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Semistate Equivalency: The Lewis Realization*

R. W. Newcomb, C. Wooten and B. Dziurla
Microsystems Laboratory
Electrical Engineering Department
University of Maryland
College Park, MD 20742
phone: (301) 454-6869

Abstract

The concept of input-output equivalency of real linear lumped time-invariant (LLTI) systems is formulated in terms of semistate theory. Then this equivalency is tied to the Lewis backward-forward semistate decomposition which is reviewed and rederived. Finally a system synthesis based upon this Lewis decompostion is given which allows physical realization of any LLTI semistate described regular system via its Lewis equivalent system.

I. Introduction

It is well known [1, p. 51] that any regular continuous time real linear lumped time-invariant (LLTI) system with input u and output y can be described by a rational transfer function matrix, H(s), for which

$$y = H(s)u \qquad (1)$$

with s the derivative operator; here the input u and output y are real valued time-dependent vectors. It is also known [2, p.243][3] how to find a canonical semistate realization of the system of the form

$$Esx = Ax + Bu \qquad (2a)$$
$$y = Cx \qquad (2b)$$

where A,B,C,E are the real constant realization matrices and x is the semistate, a real-valued time-dependent vector. By eliminating the semistate x from (2) we know that the input-output description (1) and the semistate realization description (2) are related by

$$H(s) = C[Es-A]^{-1}B \qquad (3)$$

where, by the very definition of the word, regularity guarantees the existence of the inverse of the system matrix Es-A.

Fortunately for design in the real world there are infinitely many semistate realizations (2) which give the same transfer function H(s) as one often desires the most convenient one for construction with practical components. Consequently, we have an equivalence class of realizations with equivalence defined by (3). That is, given two semistate realizations $R_1=\{A_1,B_1,C_1,E_1\}$ and $R_2=\{A_2,B_2,C_2,E_2\}$ we say that they are defined to be equivalent, $R_1 \equiv R_2$, if and only if $H_1(s)=H_2(s)$. It should be noted that our equivalence is equivalence between input output pairings, and, hence, properly called input-output equivalence but we drop the modifier "input-output" since we only discuss this equivalence in this paper.

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Usually one can find at least one realization, for example by the method of [2], and one would like to generate from it all of the possible equivalent realizations in an effort to find all the hardware constructions that will yield the given transfer function and from which one can choose the most desirable. However, here we treat a more limited problem, that of finding a particular equivalent which we call the Lewis realization. To lead into the Lewis realization we first note a method to find a very large number of equivalents given one. For this we premultiply (2a) by P and let x=QX, where P and Q are nonsingular matrices, so that (2) becomes

$$PEQsX = PAQX + PBu \qquad (4a)$$
$$y = CQX \qquad (4b)$$

Clearly the realization {A,B,C,E} is equivalent to the realization {PAQ,PB,CQ,PEQ} since, reasoning on physical grounds, we have not destroyed the input-output relationship but only rearranged the equations and semistate components among themselves. Mathematically we can check this readily by noting

$$H(s)=C[Es-A]^{-1}B = CQ[PEQs-PAQ]^{-1}PB \qquad (5)$$

In the following we will choose P and Q to obtain the Lewis forward-backward realization.

II. The Lewis Forward-Backward Realization

In an interesting short paper Lewis [4] has shown for the purpose of solving semistate described systems that it can be convenient to decompose the solutions into two portions, one which proceeds forward in time and one that proceeds backward in time. To achieve this he uses a decomposition of the solution space of semistates. Here we translate this decomposition of the domain of semistates into matrix operations on the realization equations (1) as represented in (4).

The Lewis realization is characterized by

$$E_L = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & E_N \end{bmatrix} = I \oplus I \oplus E_N \qquad A_L = \begin{bmatrix} A_N & 0 & 0 \\ 0 & A_1 & 0 \\ 0 & 0 & I \end{bmatrix} = A_N \oplus A_1 \oplus I \qquad (6a,b)$$

where I and 0 denote identity and zero matrices, respectively, of appropriate sizes, $\oplus$ denotes the matrix direct sum, A_1 is a nonsingular matrix and E_N and A_N are nilpotent matrices (meaning that some power of them is zero). These latter two matrices can be written as the direct sum of matrices of the form

$$\begin{bmatrix} 0 & I \\ \cdot & \\ \cdot & \\ 0 \cdot \cdot \cdot & 0 \end{bmatrix} \qquad (6c)$$

where the zeros are scalar zeros. In (6a,b,c) any of the rows and columns may be absent if the corresponding portion of the decompostion is not present (for example if A has rank zero then only A_N is present and it is the direct sum of zeros). The transformations P and Q place also a transformation on the semistate and the input and outptu matrices B & C. For the semistate we break it into its forward, forward-backward, and backward parts as follows

$$X = Q^{-1}x = \begin{bmatrix} X^f \\ X^{fb} \\ X^b \end{bmatrix} \qquad (6d)$$

Here we have partitioned X as we have E_L and A_L.

To obtain this Lewis realization from any other (real) realization we need to find the associated P and Q. These transformation matrices can be found as follows. We assume given Es-A with this being a nonsingular differential operator matrix. Next we decide to treat s as a complex variable, the Laplace transform variable being a possible interpretation. Consequently, we choose some real value c of s for which Ec-A is a real nonsingular matrix. Using only real P_i & Q_i, i=1,...,6, in the following, we first transform by $Q_1=I$ and $P_1=(Ec-A)^{-1}$ to get

$$\begin{aligned} P_1(Es-A)Q_1 &= P_1[(Ec-A)+E(s-c)]Q_1 \\ &= I+(s-c)(Ec-A)^{-1}E \qquad (7a) \end{aligned}$$

Next we transform by $Q_2=P_2^{-1}$ where P_2 is chosen to bring $(Ec-A)^{-1}E$ into a direct sum of a nonsingular matrix J_1 and a nilpotent matrix J_N [5, pp. 301-306]. Then

$$\begin{aligned} &P_2P_1(Es-A)Q_1Q_2 = \\ &\quad [sJ_1-(cJ_1-I)]\oplus[sJ_N-(cJ_N-I)] \qquad (7b) \end{aligned}$$

This we in turn multiply by $P_3=J_1^{-1}\oplus(cJ_N-I)^{-1}$, with the indicated inverses existing by the nature of J_1 and J_N, and $Q_3=I$. The matrix $J_1^{-1}(cJ_1-I)$ is further brought to a nilpotent, K_N, and nonsingular, K_1, direct sum by a matrix P_4 with $Q_4=P_4^{-1}$; the matrix $(cJ_N-I)^{-1}J_N$ is similarly brought to a direct sum $L_1\oplus L_N$ with L_1 nonsingular and L_N nilpotent, by P_5 and $Q_5=P_5^{-1}$. At the end L_1 is brought to the identity by $P_6=L_1^{-1}$, $Q_6=I$. Thus

$$\begin{aligned} P(Es-A)Q &= \{[P_4\oplus(P_6P_5)]P_3P_2P_1\}\cdot \\ &\quad (Es-A)\{Q_1Q_2Q_3[Q_4\oplus(Q_5Q_6)]\} \qquad (7c) \\ &= (Is-K_N)\oplus(Is-K_1)\oplus(Is-L_1^{-1})\oplus(L_Ns-I) \qquad (7d) \\ &= (Is-K_N)\oplus(Is-[K_1\oplus(L_1^{-1})])\oplus(L_Ns-I) \qquad (7e) \end{aligned}$$

The last is in the form of (6a,b) with $A_1=[K_1\oplus L_1^{-1}]$ and P and Q the matrices in braces. The form of (6c) follows if we choose the Jordan form for the nilpotent parts $K_N=A_N$ and $L_N=E_N$, as is always possible ([5, p.301] where our form has zero eigenvalues for the nilpotent matrices).

III. Synthesis via the Lewis Equivalent

Given the Lewis realization formulated in the last section we can proceed toward a system construction. This is best looked at in signal flow graph form with the graph elements resulting from writing the canonical semistate equations of the Lewis realization as follows.

$$\begin{aligned} X^f &= (I/s)(A_NX^f + B^fu) \qquad (8a) \\ X^{fb} &= (I/s)(A_1X^{fb} + B^{fb}u) \qquad (8b) \\ X^b &= sE_NX^b - B^bu \qquad (8c) \\ y &= [C^f, C^{fb}, C^b]X \qquad (8d) \end{aligned}$$

Equations (8) have been written so as to express all components of X in a dependent manner which will allow them to be recovered in a system realization. In so doing the forward and forward-backward portions are expressed in terms of integration operators whereas the backward portion requires differentiators. Especially we note that the key is that in (8c) we have made X^b the dependent quantity so that the left hand side has a nonsingular coefficient matrix, I in this case. Figure 1 shows a signal flow graph implementation of (8).

IV. Discussion

Given a rational transfer function with real coefficients we have shown that there is always a Lewis realization and we have given the method to find that realization. This is accomplished by finding any realization and converting it via the transformations of (7) to the forward-backward form. The advantage of the Lewis realization is that it isolates that portion which requires differentiators, the backward portion. The Lewis realization could then be further transformed into other forms that might be more convenient for physical construction, for example by scaling the integrators to reasonable sizes.

Here we have limited the treatment to real systems, but the techniques actually could be carried over to complex valued systems if this were ever desired. Likewise we have treated continuous time systems, but by replacing the derivative operator, s, by the forward shift operator, z, we have equally treated discrete time systems. By allowing the semistate to be infinite dimensional we could extend the results to non-lumped systems and by properly considering the order of the operator s and the matrices appearing we can treat a large number of time-varying systems. It should also be noted that we can extend the form of (6a,b,c) to handle nonregular systems, that is systems for wich Es-A is singular [5. pp.35-48], essentially by inserting a direct sum of zero matrices.

Since it is possible to obtain minimal degree realizations, the technique can be used to achieve all equivalent minimal degree realizations. However, one can add internal semistates that do not affect the input-output relationship, and this theory certainly handles them. Still it should be noted that once the dimension of the semistate is fixed here, then we have not considered other dimensions of the semistate within the transformations given above. Thus, the dimensions of the semistate for the Lewis realization presented here is taken to be the dimension of the semistate of the original equivalent realization from which it is obtained.

References

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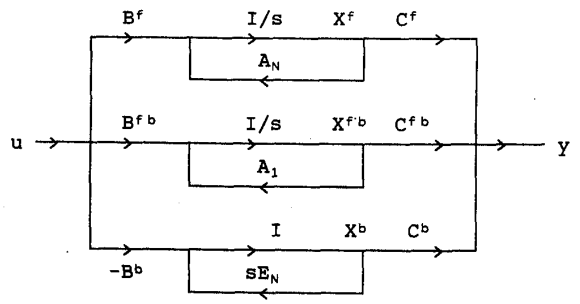

Figure 1
Signal Flow Graph for Lewis Semistate Realization