ESTIMATING BEARING ANGLES OF COHERENT SOURCES USING A SIGNAL MATRIX PAIR

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ABSTRACT

In this paper, a new approach to high resolution estimates of directions of arrival of coherent narrowband sources is presented. The properties of the largest eigenvectors of the covariance matrix estimated by the forward-backward method are discussed. A signal matrix pair is constructed from these largest eigenvectors. It is shown that the signal matrix pair handles coherent signals with a great effective aperture, and can be used to construct a matrix pencil. Thus, the new method estimates the bearing angles from the eigenvalues of the matrix pencil. It can yield high resolution estimates with low variance. Simulation results are given.

I. Introduction

In recent years, the so called signal subspace algorithms have emerged as a class of very powerful approaches for estimation of bearings via a sensor array [1-5]. Among the important members of this class of algorithms, the eigenstructure methods, pioneered by Pisarenko, Schmidt, and Reddi, have been proved successful for solving the one dimensional (1-D) spectral estimation problem and yield high resolution and asymptotically unbiased estimates. However, the ESPRIT algorithm (Estimation of Signal Parameters by Rotational Invariance Techniques), introduced recently by Paulraj and Kailath [5,6], possesses remarkable advantages over other high resolution direction-of-arrival estimations in terms of speed, storage, and indifference to calibration. But it will find difficulty when some sources are coherent. Unfortunately, coherent sources appear frequently in practical problems. Ouibrahim, et. al. have proposed a modified ESPRIT method [7] for the coherent source problem with a "moving window" operator. A disadvantage of this approach is that it reduces the effective aperture of the array.

A new method applying a subspace invariance technique to the estimation of the direction-of-arrival (DOA) of multiple narrowband sources is presented in this paper. Exploiting the forward-backward method [3] to estimate the covariance matrix and its signal eigenvectors to construct a signal matrix pair, the method of this paper can resolve partially correlated or coherent sources with a greater effective aperture than that in [7]. The signal matrix pair, which is of the form of sampled data without the additive noise, can be used to construct a matrix pencil. Like the ESPRIT method, the method estimates the bearing angles from the eigenvalues of the matrix pencil. By only utilizing several largest signal eigenvectors, however, it can yield high resolution estimates with low variance. Simulation results demonstrate the merit of the new method in comparison with the modified ESPRIT method proposed by Ouibrahim, et. al. [7].

II. Model Definition

Consider d stochastic narrowband signal sources with known center frequency Ω_0 whose emitted signals are simultaneously incident on a uniformly spaced linear array of p sensors at angles $\theta_1, \theta_2, \ldots, \theta_d$ normal to the array. Assume that the sensor noises are mutually uncorrelated. Using the analytic representation for the signal, the response measured at the i-th sensor can be written in the form

$$x_i(t) = \sum_{k=1}^{d} s_k(t)\exp(j\tau_k(i-1)) + n_i(t) \qquad (1)$$

where $s_k(t)$ is the k-th source signal observed at the first sensor, $\tau_k=(\Omega_0 D/c)\sin\theta_k$, D is the spacing between two adjacent sensors and c is the speed of propagation of plane waves. $n_i(t)$ is the additive independent noise at the i-th sensor, which is assumed to be uncorrelated with signals.

The response in (1) can also be modeled for all sensors in vector form as

$$X(t) = As(t) + N(t) \tag{2}$$

where X(t) and N(t) are px1 column vectors

$$X(t) = [x_1(t),x_2(t),\ldots,x_p(t)]^T \tag{2a}$$

$$N(t) = [n_1(t),n_2(t),\ldots,n_p(t)]^T \tag{2b}$$

s(t) is a dx1 column vector

$$s(t) = [s_1(t),s_2(t),\ldots s_d(t)]^T \tag{2c}$$

with ^{T} denoting the transpose. A is a pxd Vandemonde matrix

$$A=\begin{bmatrix} 1 & \ldots & 1 \\ \exp(j\tau_1) & \ldots & \exp(j\tau_d) \\ \vdots & & \vdots \\ \exp(j\tau_1(p-1)) & \ldots & \exp(j\tau_d(p-1)) \end{bmatrix} \tag{2d}$$

Note that each column of matrix A is associated with a relevant source. We shall refer to these columns as signal vectors. Similarly, the entry $\exp(j\tau_k)$, k=1,...,d, in a signal vector associated with the relevant direction of a source is referred to as the direction element.

The covariance matrix of signals received from the array is defined as

$$R \triangleq E[X(t)X(t)^H]$$

$$= ASA^H + \sigma^2 I = R_0 + \sigma^2 I \tag{3}$$

where ^{H} denotes conjugate transpose and E the expectation operator. $S=E[s(t)s(t)^H]$ is the covariance matrix of source signals. I is the identity matrix and σ^2 the variance of the additive noise.

It can be easily verified that with A and S being full rank, the eigenvalues and the eigenvectors of R, denoted by $(\mu_1\geq\mu_2\geq\ldots\geq\mu_p)$ and $(V_1,V_2,\ldots,V_p)$, respectively, have the following properties:

1) $\mu_d>\mu_{d+1}=\mu_{d+2}=\ldots=\mu_p=\sigma^2$ (4a)

2) $L(V_1,V_2,\ldots,V_d) = L(A)$ (4b)

$L(V_{d+1},V_{d+2},\ldots,V_p) \perp L(A)$ (4c)

where L(A) denotes the signal space spanned by the column vectors of A, and $L(V_1,V_2,\ldots,V_d)$ denotes the space spanned by vectors $V_1,V_2,\ldots,V_d$. Some eigenstructure methods [1,2] are just based on these properties.

For the case of coherent sources, properties 1) and 2) do not hold since the signal covariance matrix S is singular.

$$\text{Rank}(ASA^H) = \text{Rank}(S) = r < d \tag{5}$$

where r is the number of non-coherent sources. In this case, the eigenstructure methods will find difficulties. However, it can be proved that each of the r largest eigenvectors of R is also a linear combination of all signal vectors. This makes it possible to "decorrelate" the signals by using the largest eigenvectors.

III. Signal Matrix Pair

From Eq. (3) and the eigenstructure of R, we obtain, with $R_0=ASA^H$,

$$R_0 V_i = ASA^H V_i$$

$$= (\mu_i-\sigma^2)V_i \qquad i=1,2,\ldots,r \tag{6a}$$

$$V_i = (\mu_i-\sigma^2)^{-1}ASA^H V_i = AG_i \tag{6b}$$

where G_i is the dx1 vector

$$G_i = (\mu_i-\sigma^2)^{-1}SA^H V_i \tag{6c}$$

Eq. (6b) means that V_i belongs to the space L(A), hence

$$L(V_1,V_2,\ldots,V_r) \subset L(A) \tag{7}$$

Especially, for the coherent case, $\text{Rank}(S)=\text{Rank}(R_0)=r<d$, i.e., S is singular and we only have r (r<d) eigenvectors belonging to L(A). Thus, Properties 1) and 2) may not hold. To solve the problem, we have to add d-r other vectors $\{V_{r+1},\ldots,V_d\}\subset L(A)$ to $\{V_1,\ldots,V_r\}$ such that $L(V_1,\ldots,V_d)$ has rank d. To do so, we define a set of mx1 vectors, $d\leq m\leq p$

$$V_i^{(k)} = \begin{bmatrix} V_i(k) \\ V_i(k+1) \\ \vdots \\ V_i(k+m-1) \end{bmatrix} \qquad 1\leq k\leq p-m \tag{8}$$

and a set of mxd matrices

$$A^{(k)}=\begin{bmatrix} \exp[j\tau_1(k-1)] & \ldots & \exp[j\tau_d(k-1)] \\ \exp[j\tau_1 k] & \ldots & \exp[j\tau_d k] \\ \vdots & & \vdots \\ \exp[j\tau_1(k+m-2)] & \ldots & \exp[j\tau_d(k+m-2)] \end{bmatrix} \quad (9)$$

We have

$$V_i^{(k)} = A^{(k)}G_i = A^{(1)}\Phi^{k-1}G_i \quad (10)$$

where Φ is a dxd diagonal matrix with nonzero elements $\Phi_{ii}=\exp(j\tau_i)$.

The matrices U and W, referred to as a signal matrix pair, are defined as

$$U \triangleq [U_1, U_2, \ldots, U_r] \quad (11)$$

$$W \triangleq [W_1, W_2, \ldots, W_r] \quad (12)$$

where U_i and W_i are mx(p-m) matrices formed by the i-th eigenvector of R

$$U_i = [V_i^{(1)}, V_i^{(2)}, \ldots, V_i^{(p-m)}] \quad (11a)$$

$$W_i = [V_i^{(2)}, V_i^{(3)}, \ldots, V_i^{(p-m+1)}] \quad (12a)$$

The new method we present in this paper is based on the following theorem about the signal matrix pair.

THEOREM: The signal matrix pair (U,W) can be written in the form

$$U = A^{(1)}Q \; ; \qquad W = A^{(1)}\Phi Q$$

where Q is a dx(p-m)r matrix. Moreover, if (p-m)r is greater than or equal to d, the number of sources, then

$$\text{Rank}(Q) = \text{Rank}(U) = \text{Rank}(W) = d.$$

Note that U and W have the same form as the received array data in (2), but there is no additive noise. So, they can be used to estimate the bearing angles with better performance. In practice, the covariance matrix R has to be estimated from N snapshots. Here we can also use the forward-backward approach proposed by Utrych et. al. [3] to increase the rank of R

$$R=(2N)^{-1}\sum_{t=1}^{N} [X(t)X(t)^H+PX(t)^*X(t)^T P] \quad (13)$$

where P is the pxp permutation matrix with ones along its main antidiagonal and zero elsewhere, and * denotes conjugate.

Now using U and W calculated from R as sample data without noise, like in the ESPRIT method, we form "auto-covariance" and "cross-covariance" matrices

$$R_1 = UU^H = A^{(1)}ZA^{(1)H} \quad (14a)$$

$$R_2 = WU^H = A^{(1)}Z\Phi^H A^{(1)H} \quad (14b)$$

where $Z = QQ^H$ is a dxd matrix. Then, the following corollary provides the foundation for the results presented herein.

COROLLARY: Define Σ as the generalized eigenvalue matrix associated with the matrix pencil $\{R_1, R_2\}$. If $p \geq 3d/2$ and $(p-m)r \geq d$, then S is nonsingular, and the matrices Φ and Σ are related by

$$\Sigma = \begin{bmatrix} \Phi & 0 \\ 0 & 0 \end{bmatrix} \quad (15)$$

to within a permutation of the elements of Φ.

According to the properties mentioned above, we can estimate the DOA of coherent sources from the matrix pencil.

IV. A New Algorithm

A new algorithm for direction finding of narrowband coherent sources may be summarized as follows:

1) Estimate the covariance matrix R from sampled data of N snapshots based on (13).

2) Calculate the eigenvalues and corresponding eigenvectors of R, $\{\mu_1 \geq \mu_2 \geq \ldots \geq \mu_p\}$ and $\{V_1, V_2, \ldots, V_p\}$.

3) Using the r largest eigenvectors, $\{V_1, V_2, \ldots, V_r\}$, form the signal matrix pair {U,W} by (11,12), and then $\{R_1, R_2\}$ by (14a,b). Here, we can check the rank of R_1 as the m in (8) decreases from p to $[(p+1)/2]$, if after m_0 the rank no longer increases, in which case, $d=\text{Rank}(R_1)$. Thus the number of sources can be determined.

4) Calculate the d largest eigenvalues, $\{\epsilon_i\}_{i=1}^{d}$, of the matrix pencil $\{R_1, R_2\}$. The estimated $\{\theta_k\}_{k=1}^{d}$ are given by

$$\theta_k = \sin^{-1}\{c/(\Omega_0 D)\arg(\epsilon_k)\} \qquad k=1,2,\ldots,d \quad (16)$$

V. Simulation Results

Three narrowband sources at angles 10°, 20°, 26° impinge on a uniform linear array of 10 sensors spaced a half wavelength apart. The first two sources are coherent with each other and uncorrelated with the third one. The signal-to-noise ratio is 15 dB. The number of snapshots is 100 and 100 trials are run using independent data sets. Applying the method proposed by [7] and the new method presented here, we obtain the results in Fig. 1 and 2, respectively (In Fig. 2, we choose m=5 at (8).). It is seen that the former fails to resolve the three signals at most trials, while the latter can resolve them with less variance. Clearly, the new method can yield better performance.

VI. Conclusions

In this paper a new approach for estimating the directions of arrival of multiple narrowband sources is presented. Estimating the angles of arrival by the eigenvalues of the matrix pencil $\{R_1, R_2\}$ without any searches, it is highly efficient in computation. Furthermore, the forward-backward method is exploited to estimate R. Theoretical analysis shows that the method can be used in which the incident source signals may be uncorrelated, patially correlated or coherent and is of a greater effective aperture than that in [7].

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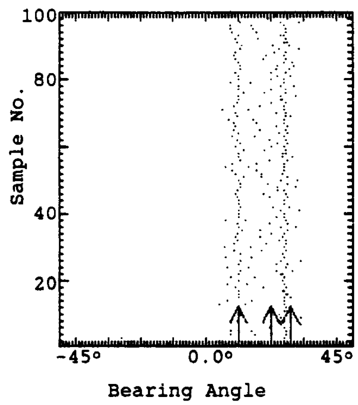

Figure 1. Three narrowband source signals impinge on a uniform linear array of ten sensors. The results are obtained by the method proposed by [7].

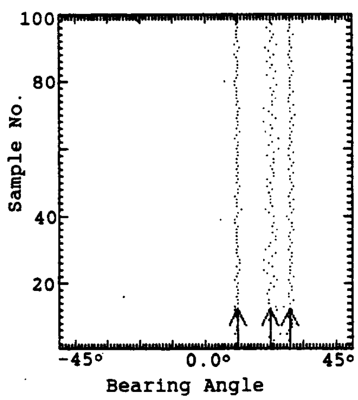

Figure 2. The sources and the array are the same as in Figure 1. The results are obtained by the method of this paper.