$$

with real coefficients. It can be represented as the scattering function of a structure shown in figure 1 having

$$T_b = \begin{bmatrix} 1 & 0 & 0 & 0 \\ p(z^{-1}) & 1 & 0 & q(z^{-1}) \\ t(z^{-1}) & 0 & 1 & w(z^{-1}) \\ d & 0 & 0 & d \end{bmatrix} \tag{7}$$

Here again d means a don't care term, and

$$\begin{aligned} a &= a_0 \\ p(z^{-1}) &= -[a_1z^{-1}+\cdots+a_nz^{-n}] \\ q(z^{-1}) &= \frac{1}{1+ac}[b_1z^{-1}+\cdots+b_mz^{-m}-cp(z^{-1})] \end{aligned} \tag{8}$$

with 'a' representing the load. We have $\delta[p(z^{-1})] \le \delta[q(z^{-1})]$, and $\delta[t(z^{-1})] \le \delta[w(z^{-1})]$. $t(z^{-1})=cp(z^{-1})$, $w(z^{-1})=cq(z^{-1})$ and c is an arbitrary non-zero constant with $c \neq -1/a$.

The proof can be derived from the signal flow of input and load section in figure 1.

The choice $t(z^{-1})=cp(z^{-1})$, $w(z^{-1})=cq(z^{-1})$ is not necessary at this point but will be needed later in the decomposition algorithms.

Decomposition Algorithms

There are two types of decomposition algorithms to be given, these being called AR and MA types. The main purpose of the algorithms is to decompose a degree n matrix into the cascade form of a degree one AR or MA integral section and a degree n-1 remainder of transfer scattering matrix T'_b.

AR type decomposition algorithm

We recall that T_b is the transfer scattering matrix for the structure without input and load sections. Theorem 1 gives a resulting T_b matrix that can be derived from a given scattering function S. Assuming initially n>m, we write for (7)

$$T_b = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \sum_{i=1}^{n} p_i z^{-i} & 1 & 0 & \sum_{i=1}^{n} q_i z^{-i} \\ \sum_{i=1}^{n} t_i z^{-i} & 0 & 1 & \sum_{i=1}^{n} w_i z^{-i} \\ d & 0 & 0 & d \end{bmatrix} \tag{9}$$

where $t_i = cp_i$, $w_i = cq_i$, c is a non-zero constant, for i=1,...,n and $p_0=q_0=t_0=w_0=0$. Then T_b can be decomposed into

$$T_b = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \sum_{i=1}^{n-1} p_i' z^{-i} & 1 & 0 & \sum_{i=1}^{n-1} q_i' z^{-i} \\ \sum_{i=1}^{n-1} t_i' z^{-i} & 0 & 1 & \sum_{i=1}^{n-1} w_i' z^{-i} \\ d & 0 & 0 & d \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -e\cdot z^{-1} \\ 0 & 0 & 1 & -g\cdot z^{-1} \\ h & 0 & 0 & z^{-1} \end{bmatrix} \tag{10}$$

with the following formulas:

$$\begin{aligned} &p_0' = q_0' = 0 && t_0' = w_0' = 0 \\ &p_i' = p_i - hq_{i+1} && -e = q_1 \\ &q_i' = q_{i+1} && g = c\cdot e \\ &t_i' = c\cdot p_i' && h = \frac{p_{n-1}}{q_n} \\ &w_i' = c\cdot q_i' \end{aligned} \tag{11}$$

The relationships $t_i = cp_i$, $w_i = cq_i$ which go with T_b in (9) are necesary in this algorithm to obtain equation (11). For this reason we maintained the relationship $t(z^{-1})=cp(z^{-1})$ and $w(z^{-1})=cq(z^{-1})$ in Theorem 1.

MA type decomposition algorithm

This decomposition method is similar to AR decomposition algorithm. We can use MA section insteads of using AR sections applying to the algorithm.

Minimal Delay Elements and Delay Free Loops

The minimal degree realization, which is often the best situation of realization, is established when the number of delay elements used in the digital filter equals the degree of the given scattering function.

The decomposition algorithms just given allow the realization of an arbitrary real ARMA scattering function via a transmission lattice filter with the minimal number of delay elements. From theorem 1, we can easily conclude that if a given scattering function has degree n, $q(z^{-1})$ will also have degree n if c is properly chosen. The AR decomposition algorithm will take out a degree one integral section from the T_b matrix and leave the remainder as a degree n-1 transfer scattering matrix. By considering an AR transformation to an MA structure, the MA decomposition does the same thing. We can conclude that the overall degree needed for realization of a given scattering function is equal to the $q(z^{-1})$ in T_b matrix, i.e. degree n. This results in a minimal degree realization.

Related to the delay free loop, a given scattering matrix without estimation terms will result in a transmission lattice filter without delay free loops if the following rules are used in the realization process.

(1) The right most integral section must always be an AR type integral section.

(2) If degree of denominator in z^{-1} > degree of numerator in the scattering function, then the first 4 by 4 section (the section next to the input section) must be an AR type integral section.

(3) If degree of denominator in $z^{-1} \leq$ degree of numerator in the scattering function, then the first 4 by 4 section needs to be a zero-degree fundamental section. The section following it will be an AR type integral section. If the special case in which $\delta[p(z^{-1})] = \delta[q(z^{-1})]$ occurs from the application of theorem 1, then a constant fundamental section is used as a transformation to make sure that $\delta[p(z^{-1})] < \delta[q(z^{-1})]$.

(4) If a given scattering function is a constant, we only need the input and load sections to represent the filter without using any integral sections. The load constant a is given by the constant scattering function S, a=S.

No integral section has a delay free loop. Consequently any cascade connection of the integral sections has no delay free loop. When we connect the input and load sections no delay free loop is introduced because of the following. With reference to figure 1, rule (1) avoids signal flow from the v_s to the v_x branch. Consequently, we only need to worry about signal fed back from the load section. The load section allows signal flow from the v_x to the v_y branch. However this v_y signal will eventually reach the input section and be fed back to the v_s branch and will meet a delay element according to rules (2) and (3). Therefore, there is no delay free loops in the filter.

Application For Ear Type Digital Filter

An application of our transmission lattice filter is for digital filters representing signals related to signal processing within the ear, since the transmission lattice filter derived in the previous section is analogous to the ear structure. Here we design an ear type digital filter based on our realization process.

The ear scattering function is given by using the estimation algorithm mentioned in Miyanaga, Nagai and Miki's paper [6]. Their estimation algorithm will obtain a linear scattering function and can be applied to various signals, particularly to Kemp echo [7] signals obtained as stimulated emissions of the ear. Using this estimation on a Kemp echo we obtain the following ear scattering function $S(z^{-1})$

$$S(z^{-1}) = \frac{0.001+0.002z^{-1}+0.001z^{-7}}{1-0.73z^{-1}-0.15z^{-2}+0.15z^{-5}+0.54z^{-6}-0.73z^{-7}} = \frac{A(z^{-1})}{B(z^{-1})} \tag{12}$$

Following the realization process in this paper and assuming $t(z^{-1})=p(z^{-1})$ in the coupling matrix T_b, the load a in theorem 1 can be obtained from the constant term of the numerator, that is a=0.001. Although a high degree of accuracy is needed in the calculations, for a presentation purposes only rounded off values are given below. In the following equation d again means a don't care term. According to theorem 1, we obtain the T_b matrix for (12)

$$T_b = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -0.002z^{-1}-0.001z^{-7} & 1 & 0 & B(z^{-1})-1 \\ -0.002z^{-1}-0.001z^{-7} & 0 & 1 & B(z^{-1})-1 \\ d & 0 & 0 & d \end{bmatrix} \tag{13}$$

The process continues by applying decomposition algorithms to decompose the T_b matrix into several basic factors. Finally we obtain our ear type transmission lattice filter shown in figure 4 and with its basic factors as follows.

$$T_b = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -0.73z^{-1} \\ 0 & 0 & 1 & -0.73z^{-1} \\ d & 0 & 0 & d \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0.54z^{-1} \\ 0 & 0 & 1 & 0.54z^{-1} \\ 0.003 & 0 & 0 & z^{-1} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0.15z^{-1} \\ 0 & 0 & 1 & 0.15z^{-1} \\ 0 & 0 & 0 & z^{-1} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & z^{-1} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0.001 & 0 & 0 & z^{-1} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -0.15z^{-1} \\ 0 & 0 & 1 & -0.15z^{-1} \\ 0.001 & 0 & 0 & z^{-1} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -0.73z^{-1} \\ 0 & 0 & 1 & -0.73z^{-1} \\ 0.0014 & 0 & 0 & z^{-1} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0.0015 & 0 & 0 & 1 \end{bmatrix} \tag{14}$$

Conclusion

We have introduced and carried out a theory for the proposed transmission lattice filter. Algorithms are provided to identify the parameters in the filter. These algorithms are based on the decomposition of the overall transfer scattering matrix into the representation of each fundamental filter section. An application of our transmission lattice filter is for digital filters representing signals related to signal processing within the ear, since the transmission lattice filter is analogous to the ear structure. The resulting filter gives a linear representation of the ear structure by a given ear scattering function.

The realization process can realize a given scattering function as a stable transmission lattice filter if the given scattering function does not have poles outside the unit circle. Naturally, an unstable scattering function will be realized as an unstable transmission lattice filter. Likewise if the given scattering function has real coefficients only real degree one sections are needed. However, the method easily extends to incorporate complex sections when there are complex coefficients in the scattering function.

Taking advantage of the simple structure, the expansion of the proposed filter to multiport networks is applicable. One of the practical applications will be to the analysis of signals coupling two ears, which is realistic to the human ear. Multiport networks increase the complexity of the network degree analysis. The other possibility of expansion is to two or even more dimensional structures. But how to solve the minimal degree realization is still an open problem. We will introduce the basic concept of the ear type transmission lattice filter. This concept will be helpful for the future expansion of either multiport networks or multi-dimensional structures.

References

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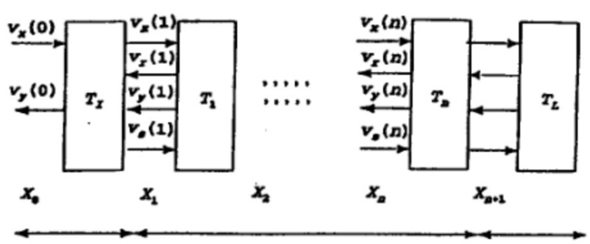

Figure 1 Transmission lattice filter and clarify ear structure

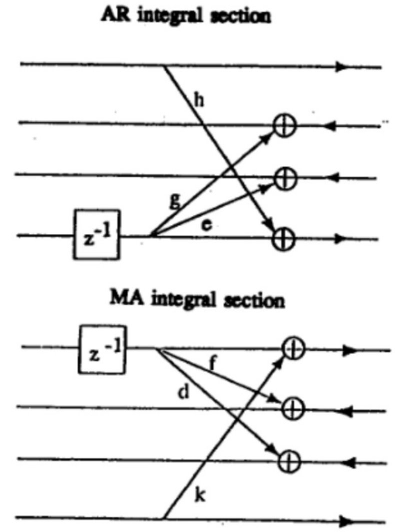

Figure 2 Basic integral sections

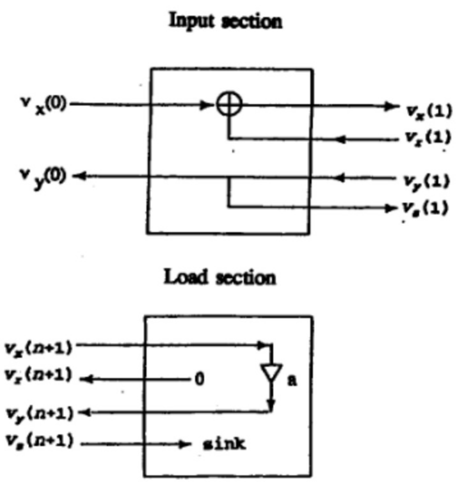

Figure 3 Input and Load sections

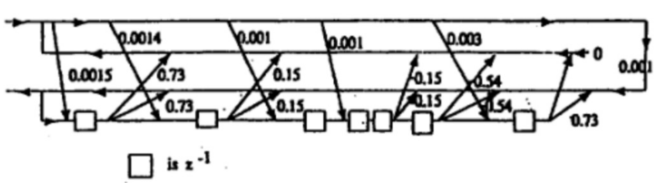

Figure 4 An ear type digital filter