Solution by Veselin Perić, Sarajevo, Yugoslavia. Let

$$f(x) = \sum a_i x_1^{\nu_{1i}} x_2^{\nu_{2i}} \cdots x_r^{\nu_{ri}}.$$

Then, for b_i with $b_i^p = a_i$ and for

$$g(x) = \sum b_i x_1^{\nu_{1i}} x_2^{\nu_{2i}} \cdots x_r^{\nu_{ri}}$$

we have $\{g(x)\}^p = f(x^p)$. Any element $a \in F$ is a polynomial $h(b_1, b_2, \cdots)$ in $b_1, b_2, \cdots$ with coefficients in k and, consequently, $a^p = h_1(b_1^p, b_2^p, \cdots) \in k$. Suppose $g(x) = \phi(x) \cdot \psi(x)$, where $\phi(x)$ and $\psi(x)$ are nonconstant polynomials in $F[x]$. Then

$$\{\phi(x)\}^p = \phi_1(x^p), \qquad \{\psi(x)\}^p = \psi_1(x^p),$$

where $\phi_1(x)$ and $\psi_1(x)$ are nonconstant polynomials in $k[x]$, for the coefficients of these polynomials are the pth powers of the coefficients of $\phi(x)$ and $\psi(x)$, and belong to k. From $f(x^p) = \{g(x)\}^p = \phi_1(x^p) \cdot \psi_1(x^p)$ we conclude that $f(x) = \phi_1(x) \cdot \psi_1(x)$ in $k[x]$, contrary to the hypothesis that $f(x)$ is irreducible in $k[x]$.

Also solved by George Bergman, L. Carlitz, and the proposer.

Dense Sets of Functions

5123 [1963, 765]. *Proposed by R. W. Newcomb, Stanford University*

Prove that the set of infinitely differentiable functions of support bounded on the left is dense in the set of distributions of support bounded on the left.

Solution by the proposer. We use the notation of L. Schwartz, *Théorie des distributions*, vol. I, 1957, p. 89, and vol. II, 1959, p. 28. An existence type proof for the problem follows immediately from theorems in Schwartz, vol. I. We offer the following constructive approach.

Consider a sequence $\{\phi_j\}$ of infinitely differentiable functions of bounded support chosen so that $\lim_{j\to\infty} \phi_j = \delta$; $\phi_j \in \mathfrak{D} \subset \mathcal{E}'$. For any $T \in \mathfrak{D}'_+ \subset \mathfrak{D}'$, form $\phi_j * T$. We know that $\phi_j * T \in \mathfrak{D}_+$ [Schwartz, v. II, p. 29]. Since $\phi_j \in \mathcal{E}'$ and $T \in \mathfrak{D}'$, it follows from the continuity of the convolution for such distributions [Schwartz, p. 13] that

$$\lim_{j\to\infty} \phi_j * T = \delta * T = T.$$

Consequently, $\mathfrak{D}_+$ is dense in $\mathfrak{D}'_+$.

Addition Chains of Vectors

5125 [1963, 765]. *Proposed by Richard Bellman, The RAND Corporation, Santa Monica, California*

It is easy to determine the minimum number of multiplications required to generate a^N from a. What is the minimum number required to generate $a^M b^N$ starting with a and b?