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SIGNAL PROCESSORS FOR NONLINEAR WAVE PROPAGATION IN THE INNER AUDITORY SYSTEM

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SUMMARY

Nonlinear phenomena may explain many interesting facts in Hearing Perception as it is well known [Kim.80, Str.86]. Through the present paper a nonlinear model for the Inner Ear based on a Transmission Line structure [Sch.83] is presented, with the nonlinearities inserted in the elastic term on the basilar membrane equivalent. This model may be seen as a chain of homogeneous sections, for each one a Korteweg-de Vries nonlinear equation may be stated showing the propagation of solitary waves in one direction [Gar.74, Abl.74]. At the boundary between two of these incremental sections a certain kind of reflective phenomena should be expected, as detected by many researchers [Kem.78, Wit.79, Wit.80]. For such, a convenient description must be found in the shape of a two way nonlinear Boussinesq's equation [Dod.84]. Our purpose is to show computational structures amenable of solving nonlinear wave propagation in one section. The interconnection of several of these sections give a global nonlinear model of the whole Inner Ear. This model could be used to adapt nonlinear theories of Hearing Perception to the design of Hearing Aids.

INTRODUCTION

The present work is related with the development of Signal Processing Algorithms amenable of being applied to Noninvasive Methods of measurement in Biomedical Systems, to Computational Perception and Robotics. Its final objective is related with the design of special purpose Signal Processors to be applied in the above mentioned areas. The specific problem to be treated is the study of Nonlinear Models in the Auditory System, which will be simulated and implemented through a certain kind of Signal Processors. These could be used in improving Hearing Aids and in Noninvasive Auditory Measurements. For such, the problems posed by the nonlinear behavior of the Auditory System [Kim.80] have been carefully reviewed. The frequency selective mechanisms in the Auditory System are not well understood yet, although many researchers deeply believe that nonlinear wave propagation phenomena are well behind them [Zwi.80]. On the other hand, it has been experimentally proved that wave propagation in the cochlea takes place with a low level of reflection and dispersion [deB.80], these two characteristics being proper of the solutions of certain kinds of nonlinear wave equations, which are known as "solitons" [Dod.84]. Through the present paper wave propagation in the Inner Auditory System under nonlinear conditions will be presented. On this basis, typical Signal Processing Structures, such as Linear Lattices, will be modified to allow their use in nonlinear problems of the before mentioned kind [Góm.83]. Finally, some words will be said on possible methods for the characterization of the nonlinear behavior of the Auditory System through a generalization of System Inversion techniques in the linear case [Son.81].

A NONLINEAR MODEL FOR THE AUDITORY SYSTEM

Through this section a nonlinear wave propagation model will be presented, from which a set of computationally tractable equations amenable of being solved by reasonably simple means will be obtained. For such we will use a unidimensional Transmission Line Model [Sch.73], assuming the nonlinear behavior to be present in the capacitive (elastic) element on the Partition Membranes. The before mentioned model may be expressed by the set of coupled equations:

$$\frac{\delta p}{\delta x} = -\ell \frac{\delta u}{\delta t} \qquad (1)$$

$$\frac{\delta p}{\delta t} = -\mu \frac{\delta^2}{\delta t^2}\left(\frac{\delta u}{\delta x}\right) - \sigma \frac{\delta}{\delta t}\left(\frac{\delta u}{\delta x}\right) - \varepsilon \frac{\delta u}{\delta x} \qquad (2)$$

where p is the differential pressure acting between both sides of the partition membranes, and u is the volumetric velocity of the fluid at a given point in any of both scales. One dimensional long wave propagation and fluid incompressibility are assumed. ℓ, μ, σ and ε are the inertia, mass, viscosity and elasticity of fluids and membranes, (1-2) being the equations for the linear case [Góm.83]. If we assume that nonlinearities are introduced by the elastic term on the membranes, then equation (2) should be slightly modified to take them into account. This can be done by allowing the elasticity ε to be a function of p as follows:

$$\varepsilon = \varepsilon_0 \, (1 - a \, p^n) \tag{3}$$

ε_0 being the linear value for the elastic term which in general depends on distance, a being a proportionality factor, and n being the order of the nonlinearity, assumed to be an integer taking the values 1 or 2. Equations (1-3) are difficult to deal with. A very suggestive approach may be derived if a perturbational method is used. This method assumes long waves in the sense that the dispersive relationship between frequency and wave number may be expanded for small values of the wave number (η) as a power series, taking only the first two terms into account. Besides that, the method assumes small amplitudes in the dynamic variables (pressure p and flux u) this fact being equivalent to consider a weak nonlinearity. To know what terms can be disregarded as second order effects the natural scales of space and time in the problem must be changed in equations (1-2). The dispersive relationship in the linearized case may be stated as:

$$w(\eta) = \frac{1}{[\ell \, \varepsilon_0]^{\frac{1}{2}}} \, \frac{\eta}{[1 + \mu/\ell \, \eta^2]^{\frac{1}{2}}} \approx$$

$$\approx a_1 \, \eta - a_3 \, \eta^3 \tag{4}$$

We will introduce this approximation into the phase relation:

$$\eta \, x - w \, t = \eta \, (x - a_1 \, t) + $$

$$+ \, a_3 \, \eta^3 \, t \tag{5}$$

This expression suggests us the change of variables given by:

$$\xi = \chi^q \, (x - \alpha \, t) \tag{6}$$

$$\tau = \chi^{3q} \, t \tag{7}$$

A subsequent power expansion of the dynamic variables in χ must be introduced in equations (1-2). After some algebraic manipulations, following Dodd [Dod.84], these may be reduced to the well-known Korteweg-de Vries (KdV) kind of structure [Gar.74]:

$$u_t + \alpha \, u_x + \beta \, u^n \, u_x + \gamma \, u_{xxx} = 0 \tag{8}$$

where we have taken u_t, u_x and u_{xxx} as the first time and space and the third space partial derivatives, respectively. The parameters α, β and γ are given by:

$$\alpha = 1/(\ell \, \varepsilon_0)^{\frac{1}{2}} \tag{9}$$

$$\beta = \frac{a \, \varepsilon_0 \, \alpha}{2 \, \ell} \, (\alpha \, \ell)^{n+2} \tag{10}$$

$$\gamma = \alpha \, \mu/(2 \, \ell) \tag{11}$$

This structure explains one-directional wave propagation only. For two-way wave propagation, a reformulation of KdV equation leads to Boussinesq's equation:

$$u_{tt} = \left[\alpha^2 \, u + \frac{2 \, \beta \, \alpha}{n + 1} \, u^{n+1} + 2 \, \gamma \, \alpha \, u_{xx}\right]_{xx} \tag{12}$$

This equation may be useful to study bidirectional wave propagation in the Inner Auditory System, given the great interest shown by many researchers in the field [deB.80]. Our aim will be to study the implementation of computing structures simulating the behavior of equation (8) to explain wave propagation in the Inner Auditory System by discrete methods.

NUMERICAL METHODS AND SIGNAL PROCESSORS

In the present work we will show the methods employed in the solution of the KdV equation by finite elements [Pot.73, Duc.86], which are currently being carried out. We will solve the problem of nonlinear wave propagation in one dimension taking a segment of homogeneous transmission line of a given length L, and assuming both spatial and temporal sampling processes on the wave function $\varphi(x,t)$ as follows:

$$\varphi(k,n) = \varphi(x{=}k\delta, t{=}n\theta); \; 0 \le n \le N{-}1; \; 0 \le k \le K{-}1 \tag{13}$$

where δ and θ are the space and time sampling intervals. In these conditions the "virtual domain" in which we will have to solve the problem, assuming resting conditions for $t \le 0$, will be the one given by Fig 1.

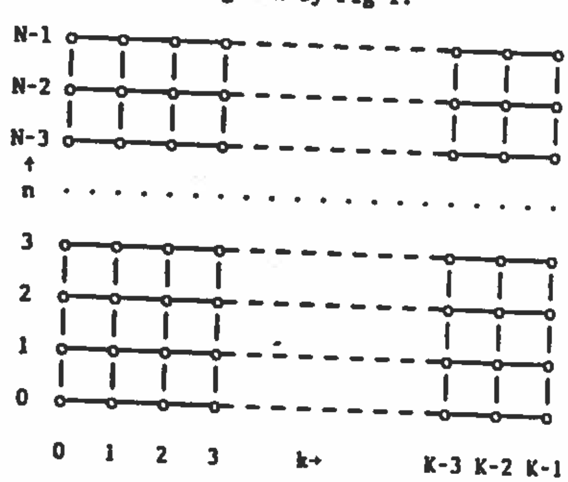

Fig. 1. Virtual Space-Time Domain of resolution.

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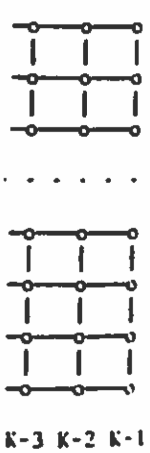

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We refer to this as the "virtual domain" (VD) because this time-domain grid of data given by the K·N matrix in (13) must be mapped into a smaller capacity "physical processor" (PP), which will not be able in general of handling the whole data structure at a time. To solve this problem a "mapping strategy" must be designed to carry data to and extract data from the physical processor and the "virtual memory" (VM). To translate KdV equation into finite elements on the domain defined in Fig. 1, we will find estimates for the derivatives in time and space. If we use the First Eulerian Differences (FED) we may state:

$$\frac{\delta\varphi}{\delta x} = 1/\delta\,[\varphi(k,n) - \varphi(k-1,n)] \qquad (14)$$

$$\frac{\delta\varphi}{\delta t} = 1/\theta\,[\varphi(k,n) - \varphi(k,n-1)] \qquad (15)$$

and extending this same concept to higher order derivatives we will translate the normalized KdV equation to:

$$\varphi^2(k,n) + \varphi(k,n)[2 - \varphi(k-1,n)] - 3\,\varphi(k-1,n) +$$

$$+ 3\,\varphi(k-2,n) - \varphi(k-3,n) - \varphi(k,n-1) = 0 \qquad (16)$$

This seems is a quadratic form in the highest space and time order sample of the wave function, $\varphi(k,n)$. The nonlinear term in itself is given by $\varphi(k,n)\cdot\varphi(k-1,n)$. In order to obtain a solution for this equation we need to solve (16) for $\varphi(k,n)$, as:

$$\varphi(k,n) = -1 + 1/2\,\varphi(k-1,n) \pm R^{\frac{1}{2}} \qquad (17)$$

with R given by:

$$R = 1 + 1/4\,\varphi^2(k-1,n) + 2\,\varphi(k-1,n) -$$

$$- 3\,\varphi(k-2,n) + \varphi(k-3,n) + \varphi(k,n-1) \qquad (18)$$

Expressions (17-18) will be referred to as the "solving recursion" (SR). As it can be easily inferred, this recursion may be implemented by the "minimal virtual processor" (MVP) in Fig. 2.

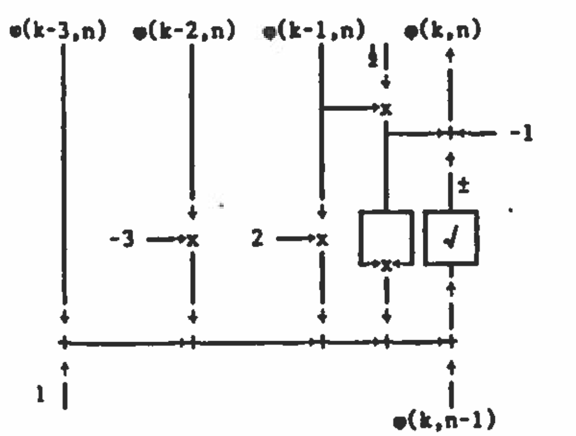

Fig. 2. Structure of the Minimal Virtual Processor.

We refer to this structure as the "minimal virtual processor" (MVP) because this is the lowest order arithmetic structure being able of computing at least one data point of the solution in a sampling interval. In fact one of these MVP's may be implemented by several "physical processors" depending on how much computational power we may physically assign to solve the problem. To save the gap between the virtual domain and the physical domain, we must use mapping concepts, as stated before. Some other MVP may be obtained in the case that representations other than FED would be used for time and space derivatives from Numerical Analysis [Ger.84], from which a family of Signal Processors may be obtained with different degrees of accuracy. A Minimal Physical Processor (MPP) is under design, this structure being the minimal amount of arithmetic hardware and memory which allows the implementation of one MVP, using the principle of "maximal compression", which means that if an arithmetic operator (let's say addition) is found p times in the MVP, then we will implement physically only one such function, which will be called p times on different data to implement one MVP. Of course, this need not be the best solution by far, but gives a hint on the complexity of the physical implementation problem, the mapping strategies to be used, and the amount of area and power consumption to be expected at the time of integrating the result as a VLSI structure [Góm.87]. The implementation of the Arithmetic Unit in the MPP can be done as suggested by Zurawski [Zur.87].

NONLINEAR INVERSE SCATTERING METHODS

In a rather different approach, the present study may be generalized to other problems of the same kind by standard Spectral Analysis methods as in the linear case, if a counterpart of these methods can be found in the nonlinear case. This counterpart is currently known as Inverse Scattering Transform Methods (ISTM) [Abl.74], and constitute a generalization of Fourier Methods to the nonlinear domain. These methods may produce lattice-type algorithms, presenting the additional advantage of being strongly connected with the problem of Line Transmission Inversion [Son.81]. The general idea behind the IST is to treat the waves travelling along the nonlinear medium as dispersive potentials for some ficticious sinusoids. We would find the transmission -or reflection- coefficients for this waves as a function of their wave number in a given instant of time (the Scattering data), and solving the (linear!) time evolution equations for the scattering data at a further instant from which we would finally infer the scatering potential (the state of the wave under propagation). This is considered as a generalization of Fourier methods because in the limit, as far as small amplitudes are involved (weak nonlinear effects) the IST can be seen as the space Fourier transform of the wave, solving the evolution equation for each wave number and reconstructing the wave by inverse space Fourier Transform. The main advantage of the IST comes from the fact that in Linear System Theory it is straight forward to treat discontinuities by means of the reflection coefficients at the points where these are met, so it could also be possible to define some kind of reflection functions for nonlinear problems which would enable us to treat wave propagation in nonlinear non-homogeneous media in a similar way.

DISCUSSION AND APPLICATIONS

The present model is being studied using finite element methods. In a first step the study has been extended only to homogeneous media with step-like discontinuities, in order to trace the reflection phenomena. The next steps to be covered will deal with the simulation of non-homogeneous cases, to be precisely represented by empirical formulation through trial and error methods and by Mathematical Analysis. The results will be checked against data collected experimentally, such as "Kemp Echoes" [Kem.78]. Other activities to be further developed are the study of the optimum physical processors amenable of being implemented as VLSI structures, and the mapping techniques associated. More research need to be done in testing the numerical behavior and stability of the FED processor. The expected results will help to improve the design of Hearing Prostheses and to develop the study of Nonlinear Lattices. A better understanding of the Auditory System behavior is also expected.

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