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5123

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*Solution by Veselin Perić, Sarajevo, Yugoslavia.* Let

$$f(x) = \sum a_i x_1^{p_{1i}} x_2^{p_{2i}} \cdots x_r^{p_{ri}}.$$

Then, for  $b_i$  with  $b_i^p = a_i$  and for

$$g(x) = \sum b_i x_1^{p_{1i}} x_2^{p_{2i}} \cdots x_r^{p_{ri}}$$

we have  $\{g(x)\}^p = f(x^p)$ . Any element  $a \in F$  is a polynomial  $h(b_1, b_2, \dots)$  in  $b_1, b_2, \dots$  with coefficients in  $k$  and, consequently,  $a^p = h_1(b_1^p, b_2^p, \dots) \in k$ . Suppose  $g(x) = \phi(x) \cdot \psi(x)$ , where  $\phi(x)$  and  $\psi(x)$  are nonconstant polynomials in  $F[x]$ . Then

$$\{\phi(x)\}^p = \phi_1(x^p), \quad \{\psi(x)\}^p = \psi_1(x^p),$$

where  $\phi_1(x)$  and  $\psi_1(x)$  are nonconstant polynomials in  $k[x]$ , for the coefficients of these polynomials are the  $p$ th powers of the coefficients of  $\phi(x)$  and  $\psi(x)$ , and belong to  $k$ . From  $f(x^p) = \{g(x)\}^p = \phi_1(x^p) \cdot \psi_1(x^p)$  we conclude that  $f(x) = \phi_1(x) \cdot \psi_1(x)$  in  $k[x]$ , contrary to the hypothesis that  $f(x)$  is irreducible in  $k[x]$ .

Also solved by George Bergman, L. Carlitz, and the proposer.

**Dense Sets of Functions**

5123 [1963, 765]. *Proposed by R. W. Newcomb, Stanford University*

Prove that the set of infinitely differentiable functions of support bounded on the left is dense in the set of distributions of support bounded on the left.

*Solution by the proposer.* We use the notation of L. Schwartz, *Théorie des distributions*, vol. I, 1957, p. 89, and vol. II, 1959, p. 28. An existence type proof for the problem follows immediately from theorems in Schwartz, vol. I. We offer the following constructive approach.

Consider a sequence  $\{\phi_j\}$  of infinitely differentiable functions of bounded support chosen so that  $\lim_{j \rightarrow \infty} \phi_j = \delta$ ;  $\phi_j \in \mathcal{D} \subset \xi'$ . For any  $T \in \mathcal{D}'_+ \subset \mathcal{D}'$ , form  $\phi_j * T$ . We know that  $\phi_j * T \in \mathcal{D}_+$  [Schwartz, v. II, p. 29]. Since  $\phi_j \in \xi'$  and  $T \in \mathcal{D}'$ , it follows from the continuity of the convolution for such distributions [Schwartz, p. 13] that

$$\lim_{j \rightarrow \infty} \phi_j * T = \delta * T = T.$$

Consequently,  $\mathcal{D}_+$  is dense in  $\mathcal{D}'_+$ .

**Addition Chains of Vectors**

5125 [1963, 765]. *Proposed by Richard Bellman, The RAND Corporation, Santa Monica, California*

It is easy to determine the minimum number of multiplications required to generate  $a^N$  from  $a$ . What is the minimum number required to generate  $a^M b^N$  starting with  $a$  and  $b$ ?