ESTIMATING 2-D ANGLES OF ARRIVAL VIA TWO PARALLEL LINEAR ARRAYS

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ABSTRACT

A new approach of high resolution estimates of two dimensional (2-D) angles of arrival of multiple narrowband sources for two parallel linear arrays is presented in this paper. By fully using the properties of the auto-covariance and cross-covariance matrices, we construct a new matrix called the direction-of-arrival (DOA) matrix. The theoretical analysis shows that the eigenvalues and corresponding eigenvectors of the DOA matrix are related to the 2-D angles of arrival of sources. Thus, they can be used to estimate 2-D angles of arrival separately, and the large computations required for 2-D searches can be saved. Simulation results are given.

I. Introduction

The localization of radiating narrowband sources is one of the central problems in radar, sonar, radio-astronomy and seismology. This problem, and especially that of estimation of the direction-of-arrival (DOA) from data sampled at a sensor array system, has received considerable attention in the last twenty years, resulting in a variety of techniques and algorithms [1-3]. The sensor arrays may not be uniformly spaced and linear. But, with the uniformly spaced linear sensor array system, a very important array system in practice, we can save a lot of computations and storages. Here, we discuss the linear sensor array system.

In recent years, there has been a growing interest in eigenstructure based methods. These methods, pioneered by Pisarenko [1], Schmidt [2] and Kumaresan [3], are known to yield high resolution and asymptotically unbiased estimates. Furthermore, Paulraj, et. al., [4,5] have proposed a subspace rotation approach called ESPRIT, which has better performance and less computations than previous ones. But when applying these one dimensional (1-D) methods to linear sensor arrays, we have to assume that all radiating sources are located in the same plane. On the other hand, most available 2-D methods need a large rectangular planar sensor array and have to perform 2-D searches with large computations.

In this paper, a new approach for estimating the DOA of narrowband sources is proposed. The method is similar to ESPRIT in that it exploits the underlying data received from two subarrays to generate asymptotically unbiased and efficient estimates. However, in the new method, eigenstructures of both the auto-covariance matrix and the cross-covariance matrix, formed from the two linear subarrays, are fully used to construct a DOA matrix, the largest eigenvalues and corresponding eigenvectors of which are utilized to estimate the 2-D DOA of sources. Without the 2-D searches, it is highly efficient in computation. Unlike ESPRIT, this method can be applied not only in the case that all sources are located in the same plane, but also in the general case that they are located anywhere in the far-field. Some simulation results are presented to illustrate the performance of the new algorithm.

II. Problem Formulation

We consider an array system consisting of two uniformly spaced linear subarrays of p sensors spaced D apart as shown in Fig. 1. The two subarrays, the subarray X_a and the subarray Y_a, are parallel with subarray X_a lying on the X axis and starting at the origin; d is the distance between them as measured by the Y axis intercept of subarray Y_a. Assume that plane waves with known center frequency Ω_0 emitted by K narrowband sources impinge on this array system and that the DOA of the sources are $\{\theta_1, \theta_2, \ldots, \theta_K\}$, where $\theta_k=(\alpha_k, \beta_k)$, α_k is the DOA of the k-th source relative to the X axis and β_k relative to the Y axis as shown in Fig. 1. Thus, γ_k, the DOA of the k-th source

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relative to the Z axis, can be given by.

$$\cos^2\alpha_k+\cos^2\beta_k+\cos^2\gamma_k=1 \quad (1)$$

The output of the i-th sensor in the subarray X_a can be expressed as

$$x_i(t)=\sum_{k=1}^{K} s_k(t)\exp[j\Omega_0(i-1)\tau_{Dk}]+n_{xi}(t) \qquad i=1,2,\ldots,p \quad (2)$$

where $s_k(\cdot)$ denotes the signal emitted by the k-th source observed at sensor one of the subarray X_a; τ_{Dk} is the inter-element path delay of the plane wave from the k-th source, $\tau_{Dk}=(D/c)\cos\alpha_k$, where c is the wave propagation velocity; $n_{xi}(t)$ is the additive noise on the i-th sensor of the subarray X_a, which is assumed to be a zero-mean white Gaussian stationary random process that is independent from sensor to sensor. Similarly, the output of the i-th sensor in the subarray Y_a can be expressed as

$$y_i(t)=\sum_{k=1}^{K} s_k(t)\exp\{j\Omega_0[\tau_{dk}+(i-1)\tau_{Dk}]\}+n_{yi}(t) \qquad i=1,2,\ldots,p \quad (3)$$

where τ_{dk} is the path delay of the plane wave of the k-th source between the two subarrays, $\tau_{dk}=(d/c)\cos\beta_k$; $n_{yi}(t)$ is the additive noise, with the same properties as the $n_{xi}(t)$ in Eq. (2).

Let $X^T(t)=[x_1(t),\ldots,x_p(t)]$ and $Y^T(t) = [y_1(t),\ldots,y_p(t)]$ be the simultaneously sampled vectors (snapshot) of array signals, where ^{T} denotes the transpose. Equations (2) and (3) can be written for all sensors in vector form as

$$X(t) = As(t) + N_x(t) \quad (4)$$

$$Y(t) = A\Phi s(t) + N_y(t) \quad (5)$$

where $s^T(t)=[s_1(t),\ldots,s_K(t)]$; $N_x{}^T(t)=[n_{x1}(t),\ldots,n_{xp}(t)]$ and $N_y{}^T(t)=[n_{y1}(t),\ldots,n_{yp}(t)]$; A is a pxK Vandermonde matrix

$$A=[a(\alpha_1),\ a(\alpha_2),\ \ldots\ ,\ a(\alpha_K)] \quad (6)$$

with k-th column

$$a^T(\alpha_k)=[1,\ldots,\exp(j\Omega_0(p-1)\tau_{Dk})] \quad (7)$$

and Φ is a KxK diagonal matrix

$$\Phi=\mathrm{diag}[\exp(j\Omega_0\tau_{d1}),\ldots,\exp(j\Omega_0\tau_{dK})] \quad (8)$$

Note that the k-th column of the matrix A, i.e. the $a(\alpha_k)$, and the k-th element, $\exp(j\Omega_0\tau_{dk})$, of the diagonal matrix Φ are associated with the DOA, i.e. (α_k,β_k), of the k-th source. These parameters are what are to be estimated.

The auto-covariance matrix of X(t) is given by

$$R_{xx} = E[X(t)X(t)^H] = ASA^H + \sigma^2 I \quad (9)$$

where ^{H} denotes conjugate transpose and E the expectation operator. $S=E[s(t)s(t)^H]$ is the covariance matrix of source signals, I is the identity matrix, and σ^2 is the variance of the additive noise. The cross-covariance matrix of the vector Y(t) and X(t) is given by

$$R_{yx}=E[Y(t)X(t)^H] = A\Phi SA^H+E[A\Phi s(t)N_x(t)]+E[N_y(t)s(t)^H A^H] + E[N_y(t)N_x(t)^H] \quad (10)$$

We assume that the additive noises are uncorrelated with signals and with each other, in which case Eq. (10) can be written as

$$R_{yx} = A\Phi SA^H \quad (11)$$

For a uniformly spaced linear array system, many eigenstructure methods, such as MUSIC [2], only estimate the parameters, $\{\alpha_k\}$, by using the eigenvectors of R_{xx}. The ESPRIT method [4,5] estimates the parameters, $\{\beta_k\}$, by using both R_{xx} and R_{yx}. Here, we propose a new method to estimate both parameters, $\{\alpha_k\}$ and $\{\beta_k\}$, by using both R_{xx} and R_{yx}.

III. Direction-of-Arrival Matrix

In Eq. (9), the auto-covariance matrix R_{xx} has two terms. The one associated with source signals is denoted by

$$R_{xx0} = R_{xx} - \sigma^2 I = ASA^H \quad (12)$$

Obviously, the rank of R_{xx0} is equal to K, the number of sources if there are no fully correlated (coherent) sources. Let $\{\mu_1\geq\mu_2\geq\ldots\geq\mu_p\}$ and $\{v_1,v_2,\ldots,v_p\}$ be the eigenvalues and corresponding eigenvectors of the matrix R_{xx0}, respectively, i.e.

$$R_{xx0} = \sum_{i=1}^{p} \mu_i v_i v_i^H \quad (13)$$

With the assumptions that p>K and S is nonsingular, it can be easily verified [6] that R_{xx0} has the following properties:

i) the minimal eigenvalue of R_{xx0} is zero with multiplicity p-K, $\mu_{K+1}=\mu_{K+2}=\ldots=\mu_p=0$;

ii) the eigenvectors corresponding to the minimal eigenvalue are orthogonal to the columns of the matrix A, i.e.

$$L\{v_{K+1}, v_{K+2}, \ldots, v_p\} \perp L\{A\} \tag{14}$$

Based on the orthogonal property, we have that

$$A^H[\sum_{i=K+1}^{p} v_i v_i^H]A = 0 \tag{15}$$

Thus

$$A^H[\sum_{i=1}^{K} v_i v_i^H]A = A^H[\sum_{i=1}^{K} v_i v_i^H + \sum_{i=K+1}^{p} v_i v_i^H]A$$
$$= A^H A \tag{16}$$

Now define a pxp matrix R referred to as the direction-of-arrival (DOA) matrix

$$R = R_{yx} R_{xxo}^{\#} \tag{17}$$

where $R_{xxo}^{\#}$ denotes the pseudoinverse of R_{xxo}, as defined in (22). The following theorem provides the foundation for the results presented herein.

Theorem: If S is nonsingular, the DOA matrix R has its K non-zero eigenvalues equal to the K diagonal elements of Φ and corresponding eigenvectors equal to the K column vectors of matrix A, i.e.

$$RA = A\Phi \ . \tag{18}$$

Proof: From Eq. (12) we obtain

$$SA^H = (A^H A)^{-1} A^H R_{xxo} \tag{19}$$

Substitute Eq. (19) into Eq. (11)

$$R_{yx} = A\Phi(A^H A)^{-1} A^H R_{xxo} \tag{20}$$

Using the relationships in (15), (16) and the pseudoinverse $R_{xxo}^{\#}$ of the matrix R_{xxo}, we obtain

$$R_{yx} R_{xxo}^{\#} A = A\Phi(A^H A)^{-1} A^H R_{xxo} R_{xxo}^{\#} A$$
$$= A\Phi(A^H A)^{-1} A^H A$$
$$= A\Phi \tag{21}$$

where $R_{xxo}^{\#}$ is constructed by the non-zero eigenvalues, μ_i, and corresponding eigenvectors, v_i, of R_{xxo}.

$$R_{xxo}^{\#} = \sum_{i=1}^{K} \mu_i^{-1} v_i v_i^H \tag{22}$$

From Eq. (17) and Eq. (21), we have

$$RA = A\Phi \quad \blacksquare$$

Thus, we can estimate the DOA of radiating sources, (α_k, β_k), from the eigenvectors and the eigenvalues, respectively.

IV. A New Algorithm for Direction Finding

According to the eigenstructure of the matrix R, we obtain a new algorithm of array signal processing. It may be summarized as follows:

1) Estimate the pxp auto-covariance matrix R_{xx} and cross-covariance matrix R_{yx} using N snapshots. Here, well-known asymptotically unbiased estimates of R_{xx} and R_{yx} can be used [7]

$$R_{xx} = N^{-1} \sum_{t=1}^{N} X(t)X(t)^H \tag{23}$$

$$R_{yx} = N^{-1} \sum_{t=1}^{N} Y(t)X(t)^H \tag{24}$$

2) Compute the eigenvalues, $\{\epsilon_1 \geq \epsilon_2 \geq \ldots \geq \epsilon_p\}$, and the eigenvectors, $\{v_1, v_2, \ldots, v_p\}$, of R_{xx}.

3) Determine K, the number of sources. Based on property (i), the smallest eigenvalue is equal to the noise variance, and there are p-K smallest eigenvalues, i.e., $\epsilon_{K+1} = \epsilon_{K+2} = \ldots = \epsilon_p = \sigma^2$. In practice, K can be determined simply by observing the smallest eigenvalues, or by using such hypothesis testing as AIC or MDL [9].

4) Construct the DOA matrix R from Eq. (17) and Eq. (22).

5) Compute the eigen-decomposition of R to get the K largest eigenvalues σ_i and corresponding eigenvectors u_i.

6) From the Theorem, the estimated α_i can be given by calculating the search function

$$E_i(\alpha) = |u_i^H a(\alpha)| \quad i=1,2,\ldots,K \tag{25}$$

Here α_i is chosen as the angle for which E_i is a maximum, and the estimated β_i is given by

$$\beta_i = \mathrm{Sin}^{-1}\{c/(\Omega_0 d)\arg(\sigma_i)\} \quad i=1,2,\ldots,K \tag{26}$$

Note the new algorithm proposed can estimate both angles, α_i and β_i, simultaneously. Clearly, in the special case that all the sources are located in the same plane, it can also be used by simply calculating the α_i or β_i, like the MUSIC method or the ESPRIT method.

V. Simulation Results

We use the example of two uncorrelated narrowband signals that impinge from the directions $\alpha_1=75^\circ$, $\beta_1=80^\circ$, $\alpha_2=85^\circ$, $\beta_2=70^\circ$ on a linear array system with two parallel uniform linear subarrays of eight sensors (p=8), where $D=d=\pi c/\Omega_0$. The number of snapshots is 100. To obtain a measure of statistical repeatability, we make one hundred independent trials in each Figure. In Figure 2 for 5 dB SNR values, it is shown that the new method can yield excellent estimates with low variance. When we reduce the SNR to -5 dB, in Figure 3, the new method can still resolve the two incident plane waves in such a low SNR.

VI. Conclusions

Through theoretical analysis, we propose a new method for estimating the directions of arrival of multiple narrowband sources. Fully using the eigenstructures of the auto-covariance matrix and cross-covariance matrix, we construct a direction-of-arrival matrix. The matrix has its largest eigenvalues and corresponding eigenvectors associated with a pair of bearing angles of the sources. Unlike the ESPRIT method, the new method can be used without much more extra computations in the case that sources are not in the same plane. Here the incident source signal may be uncorrelated, partially correlated. The method can be extended to solve the coherency problem, as will be discussed in [8].

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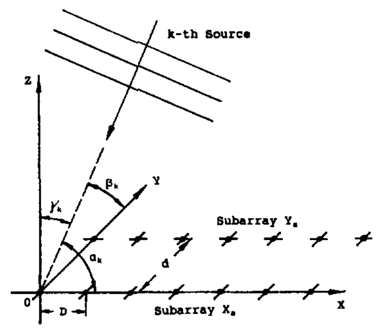

Figure 1. The geometry of the array system

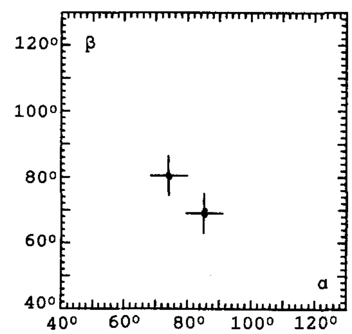

Figure 2. Two uncorrelated signals impinge from the directions $\alpha_1=75°$, $\beta_1=80°$, $\alpha_2=85°$, $\beta_2=70°$ on the parallel uniform linear array system (p=8). SNR=5dB, N=100. Results consist of 100 independent trials.

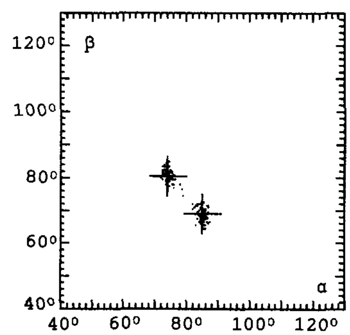

Figure 3. The sources and the array are the same as in Figure 2. SNR=-5dB, N=100. Results also consist of 100 independent trials.