# Summary of Microsystems Laboratory Activities in Semistate Theory\*

B. Dziurla and R. W. Newcomb Microsystems Laboratory Electrical Engineering Department University of Maryland College Park, MD 20742 Phone: (301) 454-6869

Abstract':

The activities of the Microsystems Laboratory, University of Maryland, are summarized with emphasis upon the main ideas and their evolution. Concentration is seen to be on the design of nonlinear hysteretic systems, especially ones of interest to neural-type microsystems

### I. Introduction

Since 1978 the Microsystems Laboratory has had ongoing activities in the area we now call semistate theory and in which our emphasis has been upon circuit theory developments. This activity was triggered by our 1978 meeting in Gdańsk, Poland, under the joint Polish-American Program on Active Microelectronic Systems which concentrated upon neural-type electronic circuits. We had come across the paper of Campbell, Meyer, and Rose [B-1] which used the Drazin inverse to solve semistate equations in one-fell-swoop and found that this would be useful for analyzing and characterizing the circuits that were under development in the Program. This is because electronic circuits generally have resistive components which lead to nondynamic portions of the describing equations; to obtain statevariable equations for these circuits generally means reductions of the equations by elimination of the nondynamic portions which in turn can lead to the use of various tricks and introduction of errors. Since work was also being done on hysteresis, it soon became evident that the semistate theory offered considerable advantage for the characterization and solution of hysteretic systems and with them neural-type microsystems in which hysteresis may be a key design component. At that 1978 meeting the first of us suggested, from which we settled upon, the word "semistate" to describe the variables used in the resulting description since the word "state" was inappropriate and some suggestive word was needed. However, it should be noted that the concept had been in use by the second of us since at least 1966 when stability of time-variable passive circuits had been proven using semistate equations, which at that point were called "pseudostate equations" [1, p.72].

#### II. Results

Our first joint paper [2] was at the 1979 MTNS in Delft where we introduced the semistate as a concept into the circuits and systems area. The main idea was to call attention to the fact that, using a formulation similar to that of Desoer [B-2], linear circuit equations could be written in the form used by Campbell, et. al. [B-1], and that this allowed for direct solution through the Drazin inverse. In essence this was a small first step since at first glance not too much is

gained by such a description of linear circuits for which many techniques were already available. However, it quickly became apparent that for nonlinear circuits, and especially ones with hysteresis, the semistate description had something new and powerful to offer [3]. Consequently, a study was made [4] which showed that almost every finite lumped circuit has a description in the semistate canonical form

$$Edx/dt = A(x,t) + Bu$$
 (1a)  
y = Cx (1b)

Here B, C, and E are constant matrices and various equivalences may be needed to place all of the nonlinearities and time, t, variations into the A(.,.) term. The input vector is u, the output vector is y, x is called the semistate vector and we have changed the notation from the original to follow the more recent linear case notation of Lewis [B-3,p.17] which is more in line with that of state-variable works and also easier to print on our letter quality printer. It should be noted that the form of equation (1) is close to that of Dolezal [B-4,p.31] except that we have taken all time variation out of E. Once (1) is obtained further manipulation can bring it into other forms that may be of interest for certain studies. For example, by constant matrix transformations on x and on (la), E can be made a direct sum of an identity and a zero matrix. In any event, because of the apparent systems theory use of the canonical equations we were led to study the behavior of the canonical equations under cascade and feedback connections [5].

The real advantage of (1) comes about in the study of nonlinear circuits and especially ones with hysteresis. For the latter we often do not have valid alternate characterizations since input-output descriptions are multivalued. However, (1) allows some classes of hysteresis to be described by single valued functions, it being the elimination of some of the semistate variables in obtaining state-like descriptions that leads to multivalued solutions including hysteresis. This led us to obtain designs of some interesting classes of hysteresis [5][7]. And since the canonical equations are of such use for systems with hysteresis, it seemed necessary to obtain convenient means to solve the equations. This led us to the very interesting continuationtype of method to solve the canonical equations [8] for which an example [11] shows that the method could be effective for nonlinear system analysis.

As mentioned above the research of the US-Poland Program concentrated upon neural-type microsystems. Pushed by the recent interests in neural networks, this original inter-

est has been rekindled. Since the basic neural-type cell of our theory depends upon hysteresis, semistate theory has turned out to be a natural for characterizing and designing our neural-type circuits. Three specific works from the Microsystems Laboratory have been presented. The first is by Professor DeClaris and A. Rindos, a student working with him, on the analysis of "Aplysia Californica" [21] in which clusters of neurons are described in semistate form. The other two papers consider the analysis of two specific transistor circuits, one being a neural-type junction [16] where neural-type pulses are combined and processed and the other being of a neural-type cell [17] where hysteresis is used to process, generate, and code information onto the neural-type pulses.

The above mentioned researches form the core of our studies, but some other related works have been undertaken, as per the following discussion.

A theme of circuit theory folklore is that by introducing parasitic effects into a system the semistate variables become state variables. Interestingly, by an example [9] of some practical importance, we have shown that folklore may not be fact in that the semistate equations do not directly become state variable equations by the introduction of realistic parasitics. On the other hand state variable equations do not exist if one is treating a differentiator while one would like to make differentiators as practical devices. But semistate equations can be created to give transfer functions with arbitrary poles at infinity, including a simple one for a differentiator. Thus, in [12] we have shown that, by some intriguing manipulations of Zaghloul, a differentiator can be stably designed through the use of integrators.

PARCOR lattices are of considerable interest in the area of digital signal pro-PARCOR lattices cessing where, with our affiliated group in Spain, we have been able to use them for such things as noninvasive analysis of the ear [B-5]. Consequently, we investigated the forward-backward decomposition of semistate equations [B-6] as they relate to the forwardbackward signals of PARCOR lattices [15]. Likewise sensitivity is of considerable interest in the design of electronic circuits since circuits that are too sensitive to parameter changes may not perform as designed. This led us to a study of sensitivity through the semistate equations [18].

An interesting area for the application of semistate theory is to that of robotic system design since there are kinematic as well as dynamic variables. In this area some of our research investigates the design of knot tying robots for which a semistate theory of knots has been developed [20]. The need for semistate theory in knot characterization is to perform a reduction from four dimensions, where two 2-dimensional circles are dynamically created by oscillators in direct product form, into real 3-dimensional physical space. Specifically two 2-dimensional circles are dynamically created by oscillators in direct product form with the reduction done via a nondynamical algebraic equation. Thus, we have

| dx/dt | = | У     | (2a) |
|-------|---|-------|------|
| dy/dt | = | -m2 x | (2b) |
| dz/dt | = | W     | (2c) |
| du/d+ | - | -527  | 1241 |

|   | $D = w + \delta$ , $\delta$ =constant           | (2e) |
|---|---|------|
|   | $R_1 = [(mx)^2 + y^2]^{1/2}$                    | (2f) |
|   | $R_2 = [(nz)^2 + w^2]^{1/2}$                    | (2g) |
| X | = mx/D  | (2h) |
|   | = y/D   | (2i) |
| Z | = $[nz/D] \cdot [R_1/(\delta^2 - R_2^2)^{1/2}]$ | (2j) |

Even though these may look formidable they are readily interpreted to give an (m,n)-torus knot in three diminsional XYZ-space. Thus, (2a-d) give two uncoupled linear oscillators of radian frequency m and n. (2h-j) reduce these from four dimensions to three with the key term being defined by the algebraic constraint of (2e);  $R_1$  &  $R_2$  are the radii of the xy and zw circles that are in direct product form in four dimensions. The result is a knotted trajectory on the torus

$$Z^2 + [R-(X^2+Y^2)^{1/2}]^2 = r^2$$
 (2k)

of meridian circle radius  $r=R_1R_2/[\delta^2-R_2^2]$  revolved around an axial circle of radius  $R=\delta R_1/[\delta^2-R_2^2]$ . Practically the knot is better realized using van der Pol oscillators rather than linear ones, however.

Since many knots are connected sums of other knots, hysteresis can be used to form the connected sum given two knots which are already realized [14]. Another use for the hysteresis that is conveniently synthesized via semistate theory is that of chaos generation [8-7]

Finally we mention that almost all semistate described systems so far studied have been regular, that is, in the linear case sE-A is nonsingular. Since nonregular systems will either have no solution or an infinity of solutions, nonregular systems are less practical than curious. Still by exhibiting some nonregular systems [19] we have shown that through the incorporation of nullators and/or nullators in a design, as has been done in certain transistor and op-amp circuit designs, if one is not careful one may meet nonregular systems in practice.

## III. The Future

At this point it seems clear that semistate theory gives a general framework with distinct advantages for the design of practical nonlinear systems, especially of ones in which both dynamic and algebraic constraints arise, such as those describing robots, knots, and neural-type electronic circuits. therefore, foresee the need for the development of design techniques that take into account the basic elements of these classes of systems within the framework of semistate theory. Indeed, in the presence of certain classes of hysteresis there seems to be few alternative mathematical descriptions available, giving semistate theory an unequaled position. Consequently, semistate theory should have a bright and promising future in the area of engineering design.

The attached figure interrelates the activities mentioned above.

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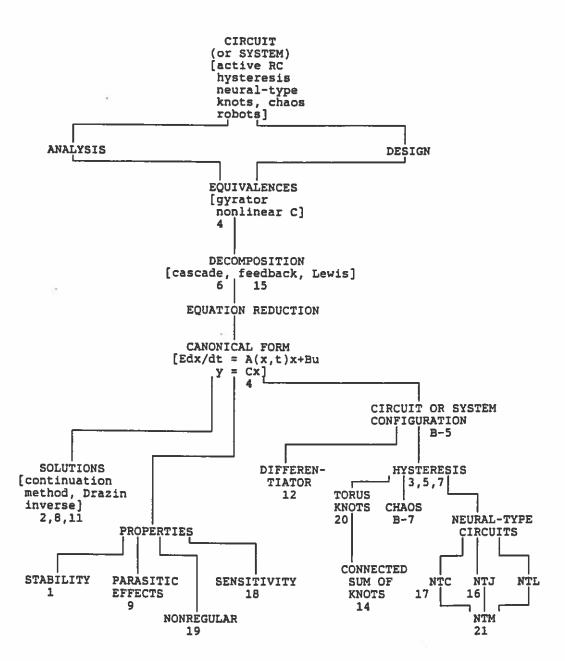
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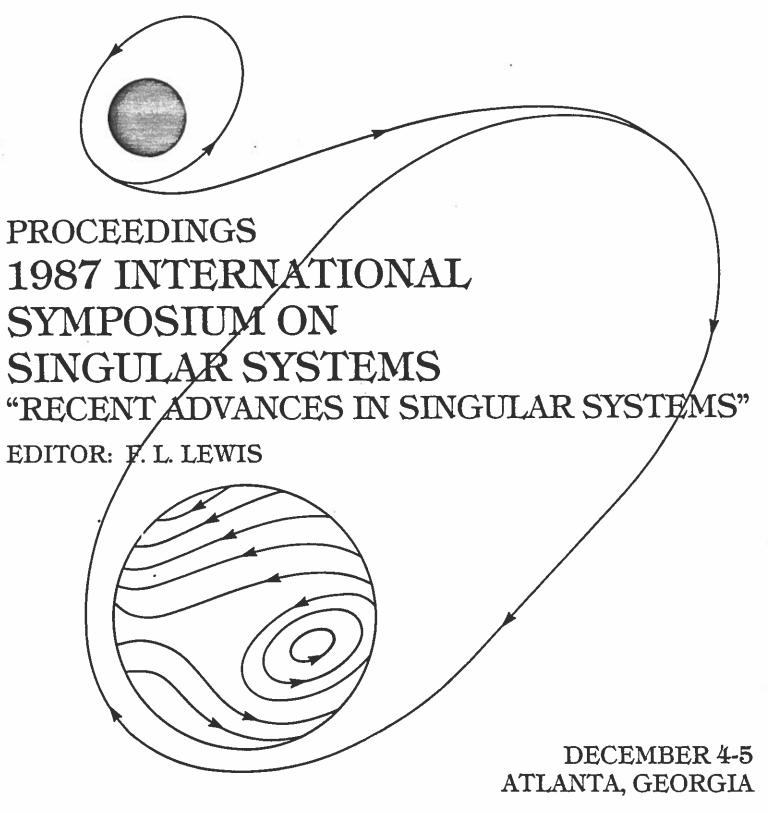
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