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A Semistate Model for Neural-Type Junction Circuits*

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This paper sets up the canonical semistate description for a circuit that behaves as a neural-type junction. The system takes n neural-type inputs and processes these to give an appropriate neural-type output. As such it is a system of degree one with n+1 nonlinearities. A solution technique is presented and a reduced noncanonical semistate formulation given. When combined with the companion work on neural-type cells, this paper illustrates the power of semistate techniques for description of neural-type electronic systems, which, for example, could mimic neural behavior in the coding of movement direction.

I. Introduction

Neural-type electronic systems are ones that have the important signal processing properties of biological neural systems [1]. In construction of neural-type systems various subsystems have been isolated so that standard subsystems can be constructed and then combined to obtain a full system. Included among the subsystems are the neural-type junctions (NTJ's) where neural-type signals are combined [2]. Indeed the NTJ's are rather important since it is in the combining of neural pulses that information is coded in biological neural systems, as for example in the coding of movement direction [3] which may be of direct interest in realizing robot motion based upon neural-type signals. And since an NTJ would normally feed to a Neural-Type Cell, NTC, it is of use to have similar types of descriptions for NTC's and NTJ's. As shown in a companion paper [4] the natural description for NTC's is the semistate one since normally hysteresis is present, ruling out other more familiar descriptions such as state-variable ones. Consequently, we here present the semistate description of an MOS NTJ circuit, this latter being a simplified version of one given in [5].

II. The NTJ

The MOS NTJ of interest is shown in Fig. 1 where there are n input voltages v_1, ..., v_n to the left hand n-channel transis-

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tors; normally these will be neural-type pulse trains in which the neural-type pulses are assumed to be positive. These input transistors sum their drain currents in the resistor R_I which in turn feeds its voltage to the output p-channel transistor on the right, the output circuit and its behavior being identical to the output stage of the companion NTC [4]. Indeed the circuit of Fig. 1 can be considered as a multi-input NTC except that the feedback necessary to do pulse coding is absent since the function of the NTJ is simply to combine signals. Taken in conjunction with the nonlinear behavior of Q_y the R_c-C combination is, therefore, to shape the output into neural-type pulses, with the output being taken across R_y.

III. The Semistate Equations

We consider the NTJ circuit of Fig. 1 where we measure all voltages with respect to ground. As discussed above, this circuit is to be considered as the cascade of two subcircuits, an input circuit which develops the voltage v_I, by summing the drain currents of transistors Q_1 through Q_n in R_I, and an output circuit which develops the voltage v_y by dynamically forming the drain current i_y of Q_y. To write the semistate equations we first need the laws for the transistors. These we take to be

$$i_g = 0 \qquad (1a)$$

$$i_d = f(v_{gs}, v_{ds}) \qquad (1b)$$

where v_{gs} is the gate to source voltage and v_{ds} is the drain to source voltage, these being taken with the appropriate signs to use the same law for complementary n- & p-channel transistors. To distinguish the f(.,.)'s for the different transistors we will use an appropriate subscript.

Writing first the dynamical equation, we have (with $G_c=1/R_c$)

$$C[d(V_{dd}-v_c)/dt] + G_c(V_{dd}-v_c) = i_y \qquad (2a)$$

$$i_y = f_y(v_c-v_I, v_c-v_y) \qquad (2b)$$

Next we note that

$$V_{dd} - v_I = R_I \sum_{j=1}^{n} f_j(v_j, v_I) \qquad (2c)$$

and that the output y and inputs u_1 through u_n are given by

$$y = v_y = R_y i_y \qquad (2d)$$

$$u_j = v_j \quad \text{for } j=1,\dots,n \qquad (2e)$$

Equations (2) comprise the canonical semistate equations for which we see that we wish to define the semistate (n+3)-vector x and the input n-vector u as (superscript T denoting matrix transposition)

$$x^T = [V_{dd}-v_c,\ i_y,\ V_{dd}-v_I,\ v_1,\ \ldots,\ v_n] = [x_1, \ldots, x_{n+3}] \quad (3a)$$

$$u^T = [u_1,\ \ldots,\ u_n] \quad (3b)$$

In matrix form these semistate equations are rewritten as

$$\begin{bmatrix} C & 0 & 0 \ldots 0 \\ 0 & 0 & 0 \ldots 0 \\ 0 & 0 & 0 \ldots 0 \\ 0 & 0 & 0 \ldots 0 \\ & \vdots & \\ 0 & 0 & 0 \ldots 0 \end{bmatrix} \dot{x} + \begin{bmatrix} G_c & -1 & 0 & 0 \ldots 0 \\ 0 & 1 & 0 & 0 \ldots 0 \\ 0 & 0 & 1 & 0 \ldots 0 \\ 0 & 0 & 0 & 1 \ldots 0 \\ & & \vdots & \\ 0 & 0 & 0 & 0 \ldots 1 \end{bmatrix} x - \begin{bmatrix} 0 \\ f_y(x_3-x_1, V_{dd}-x_1-R_y x_2) \\ R_I \Sigma f_j(x_{j+3}, V_{dd}-x_3) \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ \vdots \\ 1 \end{bmatrix} u \quad (3c)$$

$$y = [0,\ R_y,\ 0, \ldots, 0]x \quad (3d)$$

Equations (3) are in the canonical form

$$E\dot{x} - A(x) = Bu \quad (4a)$$

$$y = Cx \quad (4b)$$

where in (3c) the linear and nonlinear portions of the nondynamical operator A(.) have been separated with the lower $n \times n$ submatrix of the linear portion being the identity in order to realize (2e).

By eliminating the semistate components v_1 through v_n, reduced but noncanonical semistate equations can be obtained, these being

$$x_r^T = [V_{dd}-v_c,\ i_y,\ V_{dd}-v_I] = [x_1,\ x_2,\ x_3] \quad (5a)$$

$$\begin{bmatrix} C & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \dot{x}_r + \begin{bmatrix} G_c & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} x_r = \begin{bmatrix} 0 \\ f_y(x_3-x_1, V_{dd}-x_1-R_y x_2) \\ R_I \sum_{j=1}^{n} f_j(u_j, V_{dd}-x_3) \end{bmatrix} \quad (5b)$$

$$y = [0,\ R_y,\ 0]x_r \quad (5c)$$

To obtain semistate solutions, one must solve the last of (5b) for x_3 which when substituted in the middle equation allows x_2 to be determined in terms of x_1 and u. This in turn allows the first equation of (5b) to become a state-variable equation, with x_1 the state variable.

III. The Junction Effect

To be effective one desires the NTJ to give an output when the inputs sum to above a desired threshold. This phenomena is achieved through the input stage with the desired effect showing up in the summation voltage v_I. When this summation voltage falls below the threshold of the output stage a neural-type pulse is triggered. Consequently, we investigate this summation voltage for which we assume the following law for the transistor drain current [6].

$$f(x,y) = \begin{cases} 0 & 0 \geq x-V_T & \text{off} \\ \beta[2(x-V_T)y-y^2] & 0 \leq x-V_T \geq y & \text{ohmic} \\ \beta(x-V_T)^2 & 0 \leq x-V_T \leq y & \text{saturation} \end{cases} \qquad (6)$$

β and V_T are device gain and threshold voltage constants that depend upon the processing used to construct the transistor. Again appropriate subscripts will be used corresponding to a referenced transistor except that all of the n input transistors will be assumed identical and indicated by the same subscript i.

We desire to calculate v_I for which we assume that at a given instant there are k nonzero inputs all of the same value. If all inputs are below V_{Ti}, then no current flows in R_I and $v_I=0$. If the nonzero inputs are of value U and above threshold, then there are two possible cases for the non-off transistors, either they are all ohmic or all saturated. In the saturated case we directly find

$$v_I = V_{dd} - kR_I\beta_i(U-V_{Ti})^2 \qquad (7a)$$

while in the ohmic case we need to solve

$$v_I = V_{dd} - kR_I\beta_i[2(U-V_{Ti})v_I-v_I^2] \qquad (7b)$$

the solution of which is

$$v_I = (U-V_{Ti} + \frac{1}{2k\beta_i R_I})[1 - (1 - \frac{4k\beta_i R_I V_{dd}}{(1+2k\beta_i R_I[U-V_{Ti}])})^{1/2}] \qquad (7c)$$

Further, the transition between the two cases is when the value of the inputs satisfies $U-V_{Ti}=v_I$ which when substitited into (7a) gives

$$U_{trns} = V_{Ti} + [-1 + (1+4k\beta_i R_I V_{dd})^{1/2}]/(2k\beta_i R_I) \qquad (7d)$$

For $U<U_{trns}$ the input transistors are in the saturation region and for $U>U_{trns}$ they are in the ohmic region.

Considering the example of an MC4007 complementary transistor package, which has $\beta=0.00063$ and $V_T=2$, we choose a bias voltage of $V_{dd}=10$, and an input pulse level of $U=4.5$; R_I is a design parameter to be varied. v_I is plotted in Fig. 2, along with U_{trns} for two values of R_I to show that the latter is an effective design parameter. Thus, as shown by Fig. 2, if the summing resistor is chosen to be 500 ohms then the NTJ will only be triggered on when four or more inputs are on while if the summing resistor is chosen to be 1000 ohms then only one input transistor needs to be turned on to yield an output pulse.

IV. Discussion

Here we have presented semistate equations that describe the simple NTJ of Fig. 1, these being given in several forms the most convenient being the noncanonical form of equations (5). The dynamical behavior of this circuit is similar to that of the related NTC and, hence, not treated here. But the junction effect is well illustrated in Fig.2 where it is seen that the number of inputs needed to trigger a neural-type output pulse is determined by the summing resistor R_Σ which can, therefore, be used as a design parameter for the NTJ.

The circuit treated here is a simplified version of the NTJ of [5] where dynamics is inserted into the input transistor. This dynamics allows the circuit to respond to trains of neural-type pulses, as would be desired in decoding real neural pulses, for example in the control of limb motion. One can further insert a feedback transistor in this NTJ, as is done in the NTC of [4] in which case the NTJ can itself generate neural-type pulse trains allowing coding of information to take place directly in the NTJ. These modifications of course lead to much more complicated semi-state equations but their importance should make them subjects for future study.

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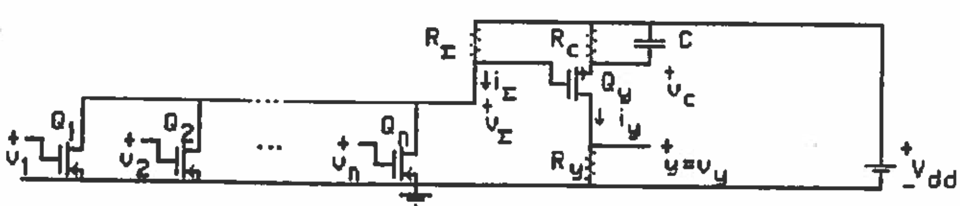
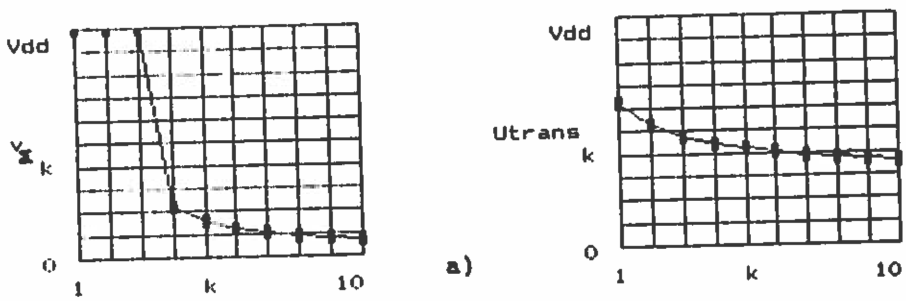


Figure 1
NTJ

R_Σ = 500

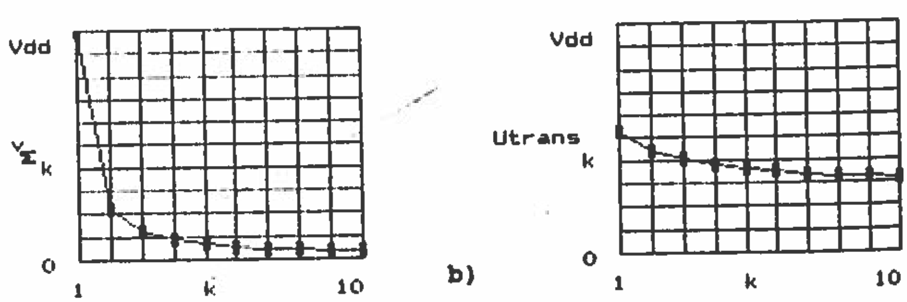
k	$v_{\Sigma k}$	$Utrans_k$
1	9.9960625	6.26637743
2	9.992125	5.26872505
3	9.9881875	4.76664794
4	2.22222222	4.4481664
5	1.55695395	4.22222222
6	1.2310142	4.05083137
7	1.0238016	3.91486965
8	0.87819922	3.80349179
9	0.76964122	3.7100165
10	0.68534655	3.63006788



a)

R_Σ = 1000

k	$v_{\Sigma k}$	$Utrans_k$
1	9.9960625	5.26872505
2	2.22222222	4.4481664
3	1.2310142	4.05083137
4	0.87819922	3.80349179
5	0.68534655	3.63006788
6	0.5626448	3.49959476
7	0.47745395	3.39673006
8	0.41477355	3.31287328
9	0.36669181	3.24277291
10	0.32862684	3.18301379



b)

Figure 2 v_Σ Versus Number of Inputs
a) R_Σ = 500 b) R_Σ = 1000

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