ESTIMATION OF 2-D ANGLES-OF-ARRIVAL FOR COHERENT SIGNALS

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ABSTRACT

The problem of high resolution estimates of two dimensional (2-D) angles of arrival of coherent narrowband sources for a two parallel linear sensor array system is investigated. We extend the spatial smoothing technique developed by Shan, et al. [3], to the 2-D case for the two parallel linear array system to improve the DOA MATRIX method we previously proposed [6]. A modified DOA MATRIX method is, therefore, presented here. It allows uncorrelation, partially correlation and coherency between sources. Without 2-D searches, the method is highly efficient in computation, and, thus, is applicable to practical multipath environments.

I. Introduction

The basic signal processing problem in direction finding is to combine the outputs of an array system of sensors and produce estimates of the directions-of-arrival (DOA) of plane waves propagating across the array system. An eigenstructure method known as MUSIC due to Schmidt [1] has become popular since it yields high resolution and asymptotically unbiased estimates. Recently, Paulraj, et. al. [3,4] have proposed a subspace rotation approach called ESPRIT, which has better performance and less computations than previous ones. But, in the 2-D case where radiating sources are not located in a same plane, MUSIC has to carry out the 2-D searches requiring a lot of computations, while the ESPRIT can not be used. In [6] we have presented a method named the DOA MATRIX method, which yields good performance with less computations, for estimating the 2-D angles of arrival using the largest eigenvalues and corresponding eigenvectors of a matrix R, called the DOA matrix. However, like the ESPRIT method, it will encounter significant difficulties when some sources are coherent.

In this paper we present a new method for the coherent case, which extends the spatial smoothing technique, proposed by Evans, et. al. [2], to a two-parallel-linear-array system to modify the method presented in [6]. Theoretical analysis shows that the new method can "decorrelate" the coherent signals effectively. It yields the estimates of 2-D angles of arrival of narrowband sources from the largest eigenvalues and corresponding eigenvectors of the modified DOA matrix R. Without 2-D searches, as in the method of [6], computations can be saved. Computer simulation results in comparison with other methods are presented to illustrate the performance.

II. 2-D Direction Finding Method Via Two Parallel Linear Arrays

For convenience, the symbols and definitions in reference [6] will be employed here. Consider two uniformly spaced linear arrays of p sensors spaced D apart as shown in Fig 1. The two arrays, X_a and Y_a, are parallel with the array X_a lying on the X axis and starting at the origin; d is the distance between them as measured by the Y axis intercept of the array Y_a. Assume that plane waves with known center frequency Ω_0 emitted by K narrowband sources impinge on this array system and that the DOA of the sources are $(\theta_1,\theta_2,\ldots,\theta_K)$, where $\theta_k=(\alpha_k,\beta_k)$, α_k is the DOA of the k-th source relative to the X axis and β_k relative to the Y axis. Thus, γ_k, the DOA of the k-th source relative to the Z axis, can be given by

$$\cos^2\alpha_k+\cos^2\beta_k+\cos^2\gamma_k=1 \tag{1}$$

Let $X^T(t)=[x_1(t),\ldots,x_p(t)]$ and $Y^T(t)=[y_1(t),\ldots,y_p(t)]$ be the simultaneously sampled vectors (snapshots) of array signals. They can be written in vector form as

$$X(t)=As(t) + N_x(t) \tag{2}$$

$$Y(t)=A\Phi s(t) + N_y(t) \tag{3}$$

where $s^T(t)=[s_1(t),\ldots,s_K(t)]$; $N_x^T(t)=[n_{x1}(t),\ldots,n_{xp}(t)]$ and $N_y^T(t)=[n_{y1}(t),\ldots,n_{yp}(t)]$ as in [6]. A is the pxK Vandermonde matrix

$$A=[a(\alpha_1),a(\alpha_2),\ldots,a(\alpha_K)] \quad (4)$$

with k-th column

$$a^T(\alpha_k)=[1,\ \ldots\ ,\exp(j\Omega_0(p-1)\tau_{0k})] \quad (5)$$

where τ_{0k} is the inter-element path delay of the plane wave from the k-th source, $\tau_{0k}=(D/c)\cos\alpha_k$, where c is the wave propagation velocity. Φ is the KxK diagonal matrix

$$\Phi=\mathrm{diag}[\exp(j\Omega_0\tau_{d1}),\ \ldots\ ,\exp(j\Omega_0\tau_{dK})] \quad (6)$$

Here τ_{dk} is the path delay of the plane wave of the k-th source between the two arrays, $\tau_{dk}=(d/c)\cos\beta_k$. Note that the k-th column of the matrix A, i.e. the $a(\alpha_k)$, and the k-th element, $\exp(j\Omega_0\tau_{dk})$, of the diagonal matrix Φ are associated with the DOA, i.e. (α_k,β_k), of the k-th source. These parameters are those to be estimated.

The auto-covariance matrix and the cross-covariance matrix are given by

$$R_{xx}=E[X(t)X(t)^H]=ASA^H+\sigma^2I=R_{xx0}+\sigma^2I \quad (7)$$

$$R_{yx}=E[Y(t)X(t)^H]=A\Phi SA^H \quad (8)$$

where ^{H} denotes the conjugate transpose and E the expectation operator. $S=E[s(t)s(t)^H]$ is the covariance matrix of source signals, $R_{xx0}=ASA^H$, I is the identity matrix, and σ^2 is the variance of the additive noise.

Using R_{xx0} and R_{yx}, we can define a matrix R, called the DOA matrix.

$$R = R_{yx}R_{xx0}^{\#} \quad (9)$$

where $R_{xx0}^{\#}$ is the pseudoinverse of R_{xx0}, which is given by

$$R_{xx0}^{\#} = \sum_{i=1}^{K} \epsilon_i^{-1}u_iu_i^H \quad (10)$$

with $\{\epsilon_1\geq\epsilon_2\geq\ldots\geq\epsilon_K\}$ and $\{u_1,u_2,\ldots,u_K\}$ denoting K nonzero eigenvalues and corresponding eigenvectors of R_{xx0}, respectively. As we know, A is a pxK Vandermonde matrix and Φ is a KxK diagonal matrix, they are full rank matrices. Thus, the rank of R_{xx0} will be the same as the rank of S, and so is the rank of R_{yx}. S is the source signal covariance matrix, and it will be full rank, Rank(S)=K, if there are no coherent sources. The following theorem summarizes a very important property of the DOA matrix.

THEOREM 1: If S is nonsingular, the DOA matrix R has its K non-zero eigenvalues equal to the K diagonal elements of Φ and corresponding eigenvectors equal to the K column vectors of matrix A, i.e.

$$RA = A\Phi \quad (11)$$

Proof: See [6].

According to the eigenstructure of the DOA matrix R discussed above, we can estimate the DOAs of non-coherent radiating sources, (α_k,β_k), from the eigenvector and the eigenvalues, respectively.

In the subsequent development, we shall consider the problem of estimating the 2-D directions of arrival of coherent sources using the results in Theorem 1. In the following section, however, we will extend the procedure proposed by Evans, et. al. [2], to obtain both smoothed auto-covariance and cross-covariance matrices with their ranks being K. With these matrices we construct a modified DOA matrix so that the property of the DOA matrix can still be used to estimate the 2-D directions of arrival of coherent sources.

III. A Modified DOA Matrix

Consider that we have the same linear arrays as described in the previous section, and also K plane waves impinge on the array system. Here, we construct p-m+1 subarrays of size m (m>K) for each linear array, X_a and Y_a, in such a way that each one shares with an adjacent subarray all but one of its sensors as shown in Fig. 1. Let $R_{xx}^{(i)}$ and $R_{yx}^{(i)}$ denote the auto-covariance matrix and corresponding cross-covariance matrix of the received signals at the i-th subarray of X_a and Y_a, respectively.

$$R_{xx}^{(i)}=A_mG^{i-1}S(G^{i-1})^HA_m^H+\sigma^2I \quad (12)$$

$$R_{yx}^{(i)}=A_m\Phi G^{i-1}S(G^{i-1})^HA_m^H \quad (13)$$

where G is the KxK diagonal matrix

$$G=\mathrm{diag}[\exp(j\Omega_0\tau_{01}),\ldots,\exp(j\Omega_0\tau_{0K})] \quad (14)$$

A_m is the mxK Vandermonde matrix and is of the same form as A in (4).

Define the modified spatially smoothed autocovariance and cross-covariance matrices $\bar{R}_{xx}$ and $\bar{R}_{yx}$

$$\bar{R}_{xx}= (p-m+1)^{-1}\sum_{i=1}^{p-m+1} R_{xx}^{(i)} = A_m\bar{S}A_m^H+\sigma^2I = \bar{R}_{xx0}+\sigma^2I \quad (15)$$

$$\bar{R}_{yx}= (p-m+1)^{-1}\sum_{i=1}^{p-m+1} R_{yx}^{(i)}= A_m\Phi\bar{S}A_m^H \quad (16)$$

where $\bar{S}$ is the modified covariance matrix of the sources, given by

$$\bar{S} = (p-m+1)^{-1} \sum_{i=1}^{p-m+1} G^{i-1} S (G^{i-1})^H \qquad (17)$$

Obviously, $\bar{R}_{xx}$ and $\bar{R}_{yx}$ still have the same structure as R_{xx} and R_{yx}. It can be shown that the modified covariance matrix of the sources, $\bar{S}$, is always nonsingular.

Lemma: Assume that there is no zero row vector in any nxn matrix Q, and F any nxn diagonal matrix with different diagonal elements, i.e.

$$F = \mathrm{diag}[\ f_1,\ \ldots,\ f_n\ ] \qquad f_i \neq f_j,\ i \neq j.$$

If Rank(Q)=r<n, then Rank[Q,FQ]≥r+1, where [Q,FQ] is a nx2n matrix consisting of the columns of Q and FQ.

The new algorithm of 2-D direction finding for coherent sources is based on following theorem.

THEOREM 2: If m is greater than K, the number of sources, and p-m+1 is greater than or equal to the number of the sources contained in the largest coherent group, then, the modified covariance matrix of sources, $\bar{S}$, is nonsingular, and

$$\mathrm{Rank}(\bar{R}_{xxo}) = \mathrm{Rank}(\bar{R}_{yx}) = \mathrm{Rank}(\bar{S}) = K. \qquad (18)$$

Proof: We first form the Cholesky factorization of the source covariance matrix

$$S = EE^H \qquad (19)$$

For the coherent case, S is a KxK singular matrix, and so is E. From (17)

$$\bar{S} = (p-m+1)^{-1} \sum_{i=1}^{p-m+1} G^{i-1} EE^H (G^{i-1})^H \qquad (20)$$

$$= (p-m+1)^{-1} [E, GE, \ldots, G^{p-m}E] \begin{bmatrix} E^H \\ (GE)^H \\ \cdot \\ \cdot \\ \cdot \\ (G^{p-m}E)^H \end{bmatrix}$$

We know that G is a diagonal matrix with different diagonal elements. There are no zero row vectors in the KxK matrix S because it is a source covariance matrix. Therefore, there is no zero row in the matrix E. Observing (20), we know that the rank of $\bar{S}$ will be the same as the rank of $[E, GE, \ldots, G^{p-m}E]$. Based on the Lemma, here replacing F by G and Q by E, it follows that $\bar{S}$ will have full rank if p-m+1 is greater than or equal to the number of sources contained in the largest coherent group. Furthermore, here we assume that m is greater than K so that the mxK Vandermonde matrix A_m has full rank. Hence

$$\mathrm{Rank}(\bar{R}_{xxo}) = \mathrm{Rank}(\bar{R}_{yx}) = \mathrm{Rank}(\bar{S}) = K$$

According to the properties mentioned above and using $\bar{R}_{xxo}$ and $\bar{R}_{yx}$, we can also define a modified DOA matrix $\bar{R}$ by

$$\bar{R} = \bar{R}_{yx} \bar{R}_{xxo}^{\#} \qquad (21)$$

Here we define

$$\bar{R}_{xxo}^{\#} = \sum_{i=1}^{K} \mu_i^{-1} v_i v_i^H \qquad (22)$$

where $\{\mu_1 \geq \mu_2 \geq \ldots \geq \mu_K\}$ and $\{v_1, v_2, \ldots, v_K\}$ are the non-zero eigenvalues and corresponding eigenvectors of $\bar{R}_{xxo}$, respectively. $\bar{R}_{xxo}^{\#}$ is, therefore, a pseudoinverse of $\bar{R}_{xxo}^{\#}$ [6].

Based on Theorem 1, $\bar{R}$ has its K non-zero eigenvalues equal to the K diagonal elements of Φ and corresponding eigenvectors equal to the K column vectors of A_m, i.e.

$$\bar{R}A_m = A_m \Phi \qquad (23)$$

Thus, we can estimate the DOA of radiating sources, (α_k, β_k), from the eigenvectors and the eigenvalues, respectively, without any 2-D searches. In [7], we will present this new algorithm in detail.

IV. Simulation Results

To illustrate the effectiveness of the method presented in this paper, we shall show some numerical examples, and compare these results with those obtained from the widely employed MUSIC method. To obtain a measure of statistical repeatability, we make one hundred independent generations of estimates for the auto-covariance and cross-covariance matrices. The one hundred resultant DOA estimates are shown in each figure associated with a different method.

Consider three narrowband sources impinging from the directions $\alpha_1=77°$, $\beta_1=70°$, $\alpha_2=70°$, $\beta_2=76°$, $\alpha_3=62°$, $\beta_3=64°$ on a linear array system with two parallel uniform linear arrays of seven sensors (p=7), where $D=d=c/(2f_0)$. The first two sources are coherent but uncorrelated with the third one. The number of snapshots is 100. SNR=18dB. The results are shown in Figs. 2 and 3. It is seen that the MUSIC method can not resolve the signal when it is coherent with other signals. But the new method presented in this paper can estimate the DOA of multiple narrowband sources regardless of their being coherent or not.

V. Conclusions

A high resolution algorithm is presented for solving the general multiple source location problem. It uses the sampled data from two parallel uniformly spaced linear subarrays for estimating the

2-D DOA parameters. A new spatial smoothing method for two parallel linear sensor subarrays is presented, which can decorrelate the signals effectively. Then, we construct a matrix, called a modified DOA matrix, from the smoothed auto-covariance and cross-covariance matrices. The incident data may be from a mixture of uncorrelated, partially correlated, or coherent source signals. It is shown that the 2-D angles of arrival of narrowband sources can be estimated from the eigenvalues and the eigenvectors of this DOA matrix. Without 2-D searches, the new method is much more efficient in computation than most other 2-D methods.

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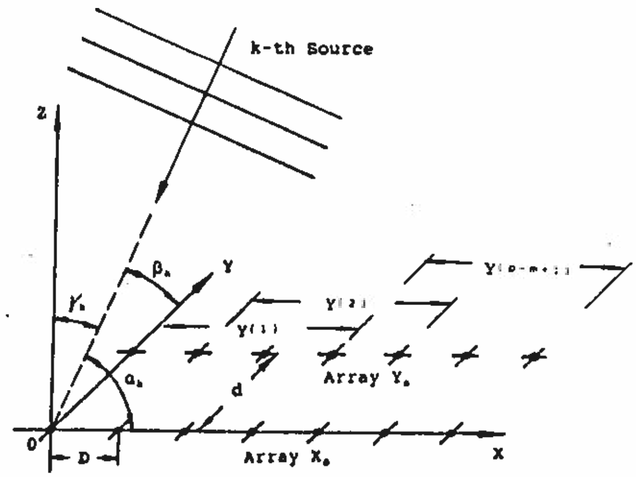

Figure 1 The geometry of the array system

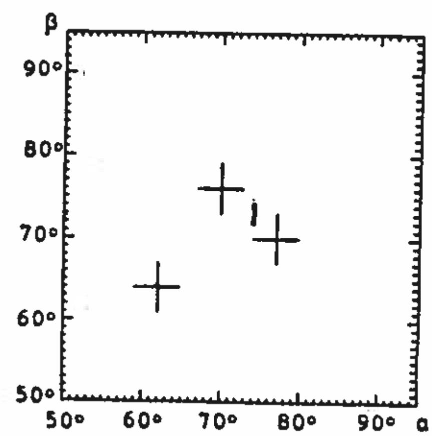

Figure 2. Three narrowband source signals impinge from the directions $\alpha_1=77°$, $\beta_1=70°$, $\alpha_2=70°$, $\beta_2=76°$, $\alpha_3=62°$, $\beta_3=64°$ on the array system shown in Figure 1. The first two sources are coherent but uncorrelated with the third one. The results are obtained by MUSIC method.

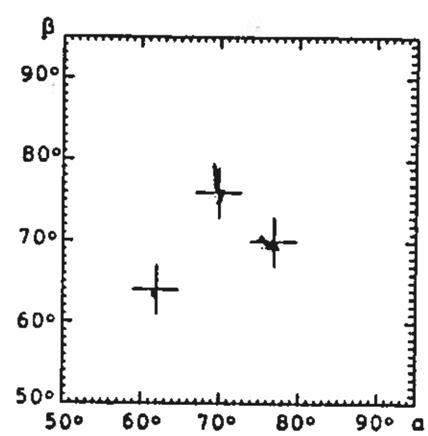

Figure 3. The sources and the array system are the same as in Figure 2. The results are obtained by the method of this paper.

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