MULTIPLE NARROWBAND SOURCE LOCATION USING A FAST CONVERGENCE ITERATIVE METHOD

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ABSTRACT

A novel fast convergence iterative method is presented for multiple source direction finding. The method is an extension and improvement of the high resolution bearing estimation technique proposed by Yeh and Bayri. A high convergence rate and a high efficiency in computation are obtained. From the examples presented, it is seen, when compared with eigenstructure methods, that the technique can yield bearing estimation with higher resolution and lower variance, especially when the number of samples is small and the SNR is low.

I. Introduction

The problem of estimating the directions of arrival of multiple sources from measurements provided by a passive array of sensors has been extensively treated in the literature [1]-[5]. With a few spatial samples, the conventional DFT method can not obtain high resolution. Thus, alternative methods that provide higher resolution have been developed, for instance, the maximum entropy (ME) method, the minimum variance (MV) method and others [1]. These methods have provided increased resolution in the spatial domain. Furthermore, in recent years, there has been a growing interest in eigenstructure based methods. These latter methods, pioneered by Pisarenko [2], Schmidt [3] and Kumaresan [5], are known to yield high resolution and asymptotically unbiased estimates [4]. These modern high resolution methods provide excellent performance often approaching the Cramer-Rao lower bound at high SNR and with long data samples [7]. Unfortunately, the performance of the methods mentioned above degrade severely when the available number of data samples is small and the SNR is low. The maximum likelihood estimation is, therefore, of interest in applications. Bresler and Macovski [7] have presented an exact maximum likelihood (ML) estimation method. And Kumaresan and Show [8] independently have derived an almost identical algorithm. The ML methods do have excellent performance at low SNR. But a lot of computation is required.

Besides these ML methods, Yeh and Bayri [6] have presented an iterative high resolution bearing estimation algorithm by Covariance Matrix Approximation (YBCMA). This algorithm is simple in structure and of low SNR threshold. It has been extended to the wideband source location case in [9], and good results have been otbtained. Based on these results, a new fast convergence iterative method is presented in this paper. With this new method, the direction of arrival of each source signal can not only be estimated independently, but also can be fully utilized in following estimates. Thus, the iterative method is of a fast convergence rate. Meanwhile, according to some properties of a matrix and its inverse to be estimated, the method substitutes a simple approach for the complex computation of finding the matrix inverse in the iteration process so that high efficiency in computation is obtained. An example is given for testing the convergence rate and it shows that the proposed method has significant improvement compared to the method presented in [6]. Finally, some computer simulation results are given to compare to two of the more popular methods as reported in [3] and [5], namely, the MUSIC (Multiple Signal Classification) and the Minimum-Norm methods.

II. Problem Formulation

For convenience, we consider now that M narrowband sources with known identical center frequency f_0 impinge from directions $\theta_1, \theta_2, \ldots, \theta_M$ on a linear array of p sensors with a uniform spacing [12]. We also assume that the source signals are stationary uncorrelated stochastic processes. The signal at the output of the ith sensor is, therefore, to be described by [4, p. 638]

$$x_i(t) = \sum_{m=1}^{M} s_m(t) \exp(-j2\pi f_0 (D/c)(i-1)\sin\theta_m) + n_i(t) \quad (1)$$

where $s_m(t)$ is the signal received by the first sensor emitted from the M-th source, D is the spacing between two adjacent sensors and c is the speed of propagation. $n_i(t)$ is additive independent noise at the i-th sensor.

Assume that the received signals are sampled simultaneously at times $t_1, t_2, \ldots, t_n$, yielding N "snapshots" with each consisting of p samples $x_i(t_k), i = 1, 2, \ldots, p$. Grouping the samples corresponding to the P sensors into a $p \times 1$ vector, we can rewrite (1) in matrix form

$$X(t) = As(t) + N(t) \quad (2)$$

where $X(t)$ and $N(t)$ are the $p \times 1$ vectors

$$X(t) = [x_1(t), \ldots, x_p(t)]^T, \quad N(t) = [n_1(t), \ldots, n_p(t)]^T$$

$s(t)$ is the $M \times 1$ vector

$$s(t) = [s_1(t), \ldots, s_m(t)]^T$$

and A is the $p \times M$ matrix

$$A = [a(\theta_1), a(\theta_2), \ldots, a(\theta_m)] \quad (2a)$$

of k-th column

$$a(\theta_k) = [1, \exp(-j2\pi f_0(D/c)\sin\theta_k), \dots, \exp(-j2\pi f_0(D/c)(p-1)\sin\theta_k)]^T \quad (2b)$$

Note that each column of A is associated with a different source. We shall refer to these columns as the signal direction vectors.

The covariance matrix of the received signals can be expressed as

$$R = E\left[XX^H\right] = ASA^H + \sigma^2 I \quad (3)$$

where ^{H} denotes the conjugate transpose and E the expectation operator, I is the identity matrix and σ^2 the variance of the additive noise, and S is the covariance matrix of the sources, i.e.,

$$S = E\left[s(t)s(t)^H\right] \quad (3a)$$

Here, we have assumed that the sources are uncorrelated, so S is a diagonal matrix with diagonal elements $s_m, m = 1, \dots, M$. The signal component is

$$R_s = ASA^H \quad (3b)$$

In practice, the matrix R can only be estimated from a set of N snapshots, i.e.,

$$R = N^{-1} \cdot \sum_{t=1}^{N} X(t)X(t)^H \quad (3c)$$

Under the assumption that the number of sources is known, we can construct a matrix $\hat{R}_s$ to approximate R in the least square error sense, subject to the constraint that these matrices are of the structure of R_s in (3b), i.e., to minimize

$$\epsilon = \| \hat{R}_s - R \|^2 = \| \hat{A}\hat{S}\hat{A}^H - R \|^2 \quad (4)$$

by choosing estimated directions $\theta_m, m = 1, 2, \dots, M$, where $\| \cdot \|$ denotes the Euclidean norm. According to this, C. Yeh and H. Bayri [6] have presented the YBCMA method. In the next section, we will extend and improve it to yield a fast convergence iterative method.

III. A Fast Convergence Iterative Method

Equation (4) can be rewritten as

$$\hat{A}\hat{S}\hat{A}^H \approx R \quad (5)$$

where $\approx$ denotes equal in the least square error sense. Set

$$Q \triangleq \hat{S}\hat{A}^H = \left(\hat{A}^H\hat{A}\right)^{-1} \hat{A}^H R \quad (6)$$

and define two $M \times M$ matrices

$$V \triangleq \hat{A}^H\hat{A} \quad (7a)$$

and

$$W \triangleq V^{-1} = \left(\hat{A}^H\hat{A}\right)^{-1} \quad (7b)$$

where the elements of V can be expressed as

$$v_{ml} = \sum_{k=1}^{p} \exp\left(j2\pi f_0(k-1)(D/c)(\sin\hat{\theta}_1 - \sin\hat{\theta}_m)\right) \quad (8)$$

Hence, each signal direction vector can be obtained from $Q = W\hat{A}^H R$ or

$$q_m = w_m\hat{A}^H R \qquad m = 1, 2, \dots, M. \quad (9)$$

where q_m and w_m are the m-th row of Q and W, respectively. The search function for $\hat{\theta}_m$ is as follows

$$E_m = | q_m a(\theta) | \quad m = 1, 2, \dots, M \quad (10)$$

Here $\hat{\theta}_m$ is chosen as the angle for which E_m is a maximum. The new $\hat{\theta}_m$ is then used in $\hat{A}$ and V to update one column in $\hat{A}$ and one row and one column in V, respectively. Then, the new W can be obtained by inverting V according to (7b).

In each step of computing W, the inverse of V is calculated by using the new iterative algorithm summarized below. Its elements change only a little at each step, which means that only one estimated bearing angle has a little change and will make a little change only in one column and one row of the matrix V. Thus, the inverse can be made by the one step iteration algorithm [11].

$$W^{(i)} = W^{(i-1)} + aW^{(i-1)}\left[I - V^{(i-1)}W^{(i-1)}\right] \quad (11)$$

where a is an acceleration factor which can be adjusted to accelerate th convergence (for simplicity, $a = 1$). The matrix V and its inverse W are Hermitian matrices, so only about half of elements in (11) need to be computed. Hence we can substitute (11) for the complex matrix inverse computation.

The new iterative algorithm is summarized as follows:

a) Choose a set of initial values $\hat{\theta}_m, m = 1, 2, \dots, M$. Set $i = 0$.

b) Compute $\hat{A}(i), V(i)$ according to (2a), (2b) and (7a).

c) Compute W from (11) except in the first step of the iteration, where the Gauss-Jordan Elimination method, or any other, may be used.

d) Compute q_m from (9).

e) Find $\hat{\theta}_m$ according to (10). Update $\hat{A}(i)$ and $V(i)$. Then, return to c) until each $\hat{\theta}_m, m = 1, 2, \dots, M$, has been estimated in the i-th iteration. Then, set $i = i + 1$.

f) Repeat from b) to e) until convergence is achieved.

IV. Convergence Rate Analysis

In the YBCMA method of [6], we have noticed that errors of the initial values $\{\theta_m\}$ will influence each other, i.e., the error of an initial value θ_k will influence estimates of all other signal direction vectors. In the whole iteration process, although each of θ_m can be estimated independently, each of the signal direction vectors is estimated without fully utilizing the preceding estimated θ_{k-1} so that these influences are severe in the signal direction estimation process.

Observing a practical iteration process, we could well verify the qualitative analysis above. In Table 1, the iteration convergence processes of two methods have been listed. Assume two ideal narrowband source signals impinge from 30° and 60° on a uniform linar array of 5 sensors with $D = c/(2f_0)$. Assume that there is no noise so that R is accurate. We set the initial values $\hat{\theta}_1(0) = 33°$ and $\hat{\theta}_2(0) = 60°$. Here $\hat{\theta}_2(0)$ is taken to be the actual value, while $\hat{\theta}_1(0)$ is taken with some error. From the YBCMA method side of the Table, it can be seen that an initial error in $\hat{\theta}_1(0)$ will influence $\hat{\theta}_2(1)$, which was initially precise, and then, they will affect each other so that we can not obtain an accurate estimate of both bearing angles even at the 10th iteration.

Unlike the YBCMA method, the new algorithm estimates each signal direction vector by fully utilizing the newest estimation of the preceding bearing angles. The influence of the error of each bearing angle of sources on the other signal direction vectors is greatly reduced. In the right side of Table 1, the same experiment is done by using the new method, and results are listed there. We see that accurate bearing angles are obtained in just the first iteration step. A lot of similar experiments have shown that the convergence rate of the new method is much faster than that of the YBCMA method. Clearly, with the convergence rate increasing and the number of iterations decreasing, the number of computations of the non-linear search function (10) will be greatly reduced. Hence, this method is of high efficiency in computation.

V. Simulation Results

Some computer simulation results are presented in order to test the algorithm proposed in this paper and compare it with the MUSIC method [3] and the Minimum-Norm method [5]. To obtain a measure of statistical repeatability, we make one hundred independent generations of the covariance matrix estimate. The one hundred resultant bearing estimates are shown in each figure associated with a different method.

Two narrowband sources impinge from 30° and 48° on a uniform linear array of 7 sensors. $SNR = 15dB$. The number of snapshots is 15. The initial bearing angles in the new method are 18° and 52°. Figure 1 has shown that the MUSIC method and the Minimum-Norm method can resolve most of these signals, while the new method can resolve them in each experiment with small variance.

To look at worse conditions we lower the signal to noise ratio and we take a smaller difference of bearing angles. This is shown in Fig. 2, where $SNR = 10dB$ and the bearing angles are 30° and 40°, other conditions being the same as in the above simulation. It is clearly seen that the MUSIC and the Minimum-Norm methods can not resolve the two sources at all in such severe conditions, but the new method still has good results.

Conclusions

Fully utilizing each preceding bearing angle estimated, we have obtained a new fast convergence iterative algorithm for estimating the directions of arrival of radiating sources. The algorithm simplifies the matrix inverse computation involved in the iterative procedure by taking advantage of the special structure of matrices involved. It is of high efficiency in computation. The simulations have shown that the algorithm has higher resolution and lower variance than the eigenstructure methods and with less computation.

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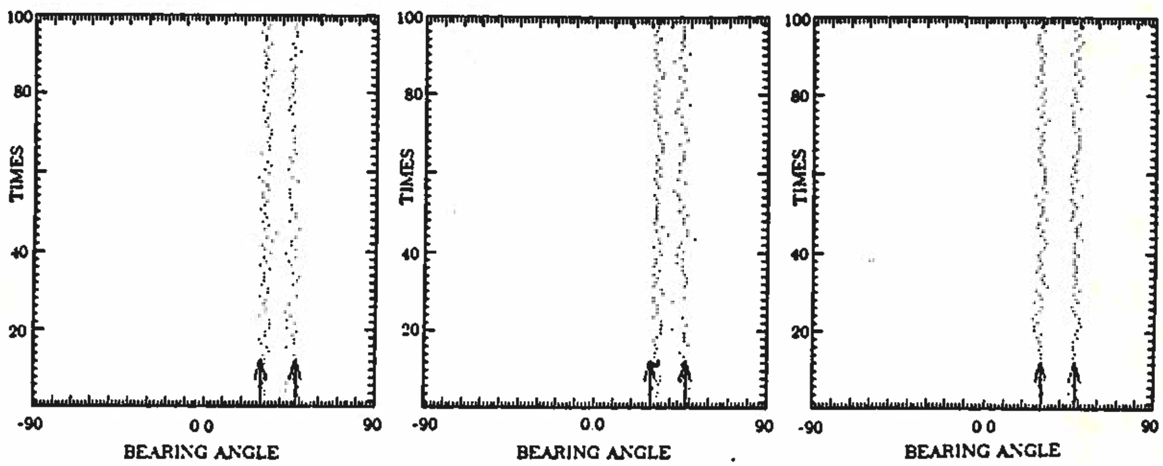
TABLE 1.

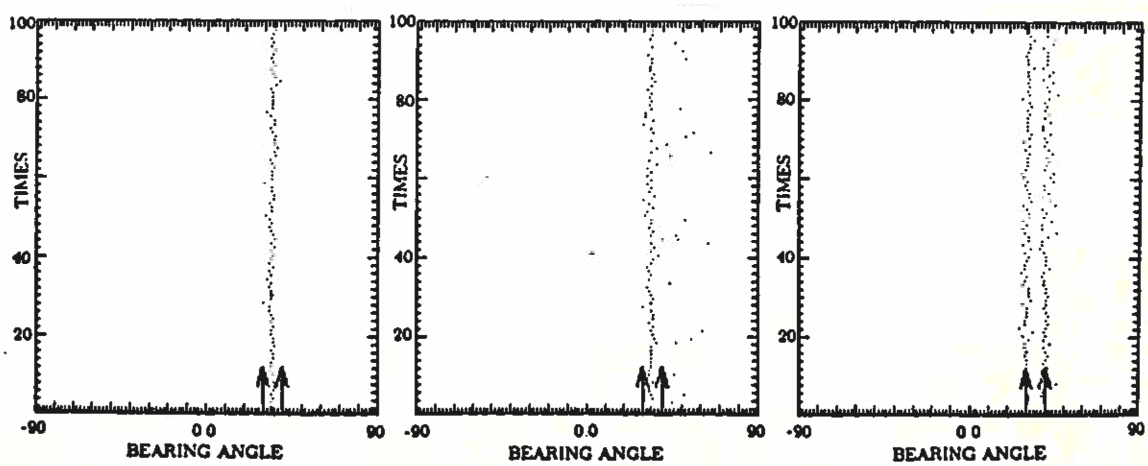
	YBCMA Method		New Method	
	$\theta_1=30°$	$\theta_2=60°$	$\theta_1=30°$	$\theta_2=60°$
0	33.0000	60.0000	33.0000	60.0000
1	30.0000	62.5000	30.0000	60.0000
2	30.6400	60.0000	30.0000	60.0000
3	30.0000	60.5210	30.0000	60.0000
4	30.1450	60.0110	30.0000	60.0000
	⋮	⋮	⋮	⋮
9	29.9990	60.0110	30.0000	60.0000
10	29.9992	60.0011	30.0000	60.0000



a. MUSIC method　b. Minimum-Norm method　c. The new method

Figure 1

Two Narrowband Source Signals Impinge from 30° and 48° on a Uniform Linear Array of 7 sensors. $SNR = 15dB$.



a. MUSIC method　b. Minimum-Norm method　c. The new method

Figure 2

Two Narrowband Source Signals Impinge from 30° and 40° on a Uniform Linear Array of 7 sensors. $SNR = 10dB$