

## Hysteresis Turn-On-Off Voltages for a Neural-Type Cell\*\*

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### Abstract

Bounds on the range of input voltages which lead to hysteresis in a neural-type cell are presented. These are important to the field of neural-type networks which operate on pulses, as do biological ones, since the presence of hysteresis allows for coding of the pulses.

### I. Introduction

Pulse coded neural networks are under considerable study since they mimic the actual operation of biological neural networks. Basic to the operation of such neural networks is the neural-type cell [1], the essential processing element of this class of networks. One of the characteristics of the electronic neural-type cell we use [2] is its voltage-controlled hysteresis which allows for the coding and generation of the neural-type pulses. For such a circuit it is important to have a design theory for the coding of the pulses so that one can do information processing with pulse coded neural-type circuits. However, as yet the severely nonlinear nature of the circuit, involving a rather complicated hysteresis, has precluded an analytic development. Here, though we are able to give a low upper bound on the smallest input voltage needed to turn on the hysteresis as well as an upper lower bound on the largest input voltage needed to turn off the hysteresis.

### II. The NTC

The CMOS circuit for the neural-type cell (NTC) under consideration here is shown in Fig. 1. For this circuit it can be seen that oscillations, as shown in Fig. 2, result at the output  $v_3$  due to constant input voltages,  $v_1$ , whenever the  $i-v$  curve seen by the C- $R_6$  contains hysteresis and  $R_6$  is appropriately chosen [3]. The appropriate choice for  $R_6$  is such that the load line it determines passes through the two vertical sections of the hysteresis thus creating unstable resting states.

### III. The Hysteresis Turn-On-Off Voltages

To obtain hysteresis in the circuit it is necessary to have both  $M_1$  and  $M_2$  pass through their ohmic regions when traversing the hysteresis curves since only then is there any

possibility of control of the output due to the feedback transistor  $M_3$ . Beyond this it is also necessary that  $M_3$  actually exerts the necessary control, which will only occur if its current can sufficiently modify the voltage across  $R_4$  to allow  $M_2$  to transition between its saturation and ohmic states. This requires that the drain current of  $M_3$  is sufficient to bring its drain to source voltage low enough to put  $M_3$  into the ohmic region but not sufficiently so as to cause it to lose control of the voltage on  $R_4$ .

Thus, as we raise  $v_1$  from 0 we first find a value, call it  $v_{1,low}$ , for which the desired hysteresis exists. As we raise it further we eventually reach a value, call it  $v_{1,high}$ , at which the hysteresis disappears. The value for  $v_{1,low}$  can be estimated by finding the smallest input voltage for which  $M_1$  and  $M_2$  simultaneously can be in their ohmic regions. If  $V_{T1} > 0$  and  $V_{T2} < 0$  are the respective threshold voltages of  $M_1$  and  $M_2$ , then this requirement is

$$v_2 \leq v_1 - V_{T1} \quad (1a)$$

$$v_4 - v_3 \leq v_4 - v_2 + V_{T2} \quad (1b)$$

This latter constraint is

$$v_3 \geq v_2 - V_{T2} \quad (1c)$$

Next we write KCL at the drain of  $M_1$  using the assumption that  $M_1$  and  $M_3$  are operating in the ohmic region [4, p.51]

$$v_2 = V_{dd} - R_4 \{ \beta_1 [2(v_1 - V_{T1})v_2 - v_2^2] + \beta_3 [2(v_3 - V_{T3})v_2 - v_2^2] \} \quad (3)$$

Substituting (1a) & (1c) with their equality signs into (3) gives

$$R_4 \{ (\beta_1 + \beta_3)(v_1 - V_{T1})^2 + [1 - 2R_4\beta_3(V_{T2} + V_{T3})] \cdot (v_1 - V_{T1}) + [V_{T1} - V_{dd} - R_4V_{T1}^2(\beta_1 + \beta_3) - 4R_4\beta_3V_{T1}V_{T3}] \} = 0 \quad (4)$$

Solving for  $v_1$  gives, using  $G_4 = 1/R_4$ ,

$$v_{1,low} = V_{T1} + \frac{[\beta_3(V_{T2} + V_{T3}) - \frac{1}{2}G_4 + \frac{1}{2}G_4\sqrt{\delta}]}{(\beta_1 + \beta_3)} \quad (5a)$$

where

$$\delta = [1 - 2R_4(\beta_1 + \beta_3)V_{T1} - 2R_4\beta_3(V_{T2} + V_{T3})]^2 - 4R_4(\beta_1 + \beta_3) \cdot [R_4(\beta_1 + \beta_3)V_{T1}^2 + 2R_4\beta_3V_{T1}(V_{T2} + V_{T3}) - V_{T1} - V_{dd}] \quad (5b)$$

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For the upper value of the input voltage that gives hysteresis we note that control via  $M_3$  on the voltage  $v_2$  is lost when  $v_2$  approaches zero. This latter occurs if the voltage across  $R_4$  approaches  $V_{dd}$  and this will in turn happen when  $M_1$  is in the saturation region and satisfies

$$V_{dd} = R_4 \beta_1 (v_1 - V_{T1})^2 \quad (6)$$

Solving for  $v_1 = v_{1high}$  we have

$$v_{1high} = V_{T1} + (G_4 V_{dd} / \beta_1)^{1/2} \quad (7)$$

The relations (5) and (7) give us an approximation of the range of the input  $v_1$  for which oscillations are present in the output  $v_3$ . Two key assumptions that are implicit in the above results are that a)  $R_4$  is not so big that  $M_1$  loses control on  $v_2$  near the high end of  $v_1$  and b)  $R_4$  is not so small that  $M_2$  can never be turned on near the low end of  $v_1$ . On recognizing these assumptions we can obtain the approximate range of  $R_4$  over which relations (5) and (7) are valid.

The calculation for the minimum value of  $R_4$  proceeds as follows: For  $R_4$  small,  $V_{dd} - v_2$  is small so  $M_2$  is cut off. Therefore:

$$I_2 = 0, v_4 = V_{dd}, v_3 = 0, v_2 \geq V_{dd} + V_{T2} \quad (8)$$

As a consequence,  $M_3$  is also cut off. Let us suppose that  $M_1$  is ohmic, we have:

$$V_{dd} - R_4 \beta_1 [2(v_1 - V_{T1})v_2 - v_2^2] \geq V_{dd} + V_{T2} \quad (9)$$

Now suppose  $v_2 = V_{dd} + V_{T2}$  to simplify, replace in (9) taken as an equality, we get

$$v_1 = V_{T1} + \frac{1}{2}(V_{dd} + V_{T2}) - V_{T2} / [2R_4 \beta_1 (V_{dd} + V_{T2})] \quad (10)$$

And we know from our simulations that  $M_1$  is ohmic for small  $R_4$  if  $v_1 \geq v_{1high}$ . Once again, taking the equality, we can combine (7) and (10) which leads to an equation of the second order in  $x = 1/R_4$ :

$$V_{T2}^2 x^2 / [4\beta_1^2 (V_{dd} + V_{T2})^2] - [(V_{dd} / \beta_1) + V_{T2} / 2\beta_1] x + \frac{1}{4}(V_{dd} + V_{T2})^2 = 0 \quad (11)$$

Equation (11) can be solved for  $x$  and therefore for  $R_4$ . We get:

$$\beta_1 (V_{dd} + V_{T2})^2 [2V_{dd} + V_{T2} - 2(V_{dd}(V_{dd} + V_{T2}))^{1/2}] \quad (12)$$

The solution of (11) with the negative sign on the radical is chosen as the other solution has little physical meaning.

Proceeding toward the maximum value of  $R_4$ , we remark that for  $R_4$  big,  $v_2$  is small when  $v_1 = v_{1high}$ . For these conditions,  $M_1$  and  $M_3$  are ohmic while  $M_2$  is saturated. Neglecting  $I_3$  with respect to  $I_1$ , we get

$$v_2 = V_{dd} - R_4 \beta_1 [2(v_1 - V_{T1})v_2 - v_2^2] \quad (13)$$

which can be solved for  $v_1$  (in order to equate it to  $v_{1high}$ )

$$v_1 = V_{T1} + \frac{1}{2}v_2 + (V_{dd} - v_2) / 2R_4 \beta_1 v_2 \quad (14)$$

We then use (7) and (14) to obtain an equation in  $x = 1/R_4$ :

$$(V_{dd} - v_2)^2 x^2 / (4\beta_1^2 v_2^2) - (V_{dd} + v_2)x / (2\beta_1) + \frac{1}{4}v_2^2 = 0 \quad (15)$$

which can be solved for  $x$  to finally give

$$R_{4max} = (V_{dd} - v_2) / [\beta_1 [V_{dd} + v_2 - 2(V_{dd}v_2)^{1/2}] v_2^2] \quad (16)$$

where the solution of (15) with a plus sign on the radical is taken as we want the maximum possible value on  $R_4$ . In evaluating (16) a nonzero value for  $v_2$  is needed; since it is small it can be chosen near  $V_{T1}$ .

#### IV. Numerical Confirmation

Two cases were considered to check the above hysteresis turn on and off input voltages.

##### Case 1:

Using the values:

$$V_{dd} = 10V, \beta_1 = 5.5E-4, \beta_2 = 2.4E-4, \beta_3 = 5.5E-4, V_{T1} = V_{T3} = 0.7V, V_{T2} = -0.9V, R_4 = 2K\Omega$$

We find  $v_{1low} = 2.52V$  and  $v_{1high} = 3.7V$ . These are to be compared with the SPICE simulation values of  $v_{1low}$  being about 2.6V and  $v_{1high}$  being about 3.6V.

For these values using equations (12) & (16), we get  $R_{4min} = 840\Omega$ ,  $R_{4max} = 3.5k\Omega$  ( $v_2 = 1V$  being chosen), which is a good approximation of those,  $R_{4min} = 750\Omega$  and  $R_{4max} = 3.2k\Omega$ , measured in PSPICE simulations.

##### Case 2:

Using the values:

$$V_{dd} = 5V, \beta_1 = \beta_3 = 5.4E-5, \beta_2 = 2.5E-5, V_{T1} = V_{T3} = 0.997V, V_{T2} = -0.8V, R_4 = 20K\Omega$$

We find  $v_{1low} = 2.03V$  and  $v_{1high} = 3.14V$ . These are to be compared with the SPICE simulation values of  $v_{1low}$  close to 2.2V and  $v_{1high}$  close to 3.5V.

#### V. Conclusions

In the above we have given the input voltages needed to guarantee the hysteresis necessary to make an NTC generate neural-type pulses. Because the voltages determined are based upon physical reasoning, they are only approximate but they do give reasonable bounds which check NTC simulations. It should be noted that hysteresis is guaranteed between the hysteresis  $v_{1low}$  and  $v_{1high}$  input voltages for suitable ranges of parameters. In the above we show the calculation to determine the range of  $R_4$  while similar derivations can be performed to determine ranges of the  $\beta$ 's.

Until now design equations have not been readily available for NTCs. Therefore we have formulated the equations for  $v_{1low}$  and  $v_{1high}$  in terms of circuit design parameters. With the voltages determined as above we are in a better position to make engineering designs of pulse coded neural-type networks.

References

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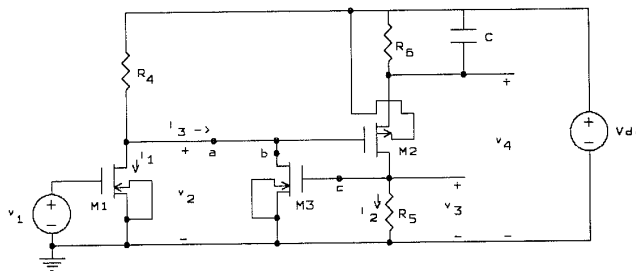
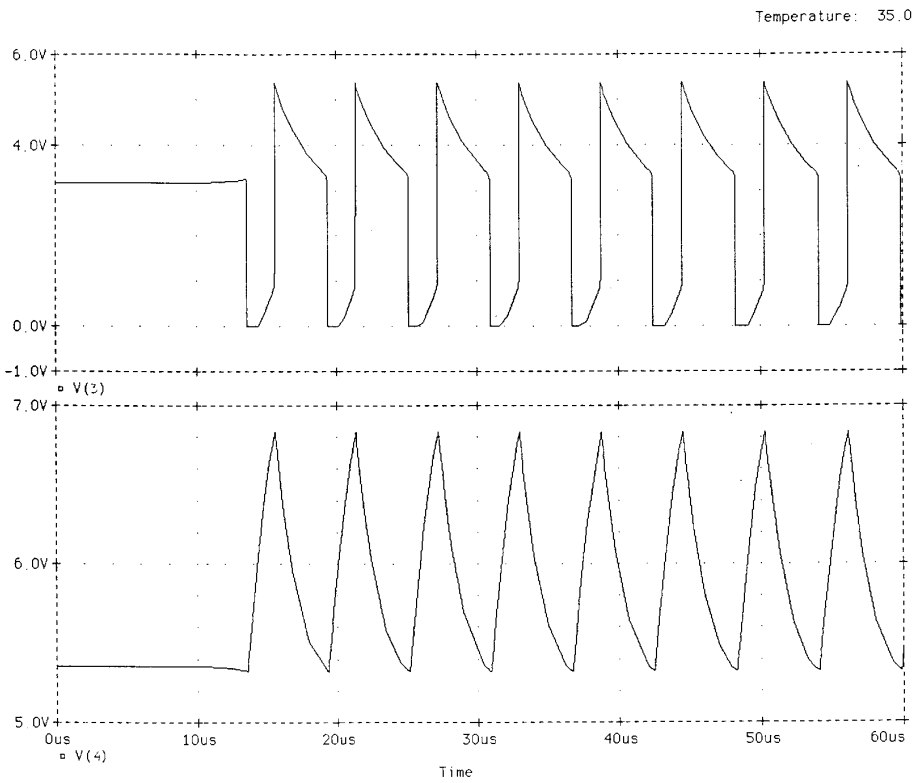
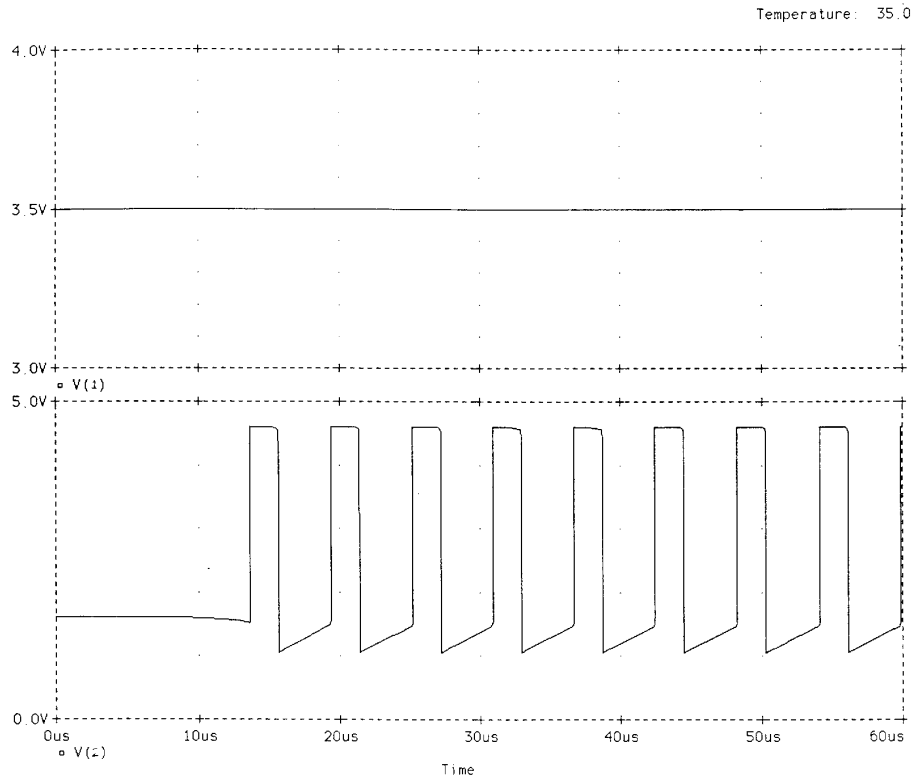


Figure 1. The Neural-Type Cell



**Figure 2. NTC Oscillations,  $v_1=3.5$   
 $v_1$  = input,  $v_3$  = output  
 $v_2$  and  $v_4$  = internal signals**