

## APPLICATIONS OF NONLINEAR LATTICES IN THE CHARACTERIZATION OF THE INNER AUDITORY SYSTEM

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### ABSTRACT

Through the present paper a model is discussed for cochlear micromechanics by means of Toda Lattices. As it is well known one of the main problems in modeling the Inner Auditory System arises with the presence of a certain degree of nonlinear behaviour in the process of wave propagation along the mentioned structure. Toda Lattices are composed of cascaded lumped cells explaining the propagation of waves by a pair of coupled nonlinear difference equations, the elastic term being derived from an exponential potential function. The natural solution for this kind of equations comes out as a solitary wave, which may be a good representation of the kind of waves traveling along the junction membranes in the inner ear. Some of the applications of such a model would be related with the study of cochlear frequency selectivity together with the stimulation of the patterns of movement in the time domain including nonlinear phenomena. Another field of application would be the processing and interpretation of auditory omissions, many of which reveal also a nonlinear behaviour. The general theory is presented, some open problems are pointed out, and a final discussion establishes the possibilities for the continuation of the research under process.

### INTRODUCTION

The Auditory System is receiving an increasing attention by nowadays researchers working in the Signal Processing aspects of Human Perception. As a matter of fact, the way in which the Auditory System works may have conditioned the way in which the Speech ability has developed in humans, and as such there is a growing tendency to better understand the behaviour of the Auditory System [1]. From this point of view, it seems critical to gain more and more insight on the way that the Auditory System collects and processes audio signals to be transformed into sensations at the cerebral level, this structure being considered as a whole in itself, as it can be seen in Fig. 1., where the

Auditory System is decomposed into the Outer Ear, comprising the Ear Canal (EC) and the Tympanic Membrane (TM); the Middle Ear, consisting mainly in the Bony Chain (BC); the Inner Ear, which is accessed through the Oval Window (OW) and essentially consists of a cavity (the Cochlea, uncalled in the representation) separated by the Partition Membranes (PM) where the Sensory Cells (SC) are supposed to be located, and a Nerve Bundle (NB), which is responsible of collecting and conveying the processed information to the brain.

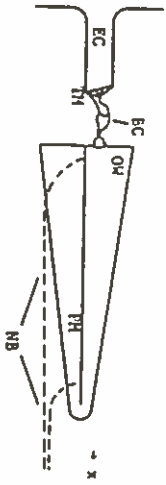


Fig. 1. Structure of the Auditory System.

The activity of the Auditory System can be considered under the systemic point of view as a cascade of different sections, each one of them being responsible for a given type of signal processing, as it may be represented in Fig. 2.

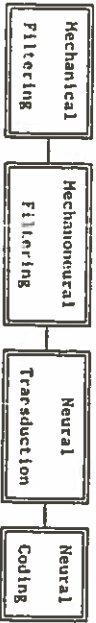


Fig. 2. Cascade Description of the Auditory System.

The Mechanical Filtering is the preliminary signal processing stage, due mainly to the mechanical properties of the Partition Membranes, and the filtering process is credited to be of passive nature. At this stage the information still propagates as mechanical waves through fluids and membranes, and it is usually being referred to as the "Macro-mechanical Filtering". The Mechanoneural Filtering, on the contrary, is referred to as the "Micro-mechanical Filtering", and is supposed to be due to the finer structure of the Partition Membranes, to the structure of the Sensory Cells and possibly to the interaction between these and the nerve connections, this last possibility accounting for the active behaviour observed in the filtering process [2]. The neural transduction is credited to be responsible for the conversion of mechanical actions into neural firing at the Nerve Bundle fibers, and its nature is not quite well understood yet. The physiology of the Sensory Cells seems to play a very special role in this kind of activity [3]. Finally, the Neural Coding is the most elusive of all these processes, in the sense that its role is not very clear by now. This process assigns neural firing

rates to a given neural fiber in the Nerve Bundle depending on the connecting place of the fiber with the Partition Membrane, on the intensity of the stimulus and on the activity of neighboring fibers, this last fact implying that some lateral inhibition mechanism may be present enhancing frequency selectivity, as it may be inferred from [4].

Any of these stages in Auditory Signal Processing is receiving much attention by very well qualified researchers, and although much knowledge has been gained on them, many points on our ability to understand the different processes still remain. Probably, one of the most passionate ones, due to the high controversy around it, will be the issue of the nonlinear behaviour detected in the Auditory System as early as in the late XVIII century [5, pg. 364]. Since that time it is well known that when the Auditory System is stimulated with simultaneous periodic sinusoidal signals, the impression for people with acute hearing perception is that a "third tone" appears to be present, and that the frequency of such tone is related with one of the first differences between the two imposed tones (2f<sub>1</sub>-f<sub>2</sub>). This fact has been confirmed nowadays through the invasive sounding of the Nerve Bundle in laboratory animals when several tones are presented [6]. In this same work a very interesting hypothesis is proposed on the possibility that compressional nonlinearities, associated with lateral inhibition could be responsible of a further enhancement of frequency selectivity. This fact brings into compressional nonlinearities would play a major role in further perception. Several explanations have been worked out to explain these facts into the Auditory System modeling, two of them being the most credited nowadays. The first one responsible of the nonlinear behaviour to the mechanical properties of the membrane in the membrane, as it does not seem clear where the nonlinearities should reside, the alternatives suggested point out to the changing parameter in the membrane mechanics, which is responsible of the losses in the system. As such, this possibility is of passive nature, in the sense that a parameter in a passive structure is credited to be responsible of the behaviour. The second one is gaining more and more attention, and it states that the nonlinearities could appear due to a feedback mechanism present in the Mechanoneural Filtering, in the sense that the Sensory Cells could behave both as Mechanoelectrical and Electro-mechanical Transducers, and as such, when electrically stimulated, are able of developing mechanical actions, this last to raise to some active "phase locking" mechanism, according to the high frequency selectivity, as reported by many authors [7]. Whether the electrical stimuli activating the Sensory Cells is due to neighbour cells or to the "reversible neural stimulation" is still an issue under much discussion. This last possibility is being supported by the well known phenomena of "spontaneous otoacoustic emissions" [8].

#### THE SUGGESTED MODEL.

Our aim through the present paper will be to present a

different point of view, which responsibilities of the nonlinear behaviour to the elastic parameter in the Partition Membranes. This approach has not yet been carried out to a well developed theory, but it seems to be a promising way to model cochlear mechanical nonlinearities in terms of mathematical structures which are receiving increasing attention, as are the so called "Toda Nonlinear Lattices" [9]. To get deeper into the question, we will present first the one-dimensional wave equation inside the Inner Ear, accounting for the behaviour of the fluids and membranes present in the system. The model is well known by now, and a description [10, pp. 137-148] on its derivation and its possibilities may be found in [10]. The mechanical filtering within the Cochlea is accounted by the pair of coupled differential equations in the Laplace frequency domain:

$$\frac{\partial p}{\partial x} = Z_s U \tag{1}$$

$$\frac{\partial U}{\partial x} = Y_p P \tag{2}$$

where P is the frequency-space differential pressure between the two sections in which the cochlea is divided by the Partition Membranes, and U is the volumetric fluid velocity in any of these sections which are called "scales". Z<sub>s</sub> and Y<sub>p</sub> are the dynamical parameters of the model, explaining the influence of the local mechanics on the movements of the fluids, and are given by:

$$Z_s = I s + r \tag{3}$$

$$Y_p = i \omega s + \sigma + c/s \tag{4}$$

s being the generalized Laplace frequency, I being the inertial component due to the fluids moving in the scales and r accounting for the losses on the walls of the scales. On the other hand,  $\sigma$  would account for the inertial movements of the Partition Membrane,  $\sigma$  for the losses appearing due to this movement, and c explains the elastical property of such a membrane.

As it was stated above, our main purpose would be in a first approach to investigate the possibility of nonlinear modeling of the mechanical filtering in the cochlea through the introduction of nonlinear behaviour on the elastical parameter of the membrane (c). For such, we will only take into account a simpler version of (1) and (2), in which we will imply no losses, and a pure elastical Partition Membrane. This approach does not seem to be very radical, because the most characteristic parameter of the membrane is its elasticity. This would imply that (1) and (2) may be written in the time domain as:

$$\frac{\partial p}{\partial x} = I \frac{\partial u}{\partial t} \tag{5}$$

$$\frac{\partial p}{\partial t} = c \frac{\partial p}{\partial x} \tag{6}$$

These last two equations account for the transmission line model of the cochlea in the linear approximation, within certain assumptions on the variation of I and c with respect to frequency. A nonlinear analytical solution was given for them [11]. A nonlinear version of (5) and (6) may be inferred from the basic theory of Toda lattices due to Toda [9]. The mentioned work has been developed to explain mechanical interaction in lattices composed of individual particles scattered at regular distances q<sub>n</sub> and momenta p<sub>n</sub> and interacting between them by means of a force derived from a nonlinear potential of exponential kind, as it can be seen in Fig. 3.



Fig. 3. A typical configuration for a Toda Lattice.

Thus, the dynamic relationships among neighbouring particles in such a one-dimensional lattice would be given by the coupled equations:

$$p_n = \frac{dp_n}{dt} \tag{7}$$

$$\frac{dp_n}{dt} = \exp[-(q_n - q_{n-1})] - \exp[-(q_{n+1} - q_n)] \tag{8}$$

where the mass of each particle and the amplitude of the external force are considered to be the unity for the sake of simplicity. The solution of equations (7) and (8) given by Toda [9, pp. 22-25] is of the kind known as a "soliton", and the most remarkable fact resides in that its behaviour is not far from the kind of traveling waves which propagate along the Partition Membranes [12], as reported by many researchers since the pioneering works of V. Bakasy [13]. According with the reference mentioned work of Toda, the shape of such a soliton will be given by:

$$\exp(-br_n) - 1 = \frac{m}{a b} \beta^2 \operatorname{sech}^2(\alpha n t) \tag{9}$$

where m is the mass of each particle, a is the amplitude of the elastical force, and b is the inverse-length constant.

adjusts the dimensionality inside the exponentials.  $\alpha$  and  $\beta$  are the wave numbers which fit the dimensionalities of both  $n$  and  $t$  in the phase of the wave. A new variable  $n_n$  has been introduced, which is related with the spatial variable  $q_n$  as:

$$n_n = q_n - q_{n-1} \tag{10}$$

having the meaning of the relative displacement between two neighbouring particles with respect to each other and out of the equilibrium position. One of the most remarkable facts in (9) among others, is the possibility for the wave to travel in both directions of the lattice at a time, this giving raise to the possibility of describing a given solution as a combination of a forward and a backward traveling wave. This fact could be used to estimate the parameters of the system by measuring and processing the reflected waves coming from the excitation with a probe signal. In the case of the Auditory System, this signals could be the nonlinear components of the Kemp-Echo as is discussed later.

It may be shown [14] that under the appropriate conditions equations (7) and (8) may be carried for the continuous case to the pair of coupled differential equations:

$$\frac{\partial p}{\partial x} = m \frac{\partial r}{\partial t} \tag{11}$$

$$\frac{\partial p}{\partial t} = a \frac{\partial}{\partial x} \{ \exp(-br) \} \tag{12}$$

for such we have assumed inertial ( $m$ ) and elastical ( $a$ ) properties. Equation (11) is essentially linear in its nature, while equation (12) is nonlinear. Based on this model a similar analogy may be hypothesized for the case of cochlear dynamics, stating clearly that in the cochlear case  $p$  will mean pressure instead of linear momentum, and  $r$  (elongation) would be substituted in its role by  $v$  (volumetric velocity). The role of  $m$  should be played by  $I$  (inertia of fluids) and that of  $a$  will be done by  $c$  (membrane elasticity). The parameter  $b$  establishes the degree of nonlinearity in the system. Under the hypothesized analogy we will be able of extending the solutions and conclusions derived for the continuous Toda lattice to the cochlear case, for which the dynamic equations should read:

$$\frac{\partial p}{\partial x} = I \frac{\partial v}{\partial t} \tag{13}$$

$$\frac{\partial p}{\partial t} = c \frac{\partial}{\partial x} \{ \exp(-bv) \} \tag{14}$$

For the general case, when we assume the line as being non-homogeneous, i.e., when  $I$  and  $c$  may vary with  $x$ , the solution

of (13) and (14) becomes a difficult problem. Nevertheless, several techniques used in other fields of research [15] may be applied, combining both analytical and numerical methods. The simplest approach would be to assume the line divided into a given set of sections of the same length in  $x$ , and assuming that both  $I$  and  $c$  remain constant within each section. For each section, a solution is being searched for  $p$  and  $v$  written in terms of a forward wave  $f$ , and a backward wave  $g$ , each of which of the general kind given by (9):

$$p = Z \{ f(\alpha x - \beta t) + g(\alpha x + \beta t) \} + Q_p(f, g) \tag{15}$$

$$v = f(\alpha x - \beta t) - g(\alpha x + \beta t) + Q_v(f, g) \tag{16}$$

$Z$  being a function of the line parameters and the terms  $f$  and  $g$  being small and declining with time. A solution of the kind above exposed would be of great convenience in the case we want to use this theory to characterize the nonlinearities in the Auditory System in terms of its reflective response. The next step would be to adjust energetically the amplitudes of both waves  $f$  and  $g$  at both sides of an interconnection between two sections, from which a given amount of the waves would be reflected, and through the concatenation of many of these sections a description of the whole structure should come out. Nevertheless, this is still an open problem and more research needs to be done in order to get positive results to be obtained.

DISCUSSION

The model shown may help greatly in understanding the nature of the nonlinear phenomena appearing inside the Auditory System as the above mentioned intermodulation tones. It may also help in the interpretation of the nonlinear behaviour of the so called "Kemp-Echoes", in the sense that an otoemission is considered to be a "Kemp-Echo" only when it exhibits certain amount of nonlinear behaviour, and this excludes other emissions with linear nature. Some words have to be said about the nature of the potential function taken for the modeling of cochlear dynamics in comparison with that proposed for Toda lattice, and this comes from the fact that Toda takes a repulsive potential, whilst for the cochlear case this may not be the best choice, and some other possibilities should be tested. A second problem arises when dealing with the otoemissions from the stimulation of the Auditory System with an impulsive sound wave, and this comes from the separation of the linear and nonlinear parts of the response. Up to this moment, researchers working in the area and study of these otoemissions have only rough methods to distinguish whether there is nonlinear behaviour or not, but they cannot go further in depth in the topic due to the lack of a good nonlinear theory to describe their propagation. In our feeling the theory of Nonlinear Lattice and Toda may be called to fill that gap. On the other hand, we have to deal with signals produced by a system with a nonlinear behaviour with algorithms based on linear theory.

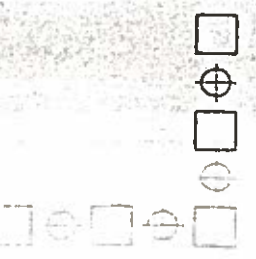
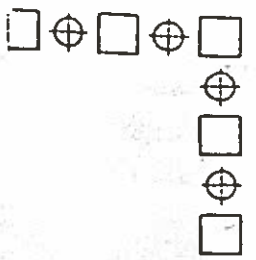
validity of the methods will be reduced to within some limits. Another line of continuation of the work as it has been introduced here has to see with the use of the partial results which would come out from this first stage of the research to generalize and extend the model to a more complete one including both inertial and viscid components, although, as it was mentioned before most of the work is still to be done.

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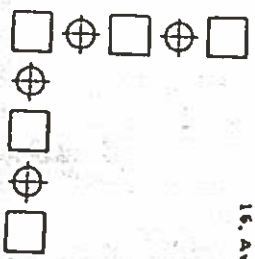
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