A TWO-DIMENSIONAL APPROACH FOR FREQUENCY-WAVENUMBER ESTIMATION

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ABSTRACT

A new approach of high resolution estimates of frequencies and wavenumbers of multiple narrow band sources is presented in this paper. By fully using the properties of the auto-covariance and cross-covariance matrices, we construct a new matrix called the signal matrix. The theoretical analysis shows that the nonzero eigenvalues and the corresponding eigenvectors of the signal matrix are related to the center frequencies and the wavenumbers of incident source signals. Therefore, they can be used to estimate those frequencies and wavenumbers. Without 2-D spectrum-peak searches, this method is highly efficient in computation. Computer simulations showing improvements on an example from the literature are given.

I. Introduction

Estimation of the frequency and the wavenumber (which yields the direction-of-arrival (DOA)) of multiple narrow band sources by passive array systems is currently a topic of considerable interest in many areas, such as radar, sonar and seismology [3-5]. Generally speaking, the problem can be referred to as a two-dimensional spectral estimation problem. In recent years several eigenstructure approaches for 2-D spectral estimation have been proposed [1,3], but few of them are efficient in computation. They have to carry out the eigenanalysis of the mpxmp covariance matrix (p is the number of sensors and m the number of delay line taps following each sensor). Then they search for the spectral peaks in a 2-D frequency domain. Both steps entail a great deal of computation. Although the method proposed by the authors [7] can separately estimate the frequency and the wavenumber without 2-D searches, it requires extra computation to correctly pair one frequency with its associated wavenumber. Thus, it will be difficult to use for multiple incident source signals.

In this paper a new high resolution method is presented for estimating the frequency and the wavenumber of multiple narrow band sources. The pxp auto-covariance matrix and the pxp cross-covariance matrix are formed from the data of a sensor array that is sampled at each sensor and after one delay element. Using these two matrices, we define a pxp matrix, called the signal matrix. It is shown that the largest eigenvalues and corresponding eigenvectors can be used to estimate the frequencies and corresponding wavenumbers, respectively, without any searches. Hence, it is highly efficient in computation. Some computer simulation results are presented to illustrate the performance of the new method.

II. Problem Formulation

We consider a uniformly spaced linear array of p identical omnidirectional sensors, each followed by a tapped delay line with T delay units, as shown in Fig. 1. This array receives d narrow band signals centered at frequencies $\Omega_1, \Omega_2, \ldots, \Omega_d$ radians (i.e. the source bandwidth is much smaller than the reciprocal of the propagation time of the signal across the array). Assume that the source signals are stationary stochastic processes and that they are not fully correlated or coherent. The signal received at the i-th sensor can be expressed, using the narrowband assumption, as

$$x_i(t) = \sum_{k=1}^{d} s_k(t)\exp[-j\Omega_k(i-1)\tau_k] + n_{xi}(t) \tag{1}$$

where $s_k(\cdot)$ denotes the signal emitted by the k-th source observed at the first sensor; $\Omega_k=2\pi f_k$, τ_k is the path delay of the plane wave from the k-th source, $\tau_k=(D/c)\sin\theta_k$, where c is the wave propagation velocity, and D is the spacing between two adjacent sensors (note that the wavenumber Γ_k is related to the angle θ_k of arrival of the k-th source by $\Gamma_k=(f_k/c)\sin\theta_k$); $n_{xi}(t)$ is the additive noise on the i-th sensor, which is assumed to be a zero-mean white Gaussian stationary random process that is independent from sensor to sensor. Similarly, the signal, $y_i(t)=x_i(t-T)$, received at the delayed tap of the i-th sensor can be written, again using the narrowband assumption, as

$$y_i(t)= \sum_{k=1}^{d} s_k(t)\exp\{-j\Omega_k[T+(i-1)\tau_k]\}+n_{yi}(t) \tag{2}$$

where $n_{yi}(t)$ is the additive noise, with the same properties as the $n_{xi}(t)$ in Eq. (1).

Let $X^T(t)=[x_1(t),\ldots,x_p(t)]$ and $Y^T(t)=[y_1(t),\ldots,y_p(t)]$ be transposes of the simultaneously sampled vectors of array signals ($X^T(t_k)$ and $Y^T(t_k)$ for k=1,2,...,N are called <u>snapshots</u>). Equations (1) and (2) can be written for all sensors in vector form as

$$X(t) = As(t) + N_x(t) \tag{3}$$

$$Y(t) = A\Phi s(t) + N_y(t) \tag{4}$$

where $s^T(t)=[s_1(t),\ldots,s_d(t)]$; $N_x{}^T(t)=[n_{x1}(t),\ldots,n_{xp}(t)]$ and $N_y{}^T(t)=[n_{y1}(t),\ldots,n_{yp}(t)]$; A is a pxd Vandermonde matrix

$$A = [a(\theta_1),\ a(\theta_2),\ \ldots\ ,\ a(\theta_d)] \tag{5}$$

with k-th column

$$a^T(\theta_k) = [1,\ \ldots,\ \exp(-j\Omega_k(p-1)\tau_k)] \tag{6}$$

and Φ is a dxd diagonal matrix

$$\Phi = \mathrm{diag}[\exp(-j\Omega_1 T),\ \ldots\ ,\exp(-j\Omega_d T)] \tag{7}$$

Note that, since τ_k varies with θ_k, the k-th column of the matrix A, i.e. the $a(\theta_k)$, and the k-th element, $\exp(-j\Omega_k T)$, of the diagonal matrix Φ are associated with the DOA and frequency, i.e. (θ_k,Ω_k), of the k-th source. These parameters are to be estimated.

The auto-covariance matrix of X(t) is given by

$$R_{xx} = E[X(t)X(t)^H]$$

$$= ASA^H + \sigma^2 I = R_{xx0} + \sigma^2 I \tag{8}$$

where ^{H} denotes conjugate transpose and E[.] the expectation operator. $S=E[s(t)s(t)^H]$ is the dxd covariance matrix of source signals, I is the identity matrix, and σ^2 is the variance of the additive noise. R_{xx0} is the term in (8) associated with the source signals, $R_{xx0}=ASA^H$. The cross-covariance matrix of the vector Y(t) and X(t) is given by

$$R_{yx} = E[Y(t)X(t)^H]$$

$$= A\Phi SA^H+E[A\Phi s(t)N_x(t)]$$

$$+E[N_y(t)s(t)^H A^H]+E[N_y(t)N_x(t)^H] \tag{9}$$

We assume that the additive noises are uncorrelated with signals and with each other, in which case Eq. (9) can be written as

$$R_{yx} = A\Phi SA^H \tag{10}$$

All eigenstructure methods for estimating the DOA of narrow band sources are based on exploiting the structure of R_{xx} and/or R_{yx}. For instance, the MUSIC method [4] estimates only the parameters $\{\theta_k\}$ by using the eigenvectors of R_{xx}. The ESPRIT method [5,6] estimates the parameters $\{\theta_k\}$ from both R_{xx} and R_{yx}. The 2-D MUSIC method [3] has to carry out the eigendecomposition of a mpxmp autocovariance matrix and 2-D searches to obtain both parameters, $\{\Omega_k,\theta_k\}$. Here, we propose a new 2-D method to estimate both parameters, $\{\Omega_k,\theta_k\}$, from the two pxp matrices

R_{xx} and R_{yx}.

III. Estimating Frequency-Wavenumber via Signal Matrix

Now we define a pxp matrix R referred to as the signal matrix

$$R = R_{yx} R_{xxo}^{\#} \qquad (11)$$

where $R_{xxo}^{\#}$ denotes the pseudoinverse. The following theorem provides the foundation for the results presented herein.

Theorem: If S is non-singular, the signal matrix R has its d non-zero eigenvalues equal to the d diagonal elements of Φ and corresponding eigenvectors equal to the d column vectors of the matrix A, i.e.

$$RA = A\Phi \ . \qquad (12)$$

Proof: From $R_{xxo} = ASA^H$ we directly obtain

$$SA^H = (A^H A)^{-1} A^H R_{xxo} \qquad (13)$$

Substitute Eq. (13) into Eq. (10) to get

$$R_{yx} = A\Phi (A^H A)^{-1} A^H R_{xxo} \qquad (14)$$

Using the pseudoinverse $R_{xxo}^{\#}$ of the matrix R_{xxo}, we obtain

$$R_{yx} R_{xxo}^{\#} A = A\Phi \qquad (15)$$

where $R_{xxo}^{\#}$ is constructed by the d=Rank(S) non-zero eigenvalues, μ_i, and corresponding eigenvectors, v_i, of R_{xxo}.

$$R_{xxo}^{\#} = \sum_{i=1}^{d} \mu_i^{-1} v_i v_i^H \qquad (16)$$

From Eq. (11) and Eq. (15), we have

$$RA = A\Phi \ \blacksquare$$

Obviously, the signal matrix R has all the desired information of incident signals, and the frequencies and the wavenumbers (or DOAs) of radiating sources can be estimated from the eigenvalues and the eigenvectors, respectively, as seen from (6) and (7).

IV. A New Algorithm

According to the properties of the matrix R in (11), we obtain a new algorithm for array signal processing. It may be summarized as follows:

1) Estimate the pxp (p>d) auto-covariance matrix, R_{xx} and cross-covariance matrix R_{yx} using N snapshots by

$$R_{xx} = N^{-1} \sum_{t=1}^{N} X(t)X(t)^H \qquad (17)$$

$$R_{yx} = N^{-1} \sum_{t=1}^{N} Y(t)X(t)^H \qquad (18)$$

2) Compute the eigen-decomposition of R_{xx} with $\{\epsilon_k\}$ denoting the eigenvalues and $\{v_k\}$ representing the corresponding eigenvectors. Use $\mu_k = \epsilon_k - \sigma^2$ in (16), where σ^2 is the average of the smallest eigenvalues. Then, construct the signal matrix R from Eq. (16) and Eq. (11).

3) Compute the eigen-decomposition (12) of R to get the d largest eigenvalues $\{\sigma_i\}$ and corresponding eigenvectors $\{u_i\}$.

4) Estimate the frequencies $\{\Omega_k\}$ by [observe (7)]

$$\Omega_k = -T^{-1} \arg(\sigma_k) \qquad (19)$$

and corresponding wavenumbers $\{\Gamma_k\}$ or DOAs $\{\theta_k\}$ by [observe (5) and (6)]

$$\Gamma_k = -(p-1)^{-1} \sum_{i=2}^{p} [2\pi D(i-1)]^{-1} \arg[u_k(i)] \qquad (20)$$

$$\theta_k = \sin^{-1}(c\Gamma_k / f_k) \qquad (21)$$

respectively.

V. Simulation Results

We use the example, which is proposed by Wax et. al. [3], of two uncorrelated narrow band signals with normalized center frequencies 0.2, 0.3 and normalized wavenumbers 0.125, 0.2 that impinge on a linear array of 9 sensors, each followed by a delay line as shown in Figure 1. The number of snapshots is 100. To obtain a measure of statistical repeatability, we make one hundred independent trials in each figure. In Figure 2 for 10dB SNR values, it is shown that the new method can yield good estimates with low variance. When we reduce the SNR to 0dB, in

Figure 3, the new method can still resolve the two incident signals in such a low SNR, where as the method of [3] has great difficulty.

VI. Conclusions

Through a theoretical analysis, we propose a new method for estimating center frequencies and wavenumbers of multiple narrow band sources. It is shown that the center frequencies and wavenumbers of sources can be estimated from the non-zero eigenvalues and the corresponding eigenvectors of the signal matrix. Without any searches, this method is highly efficient in computations and resolves signals that cannot be resolved by other popular methods.

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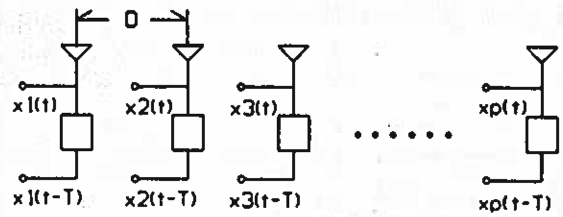

Figure 1.The geometry of the array system

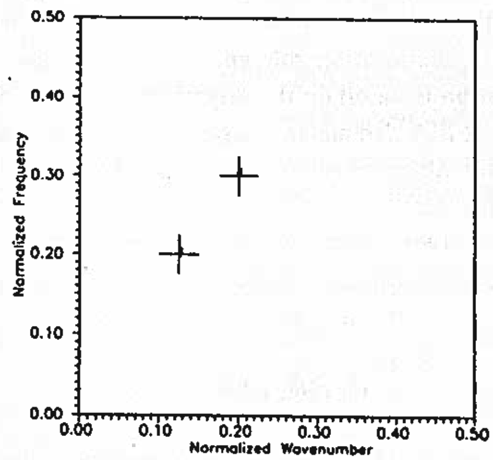

Figure 2. Two narrowband source signals with normalized frequencies 0.2, 0.3 and normalized wavenumbers 0.125, 0.2 impinge on a linear array system (p=9). N=100, SNR=10dB. Results consist of 100 independent trials.

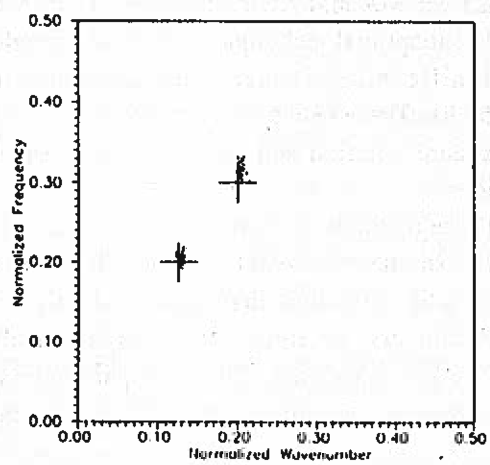

Figure 3. Situation as in Figure 2 except SNR=0dB. Results also consist of 100 independent trials.

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