for an ideal second-order bandpass filter ((1)), or for a second-order filter with high-frequency parasitic poles. Equations (13) and (14) guarantee that the filter has the right gain at resonance, and the right-3 dB frequencies; this leads indirectly to the right values for F_0 and Q. Again transfer functions are shown in Fig. 5 for tuning the circuit in Fig. 1, in the presence of an extra pole with parasitic delay τ in the transconductors. The tuned filter is now almost exactly equal to the desired filter for all values of $\omega_0\tau$; this shows the superior performance of the T23 tuning strategy. It is however clear that T23 tuning leads to a considerably more complex tuning circuit (or algorithm). Hence, this strategy is suitable for the case where tuning is performed using digital signal processing concepts, or in those cases where many different errors add together (e.g., mismatching errors). In this last case, A and Q errors do not necessarily track, and the transfer function has to be tuned in at least two frequencies, as is presented here.

Appendix 1

Here we describe how exactly the control parameters of the filter have to be adapted, to yield a stable and converging tuning process. In this section, we will use the symbols $V_{\omega 0}$, V_A, and V_Q for the control variables that control the corresponding filter parameter. K is an integration constant, which can be chosen independently in any of the following sets of equations.

T11: Single-frequency, single-objective tuning process:

Control of $g_{m1}, g_{m2}, g_{m3}, g_{m4}$:

$$V_{\omega 0} = -K\int \mathrm{Im}[H(2\pi jF_{\mathrm{ex}})]\cdot dt. \tag{A1}$$

T12: Single-frequency, double-objective tuning process:

Control of g_{m2}, g_{m3}, g_{m4}:

$$V_{\omega 0} = -K\int \mathrm{Im}[H(2\pi jF_{\mathrm{ex}})]\cdot dt. \tag{A2}$$

Control of g_{m1}:

$$V_A = -K\int\{\mathrm{Re}[H(2\pi jF_{\mathrm{ex}})]-1\}\cdot dt. \tag{A3}$$

T23: Double-frequency, triple-objective tuning process:

Control of g_{m2}, g_{m3}:

$$V_{\omega 0} = -K\int\{\mathrm{Im}[H(2\pi jF_L)]-\mathrm{Im}[H(2\pi jF_H)]-1\}\cdot dt. \tag{A4}$$

Control of g_{m1}:

$$V_A = -K\int\{\mathrm{Re}[H(2\pi jF_L)]-0.5\}\cdot dt. \tag{A5}$$

Control of g_{m4}:

$$V_Q = K\int\{\mathrm{Re}[H(2\pi jF_H)]-0.5\}\cdot dt. \tag{A6}$$

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Input–Output Pairing in LTV Semistate Systems

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***Abstract*—It is shown that linear time-varying (LTV) semistate described systems have a general description input–output pairing when the time variations can be expressed in terms of functions in a differential field. The method of obtaining this general description is given beginning with the canonical linear semistate equations described over a differential field.**

I. Introduction

The semistate description [1], [2] of electronic circuits is a natural one since it allows both the dynamic operation of capacitors and the algebraic operation of resistors and transistors to be simultaneously expressed in state-variable types of equations without the need for elimination of variables. In the linear time-invariant case, if one assumes the property of regularity, one can obtain solutions via the Drazin inverse [3] and find transfer functions in a straightforward manner. However, as we have shown before [4] and see again here in Fig. 1, innocent-looking circuits might not be regular, in which case the transfer function need not exist and one loses the property of existence and/or uniqueness of solutions. Nevertheless, as we show here, an input–output pairing does exist even in the linear time-varying (LTV) case, this being of the form of that which has previously been called the "general description." Specifically, if u and y are the input and output in the time, t, domain and s is the *derivative operator*, under very general conditions we can obtain, directly from the canonical semistate equations, the general description [5, p. 46]:

$$A(s,t)\cdot y(t) = B(s,t)\cdot u(t). \tag{1}$$

For the finite-dimensional input and output spaces that we treat here $A(s,t)$ and $B(s,t)$ are matrices that are rational in s with coefficients that we assume to be in a differential field F [6]. The specification of the treatment to this class of $A(s,t)$, $B(s,t)$ is natural since one of us has previously given a circuit synthesis of passive LTV circuits for this class [7], which is probably as general as one would desire in both theory and practice. Since we are treating time-varying circuits it should be recalled that the operator entries in $A(s,t)$, $B(s,t)$ satisfy the main differential polynomial law [8]:

$$s\cdot a(t) = a(t)\cdot s + a'(t) \tag{2}$$

where a' is the derivative of a, and by the specification to a differential field F, both a and a' lie in F. In (2) we use s to operate on things on its right; (2) also shows that multiplication of differential polynomials and rational functions in s is not commutative.

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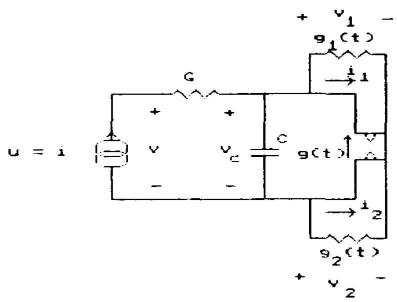

Fig. 1. Example circuit.

We also assume that the semistate equations are given in their canonical form [9]

$$Esx(t)=A(t)x(t)+Bu(t) \tag{3a}$$

$$y(t)=Cx(t) \tag{3b}$$

where x is the semistate vector, E, B, C are constant matrices, and u and y are linear combinations of voltages and currents within the circuit. Following the above comments, we assume that the entries of the matrix $A(t)$ are in the differential field F.

II. Input–Output Pairing

In line with (1) we desire to find an operator multiplier on y to equate to an operator multiplier on u. Since the equation in terms of u, (3a), and the equation in terms of y, (3b), both involve x, we search for multipliers on x to make these two equations the same. Putting both side by side we have $y=Cx$ and $[Es-A(t)]\cdot x=Bu$. If we can find operator matrices X_{21} and X_{22} such that $X_{21}\cdot C=-X_{22}\cdot[Es-A(t)]$, we have what we desire. Using the techniques of Wedderburn [10] and Ore [8], these are readily found for the LTV case by extending the technique of MacDuffee [11, p. 35] which finds the greatest common right divisor of two matrices. Thus by using elementary row operations in the differential algebra of rational functions in s with coefficients in the given differential field F we find $X(s,t)_{ij}$ to give

$$\begin{bmatrix} X_{11}(s,t) & X_{12}(s,t) \\ X_{21}(s,t) & X_{22}(s,t)\end{bmatrix}\cdot\begin{bmatrix} C \\ Es-A(t)\end{bmatrix}=\begin{bmatrix} D(s,t) \\ 0\end{bmatrix} \tag{4}$$

where the rank of $D(\cdot,\cdot)$ is equal to its number of rows. Then

$$X_{21}\cdot(s,t)\,y(t)=-X_{22}(s,t)\cdot B\cdot u(t) \tag{5}$$

since

$$X_{21}(s,t)\,y(t)=X_{21}(s,t)Cx(t)=-X_{22}(s,t)[Es-A(t)]x(t)$$
$$=-X_{22}(s,t)\,Bu(t).$$

Thus we have achieved (1) with

$$A(s,t)=X_{21}(s,t) \tag{6a}$$

$$B(s,t)=-X_{22}(s,t)\cdot B. \tag{6b}$$

We comment that the number of rows of $F(s,t)$ is the same as the number of rows of $G(s,t)$, but the numbers of their columns may differ (for example, u may be a vector of m input port voltages and y may be a vector of n output port voltages where the input and output ports differ and m need not equal n). In the manipulations to achieve (4) one must of course be very careful to observe the rules of time-variable systems, of which (2) is primary.

If the operator matrix $[Es-A(t)]$ is nonsingular, the semistate equations are said to be *regular* and have an operator transfer function

$$T(s,t)=C[Es-A(t)]^{-1}B \tag{7a}$$

in which case, as a time-domain operator equation, $y(t)=T(s,t)\cdot u(t)$. Clearly, also

$$T(s,t)=A(s,t)^{-1}\cdot B(s,t). \tag{7b}$$

In the case of nonregular LTV semistate equations the inverse used in (7a) does not exist, in which case a transfer function may or may not exist, depending upon the relation of C and B to $[Es-A(t)]$, but in any event the transfer function exists if and only if $A(s,t)$ is a nonsingular operator matrix.

III. Example

Consider the circuit of Fig. 1 where the gyrator is described by $i_1=gv_2$, $i_2=-gv_1$, $g=g(t)$ being the time-variable gyration conductance; we also assume that $g_1=g_1(t)$ and $g_2=g_2(t)$ but that G and C are constant. The portion of Fig. 1 to the right of the capacitor is created to be able to realize a norator or a nullator [5, p. 13] with the capacitor inserted to obtain dynamics. If

$$u=i, \tag{8a}$$

$$y=\begin{bmatrix} v \\ v_c\end{bmatrix} \tag{8b}$$

$$x=\begin{bmatrix} v_1 \\ v_2 \\ v\end{bmatrix} \tag{8c}$$

a canonical set of semistate equations can be written as

$$\begin{bmatrix} C & -C & 0 \\ C & -C & 0 \\ 0 & 0 & 0\end{bmatrix}sx=\begin{bmatrix} -g_1 & -g & 0 \\ -g & g_2 & 0 \\ G & -G & -G\end{bmatrix}x+\begin{bmatrix} 1 \\ 1 \\ 1\end{bmatrix}u \tag{8d}$$

$$y=\begin{bmatrix} 0 & 0 & 1 \\ 1 & -1 & 0\end{bmatrix}x. \tag{8e}$$

Heading toward (4) we use elementary transformations to arrive at

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & -Cs-g_1 & 1 & 0 & 0 \\ 0 & g-g_1 & 1 & -1 & 0 \\ -G & G & 0 & 0 & 1\end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 1 & -1 & 0 \\ Cs+g_1 & -Cs+g & 0 \\ Cs+g & -Cs-g_2 & 0 \\ -G & G & 0\end{bmatrix}$$
$$=\begin{bmatrix} 0 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & g_1+g & 0 \\ 0 & g_1+g_2 & 0 \\ 0 & 0 & 0\end{bmatrix}. \tag{9}$$

There are two essentially different cases for (4) depending upon whether the right-hand matrix has rank 2 or 3. In the rank 2 case X_{21} is 3×2 and in the rank three case it is 2×2.

Rank 2 case: $g_1=-g=-g_2$

Immediately

$$X_{21}=\begin{bmatrix} 0 & -Cs+g \\ 0 & 2g \\ -G & G\end{bmatrix},\quad X_{22}=\begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1\end{bmatrix} \tag{10a}$$

from which (6) gives

$$\begin{bmatrix} 0 & -Cs+g(t) \\ 0 & 2g(t) \\ -G & G\end{bmatrix}\cdot y=\begin{bmatrix} -1 \\ 0 \\ -1\end{bmatrix}\cdot u. \tag{10b}$$

If $g \neq 0$ this means that $v_c = v = i = 0$, as for a nullator, and an arbitrary current source can not be used to excite the circuit.

Rank 3 case:

Assume the subcase $g_1 + g \neq 0$ (otherwise we have $g_1 + g_2 \neq 0$ and can interchange g and g_2 and rows 3 and 4 before starting the following). Then, on subtracting $(g_1 + g_2)/(g_1 + g)$ times the third row from the fourth in (9) we get

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & -Cs - g_1 & 1 & 0 & 0 \\ 0 & x_{42} & x_{43} & -1 & 0 \\ -G & G & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 1 & -1 & 0 \\ Cs + g_1 & -Cs + g & 0 \\ Cs + g & -Cs - g_2 & 0 \\ -G & G & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & g_1 + g & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{11a}$$

where

$$x_{42} = \left[Cs(g_1 + g_2) + g^2(1 + g_1) \right] \cdot (g_1 + g)^{-1} \tag{11b}$$

$$x_{43} = (g - g_2) \cdot (g_1 + g)^{-1} \tag{11c}$$

for which

$$X_{21} = \begin{bmatrix} 0 & x_{42} \\ -G & G \end{bmatrix}, \; X_{22} = \begin{bmatrix} x_{43} & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{11d}$$

giving

$$\begin{bmatrix} 0 & x_{42} \\ -G & G \end{bmatrix} \cdot y = \begin{bmatrix} 1 - x_{43} \\ -1 \end{bmatrix} \cdot u. \tag{11e}$$

If $g_1 + g_2 \neq 0$ then the transfer function is readily found from (11e). However, if $g_1 + g_2 = 0$, then either (11e) or (9) directly give

$$\begin{bmatrix} 0 & g(t) - g_1(t) \\ -G & G \end{bmatrix} \cdot y = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \cdot u \tag{11f}$$

which has the further interesting subcase of $g = g_1$ for which v and i are completely independent and arbitrary, as for a norator.

IV. Discussion

We have shown how a general description can be obtained for any LTV circuit that has a canonical semistate description with time variations described by functions of time that are in a differential field, these being the functions in the semistate $A(t)$ matrix. Although we have phrased the results for electronic circuits, where it is known how to calculate the canonical semistate equations (which almost always exist), the results are true for any system described by the canonical semistate equations, for example those of time-adaptive robots.

It should be noted that the transfer function used here is a natural operator theory one, apparently first used by Darlington [12], but does differ from others in the literature, where, for example, one finds $H(t,s)$ as the Laplace transform in the τ variable of the impulse response matrix $h(t,\tau)$ [13]. Similar results could be developed for the latter type of transfer function or one could use just the impulse response matrix itself as the LTV operator [14, p. 80]. As pointed by one of the reviewers, the form of (4) is familiar to systems theorists in the time-variant case [15, p. 381], which is to be expected since time-invariant results are special cases of the time-varying ones within the operator theory development of this note.

The nonregular example given here rests upon the use of the nullator and norator, singular network elements. Others [4] use op-amps which are of the same class. This appears to be a general result since the synthesis of time-invariant square general descriptions can be done in terms of standard components and nullators and norators [5, p. 263], though the synthesis of LTV nonsquare general descriptions remains to be carried out in detail.

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A New Stability Test for Two-Dimensional Discrete Systems Using Bilinear Continuous Fractions

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Abstract **—A new algorithm for testing the stability of single-input single-output (SISO) two-dimensional (2-D) discrete-time systems is presented, using a method in the z-domain exclusively. It is based on expanding the bilinear discrete reactance function into z-domain continuous fractions in a mixed fashion. It is faster than Karan and Srivastava's algorithm and compares favorably with the commonly used tabular form of testing stability of 2-D systems. Two examples are given to illustrate the method and its practical implementation.**

I. Introduction

Stability of two-dimensional (2-D) and multidimensional discrete systems and filters has received great attention over the years. Comprehensive surveys and reviews of this field have been provided in [1]–[4]. A rather complete picture of the entire field

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