AN ALGORITHM FOR TYING A STRING ON A TORUS USING TWO SYNCHRONIZED ROBOT ARMS *

Rassa Rassai, George Syrmos and
Robert W. Newcomb
Microsystems Laboratory
Electrical Engineering Department
University of Maryland
College Park, MD 20742
Phone: (301) 405-3662

ABSTRACT

In this paper we give an algorithm for tying a knot on a torus using two synchronized robot arms. This algorithm is used in programming two Rhino XR-3 series robot arms in BASIC. The two robot arms tie a knot on a torus by following a trajectory which is the path generated from the equations of an (m_1,m_2)-torus knot.

I. INTRODUCTION

Tying a knot is a simple task that we perform everyday of our lives. For example we tie a ribbon around a package, or we tie ropes while sailing or at the dock. However, we perform these tasks without thinking about the complicated process that the flex muscles in our hands and the joints in our fingers go through in order to tie a knot. There are times that under hazardous conditions one may need robots to perform the task of knot tying, for example in nuclear reactors. In this paper with reference to the findings in [1] we give an algorithm for tying a knot on a torus using two synchronized robot arms. The robots available to us had four rotation angles, namely waist, shoulder, elbow and wrist, however, each robot had one gripper comprising two fingers. In order to compensate for the third finger missing from each robot arm which in human hands mainly is used to support the knot, we used a torus mounted on two pins with flat bases and hook like tops as a support, Fig. 1a, [1]. The torus had a rough suface to prevent the string from sliding. In part II of this paper we give an algorithm for tying a string on a torus which is positioned at a distance "a/2" from each robot and at waist height , Fig. 1a. In part II we also discuss the workspace of the two robots, and precautions in order to avoid collision between the two robot arms as well as between the robot arms, the torus, and the base which supports the torus. In part III we discuss the experimental results obtained for tying a piece of string on a torus and in particular a (2,3)-torus knot using two synchronized Rhino XR-3 series robot arms. Part IV contains some discussion and conclusions.

II. Algorithm For Tying A Torus Knot On A Torus Using Two Two Synchronized Robot Arms

In [1] we gave the semistate equations for two synchronized robot arms for tying a knot on a torus. In [1], Table 1, we also gave the coordinates of the end-effectors of the two robot arms and mentioned that the coordinates of the end-effectors of the two robot arms in synchronization with one another should satisfy the coordinates of a point on the torus. Eqs(1), below give the coordinates of the end-effectors of two robot arms at a distance "a" from one another along the Y axis of the X-Y-Z coordinate systems. In Eqs.(1), R_a is the height of the waist, R_b is the length of the upper arm, R_c is the length of the forearm and R_d is the length of the the gripper. θ_{n1} and θ_{n2} where n=0,1,2,3 are the angles of waist, shoulder, elbow, and wrist rotations, respectively for robot #1 and robot #2. D_1 is the end-effector of robot #1 and D_2 is the end-effector of robot #2, Fig.1a.

$$X_{D1}=[R_bCOS\theta_{11}+R_cCOS(\theta_{11}+\theta_{21})+ R_dCOS(\theta_{11}+\theta_{21}+\theta_{31})]COS\theta_{01} \quad (1a)$$

$$X_{D2}=X_{D1} \quad (1b)$$

$$Y_{D1}=[R_bCOS\theta_{11}+R_cCOS(\theta_{11}+\theta_{21})+ R_dCOS(\theta_{11}+\theta_{21}+\theta_{31})]SIN\theta_{01} \quad (1c)$$

$$Y_{D2}=-Y_{D1}+a \quad (1d)$$

$$Z_{D1}=R_bSIN\theta_{11}+R_cSIN(\theta_{11}+\theta_{21})+ R_dSIN(\theta_{11}+\theta_{21}+\theta_{31})+Ra \quad (1e)$$

$$Z_{D2}=Z_{D1} \quad (1f)$$

In [1] we called X_T,Y_T, and Z_T the coordinates of a point on the torus centered at $X_{CT}=0$, $Y_{CT}=a/2$, and, $Z_{CT}=R_a$. Now in order to tie a torus knot using two synchronized robot arms, we assume both robots are at an initial position which for reference we call the HOME position. We call the rotation angles at HOME position θ_{n1H} and θ_{n2H} for each robot where n=0,1,2,3. As mentioned and explained in [1], we assign a hysteresis plane to each robot, where the hysteresis jump points are $Z_T=R_a-Z_1$ and $Z_T=R_a+Z_1$, Fig.2b. Z_1 is the smallest increment which can be taken by the end-effector of each robot. This value is given in the experimental results. We assign to robot #1 the task of moving to all points on the torus where $Z_T>R_a-Z_1$; the task assigned to robot #2 is that of moving to all points on the torus where $Z_T<R_a+Z_1$, [1]. With reference to our Flowchart, Fig.3, starting at a point where $ZT>R_a-Z_1$, robot #1 is activated first. To tie a knot we use a piece of string which is long enough that if the robot holds it in the middle it can still be wrapped around the torus. Robot #1 holds the string in the middle by closing its gripper and then it moves this middle of the string to a point where $Z_T>R_a-Z_1$. That means the coordinates of the end-effector of robot #1 now should satisfy the coordinates of a point on the torus. With reference to Table 1 and Table 2, [1], new joint angles are calculated. As the final values for the previous step are the new values for the next step, we call the new joint angles θ_{n1F}, θ_{n2F} for each robot, n=0,1,2,3. The motors controlling each joint of the robot only respond to the number of steps each should take. That means after we calculate the new joint angles we should find the difference between the initial and the final values of the joint angles and convert them into the number of steps each joint should

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take. Each joint of the robot has its own conversion equation from degrees into steps, which will be given in the experimental results. We call the difference between the final and the initial values of the rotation angles, d_{n1} and d_{n2} for robot #1 and robot #2, respectively, for n=0,1,2,3. The first initial joint angles for the robots would be the joint angles at HOME position which we call $\theta_{n1I}=\theta_{n1H}$, $\theta_{n2I}=\theta_{n2H}$. For all points on the torus where $Z_T>R_a-Z_1$, robot #1 remains activated. At the time when $Z_T=R_a-Z_1$, robot #1 opens its gripper and lets go of the string and goes to HOME position. At this time robot #2 is activated and moves to the point where robot #1 let go of the string. Robot #2 remains activated for as long as $Z_T<R_a+Z_1$. At the time when $Z_T=R_a+Z_1$, robot #2 opens its gripper and lets go of the string and goes to HOME position. This process continues until the completion of the knot, Fig. 1b. Since each robot after completing its task goes to HOME position there will not be a collision between the two robots. Another very important set of factors which should be taken into consideration is the set of precautions needed to avoid collision of the robot with the base. The base here consists of two pins with hook like tops which support the torus. The height of the pins is R_a. We call the planes defining points on pin #1, P_1 and the points on pin #2, P_2. We assume that the width of the pins on the Y axis is negligible and the length of the pin on the X axis satisfies $-1<X_{P1}=X_{P2}<+1$, where X_{P1} and X_{P2} are the lengths of the two pins along the X axis. The Z coordinates of all the points on both pins are less than or equal to R_a. In order to avoid the points defining the two pins, we go to Table 1, [1] and find the equations for all the links of robot #1 and robot #2, that is, the upper arm, forearm and the end-effector of each robot. If the coordinates of the points on the two pins satisfy any of the equations obtained for the links, this would indicate that there is a collision of one or more parts of the robots with the pins supporting the torus. Therefore, the robots should take appropriate measures to bypass the pins and approach the point from a different route. In order to bypass the pins the robot, once it detects an upcoming collision, lets go of the string by opening its gripper and pulls back from the torus such that there is enough safe distance between the torus, its supporting pins, and the robot. Depending on which side of which pin is approached by which robot, the active robot rotates its waist such that the robot will be able to approach the point from the opposite side of the pin while still being at a safe distance from the torus. From this new initial position the robot approaches the point where it had let go of the string and grabs the string by closing its gripper and continues its task, as was mentioned earlier. At the completion of the procees mentioned above we have an (m_1,m_2)-torus knot wrapped around the torus. If the torus can be disconnected and pulled out of the knot, one has an open ended torus knot. The algorithm mentioned above is given in our Flowchart, Fig.3. The maximum coordinates of the workspace, [1], of robot #1, X_{Wmax}, Y_{Wmax}, and Z_{Wmax}, can be found using the coordinates of the end-effector of robot #1 from Table 1 of [1] and Eqs(1).

$X_{Wmax}=R_b+R_c+R_d$ (2a)

$Y_{max}=R_b+R_c+R_d$ (2b)

$Z_{Wmax}=R_a+R_b+R_c+R_d$ (2c)

In addition, by using the coordinates of the end-effector of robot #1 from Table 1 of [1], we can find the minimum theoretically possible distance reached by robot #1 on each axis, namely X_{Wmin}, Y_{Wmin}, and Z_{Wmin} with these evaluated via Eqs. (3):

$X_{Wmin}=-(R_b+R_c+R_d)$ (3a)

which is obtained when

$\theta_{11}=\theta_{21}=\theta_{31}=0$, $\theta_{01}=180$ or

$\theta_{01}=\theta_{21}=\theta_{31}=0$, $\theta_{11}=180$

$Y_{Wmin}=-(R_b+R_c+R_d)$ (3b)

which is found by setting

$\theta_{11}=\theta_{21}=\theta_{31}=0$, $\theta_{01}=-90$ or

$\theta_{21}=\theta_{31}=0$, $\theta_{11}=180$, $\theta_{01}=90$

$Z_{Wmin}=0$ (3c)

which occurs when

$R_b SIN(\theta_{11})+R_c SIN(\theta_{11}+\theta_{21})+$
$R_d SIN(\theta_{11}+\theta_{21}+\theta_{31})=-R_a$.

with reference to Eqs. (2) and (3), we make sure that the torus is well within the workspace of both robots. That means

$a/2-(R_1+R_2)>Y_{Wmin}$ and (4a)

$a/2+(R_1+R_2)<Y_{Wmax}$ (4b)

Now we proceed to give the experimental results.

III. Experimental Results

In our experiment we used two identical Rhino XR-3 series robot arms, a detatchable torus with rough surface (to hold the string on the torus and prevent the string from sliding) mounted on two pins with flat bases and hook like tops, and a long piece of string. In synchronization with each other the robot arms tied the string on the torus using the algorithm explained in part II. The result was an (m_1,m_2)-torus knot, [1]. Specifically, in our experiment we constructed a (2,3)-torus knot. The robots were identical with the following specifications (phrased for robot #1).

$0<\theta_{01}<360$, $0<\theta_{11}<250$, $0<\theta_{21}<270$, $0<\theta_{31}<270$ (5a)

$R_a=10.25$, $R_b=9$, $R_c=9$, $R_d=6.25$ (5b)

Gripper rotation = infinite (5c)

Also we choose the joint angles of each robot to have the following initial values

$\theta_{01I}=\theta_{02I}=0$, $\theta_{11I}=\theta_{12I}=60$, $\theta_{21I}=\theta_{22I}=-60$, $\theta_{31I}=\theta_{32I}=-90$ (6)

With reference to Fig. 1a and the requirements for the height of the base supporting the torus, we can determine the dimensions of the torus, that is, R_1 and R_2 and the distance a, in terms of the dimensions of the robot arm for safe operation. We call b_1 the minimum safe distance between the pins supporting the torus and the robots and b_2 the maximum safe distance each robot can reach. b_1 was found to be equal to 15.5 inches and b_2 was found to be equal to 22.9 inches. Using the above results, we find the distance between each robot arm and the center of the torus, a/2, as follows

$a/2=(b_2+b_1)/2=19.2$ Inches (7)

The torus should have a diameter no larger than

$(b_2-b_1)/2=3.7$ Inches (8)

Consequently, we choose R_1+R_2 less than 3.7 for which we take

$R_2=R_d/5=1.25$ and $R_1=R_c/5=1.8$ (9)

We used BASIC to program the two robots. In programming the robot one converts the rotation angles into the number of steps that each joint of the robot takes. We start at an initial point where the joint angles are known and then we increment t; We call the number of steps taken by joints of each robot N_{n1} and N_{n2}. The conversion formula from degrees into steps for each joint of a Rhino robot is as follows, [3].

$$N_{o1}=4.4d_{o1}, \quad N_{11}=8.8d_{11}, \quad N_{21}=8.8d_{21}, \quad N_{31}=8.8d_{31} \quad (10a)$$

$$N_{o2}=4.4d_{o2}, \quad N_{12}=8.8d_{12}, \quad N_{22}=8.8d_{22}, \quad N_{32}=8.8d_{32} \quad (10b)$$

where d_{o1}, d_{11}, d_{21}, d_{31}, d_{o2}, d_{12}, d_{22}, and d_{32} are the differences in initial and final values of the joint angles of each robot. From Eqs(10), the minimum increment in the waist rotation is approximately 20 degrees and for shoulder, elbow, and wrist it is 8 degrees. Once the joint angles are converted into steps, we can guide the robot to move to a certain point on the torus while pulling the string along with it by activating the waist motor first and then shoulder, elbow, wrist, and gripper motors in sequence. Fig. 1a shows the movement of two robot arms tying an (m_1,m_2)-torus knot, in particular a (2,3)-torus knot. At the completion of these moves the last activated robot opens its gripper and goes to HOME position with the other robot being already at HOME position. The result is a (2,3)-torus knot wrapped around the torus, Fig. 2a. Now after both robots are at HOME position, if one pulls the torus and its base out of the knot (asumming the torus can be pulled out of the knot, which is possible in our setup) the string forms an open ended (2,3)-torus knot, Fig. 2b. The Flowchart of Fig. 3 shows the step by step procedure taken for tying an (m_1,m_2)-torus knot. In our experiment we used an IBM compatible PC with two communications ports. Opening communication with both robots at the start of the program gave us the freedom to experiment with both robots simultaneously or one at a time.

V. Conclusions

In this paper we showed how one can tie a knot on a torus using two synchronized robot arms. In our theory if we assume that the torus can be pulled out of the knot then the result is an open ended (m_1,m_2)-torus knot; in our actual experiments this was the case. Based on the results obtained here, we are currently doing research on methods of tying any knot using two synchronized robot arms without the use of a torus and the supporting bases for the torus.

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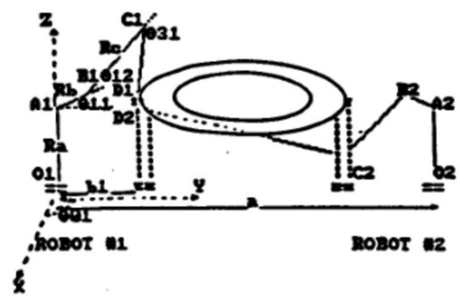

Fig.1a Two Synchronized Robot Arms

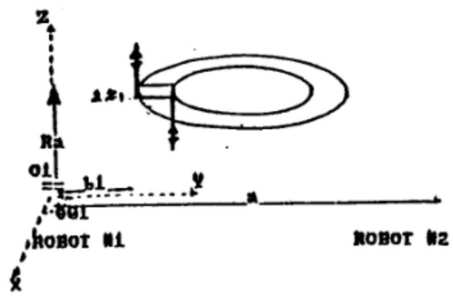

Fig.1b The Path Taken by Each Robot Arm On The Torus

Fig.2a A (2,3)-Torus knot Formed On A Torus

Fig.2b An Open Ended (2,3)-Torus Knot

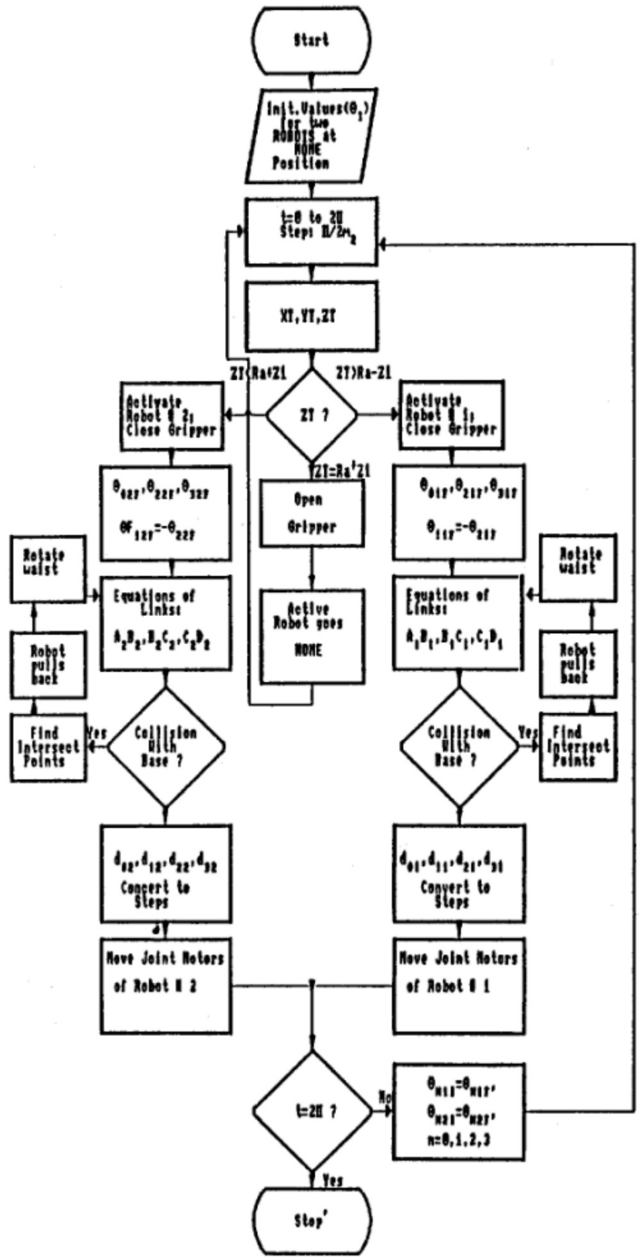

Fig. 3 The Flowchart for Tying an (m_1, m_2) Torus Knot Using Two Synchronized Robot Arms