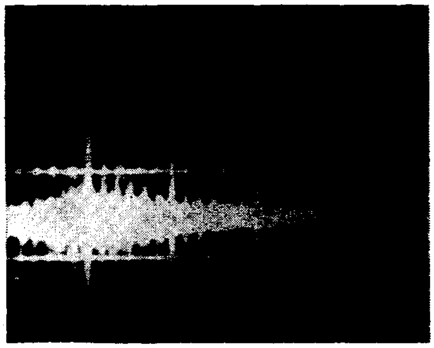

Fig. 2—Typical time and amplitude relationship of optically induced 10-Mc acoustic vibrations in glass: sweep, 50 μsec/div; gain, 10 mv/div.

elastic waves in transparent dielectrics will be submitted for publication at a later date.

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Impulse Response Stability Criteria*

A common, sometimes useful, and to many people meaningful, definition of stability is:[1,2]

Preliminary Definition:

A system is stable if an only if every bounded input yields a bounded output.

The advantage of this concept of stability over some others, such as that of the (present day) Lyapunov theory, is that it applies to general systems, perhaps not described by ordinary differential equations.[3] If the system is linear and time-invariant and described by an impulse response k then it is commonly stated that the above definition is equivalent to: [1,2]

$$\int_{-\infty}^{\infty} |k(t)|\, dt < \infty. \tag{1}$$

However, since k is generally a distribution,[4] in the sense of Schwartz, one does not always know the precise meaning of (1) or of the preliminary definition. For instance some k's cannot be convoluted with every bounded input in which case one does not know how to use the above definition; such is $k(t)=u(t)$, with an input of $u(-t)$, where u is the unit step function. Or the absolute value of some distributions cannot be defined, and one does not know how to form the integral in (1); such is the case if k is the third derivative of the unit impulse δ, $k=\delta^{(3)}$. Of course most engineers intuitively know what (1) means, and thus call a resistor stable and a differentiator unstable; nevertheless we here attempt to make (1) precise within the framework of distributions.

To make the definition more precise, we will call a function x bounded if it is a locally integrable function and $|x(t)|<c$ for all t (including $t=\pm\infty$) with c a finite fixed constant (x is called locally integrable if

$$\int_a^b |x(t)|\, dt$$

exists for all finite a and b); such functions have recently been called uniformly bounded in the engineering literature.[5] For the following we modify the definition of order given by Doležal[6] to bring it into agreement with that of Schwartz[7] and call a distribution f of order n if n is the smallest integer such that any $(n+1)$st distributional primitive,[8] $f^{(-n-1)}=F$, is a locally integrable function F. Thus the impulse is of order zero, while an infinitely continuously differentiable function has order $-\infty$. By a "linear time-invariant system" we will mean a system satisfying the assumptions A_1 through A_4 of our previous note.[4] We will assume the system to satisfy these assumptions as well as to be antecedal (causal), in which case the system is completely characterized for inputs $x\in\mathfrak{D}'_+$ by its impulse response k and $k(t)=0$ for $t<0$; here $\mathfrak{D}'_+$ is the set of distributions which are zero until a finite time.[4] Under these assumptions a primitive which is also zero for $t<0$ is

$$k^{(-1)} = k*u \tag{2}$$

where $*$ denotes convolution; in the following we will always use this primitive. Also under these assumptions

$$y = k*x, \tag{3}$$

where y is the output for any input $x\in\mathfrak{D}'_+$. For other inputs, $x\notin\mathfrak{D}'_+$, (3) cannot always be used and we are led to:

Definition:

A linear, time-invariant, antecedal system is stable if and only if every bounded input function $x\in\mathfrak{D}'_+$ yields an output which is a bounded function.

The result we wish to present is then:

Theorem:

A linear, time-invariant, antecedal system characterized by an impulse response k is stable if and only if k is at most of order zero and

$$\int_{-\infty}^{\infty} |dk^{(-1)}(t)| < \infty. \tag{4}$$

Stated more compactly, $k^{(-1)}(t)$ is a function of bounded-variation over $-\infty\le t\le+\infty$.

Proof: We first recall the meaning of (4). If g is a function of bounded variation then there exist two nondecreasing functions g_+ and g_- such that[9]

$$g(t) = g_+(t) - g_-(t). \tag{5a}$$

Although many decompositions of the form (5a) exist, we choose g_+ to be $g(-\infty)$ added to the positive variation of g, in which case g_- is the negative variation. Then the meaning of (4) is

$$\int_{-\infty}^{\infty} |dg(t)| = \int_{-\infty}^{\infty} dg_+(t) + \int_{-\infty}^{\infty} dg_-(t), \tag{5b}$$

these integrals being taken in the Lebesgue-Stieltjes sense[10] since the functions are to be considered in the distributional meaning.[11]

Since k is assumed to exist, we know an output y results from an input $x\in\mathfrak{D}'_+$ by (3). Further $x=u$ is an allowed input, being bounded, giving

$$y_u = k*u = k^{(-1)}. \tag{6}$$

Consequently $k^{(-1)}$ exists and is a bounded function; the definition of order shows that k is at most of order zero. As a consequence, $k^{(-1)}$ is necessarily of bounded variation on every finite interval,[11] and (3) can be written functionally as

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau)\, dk^{(-1)}(\tau). \tag{7}$$

To see that $k^{(-1)}$ is necessarily of bounded variation over the whole line let $x(t)=u(t)$, then from (6) and (7)

$$y_u(t) = \int_{-\infty}^{t} dk^{(-1)}(\tau). \tag{8}$$

But, if the system is stable, y_u is a bounded function with $y_u(\infty)$ being finite. That is,

$$y_u(\infty) = \int_{-\infty}^{\infty} dk^{(-1)}(t) \tag{9}$$

must exist as a Lebesgue-Stieltjes integral, which requires that (4) hold.[10] This proves the only if part.

To show the if part, we see that (7) holds when $x\in\mathfrak{D}_+'$ and if $|x(t)|<c$, then[12]

$$|y(t)| \le \int_{-\infty}^{\infty} |x(t-\tau)|\, |dk^{(-1)}(t)|$$

$$< c\int_{-\infty}^{\infty} |dk^{(-1)}(t)| < \infty \tag{10}$$

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[2] N. Wax, "A note on stable, physically realizable, linear, time invariant systems," IRE Trans. on Circuit Theory, vol. CT-9, pp. 405–408; December, 1962. See p. 405.

[3] As examples we could consider integral equations and perhaps humans.

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[6] V. Doležal, "Über die Anwendung von Operatoren in der Theorie der linearen dynamische Systeme," *Aplikace Matematiky*, vol. 6, pp. 36–67; No. 1, 1961. See p. 45.

[7] L. Schwartz, "Theorie des distributions," vols. I and II, Hermann, Paris, France, 1957 and 1959. See vol. I, p. 26.

[8] *Ibid.*, See also vol. I, p. 51.

[9] J. Burkill, "The Lebesgue Integral," Cambridge University Press, London, England, 1953. See p. 51.

[10] *Ibid.*, see also p. 74.

[11] Schwartz, *op. cit.*, see also vol. I, pp. 18 and 53.

[12] Burkill, *op. cit.*, see also p. 75.

which completes the proof.

As an example we see that a differentiator is not stable, since $k(t)=\delta^{(1)}$ is of order one. If

$$k(t) = \sum_{i=0}^{\infty} (-1)^i \delta(t - i),$$

then the system is unstable, since

$$k^{(-1)} = \sum_{i=0}^{\infty} u(t - 2i)$$

and (4) is violated. If k itself is a locally integrable function, then (4) reduces to (1). This shows that the semi-infinite RC cable is a lossy, passive, unstable system, since, for the input the "input" current and the output the "input" voltage

$$k(t) = \sqrt{\frac{R}{\pi C t}}\, u(t),$$

and (1) is violated ($x=u$ gives an unbounded y).

By the definition of a linear system which we have used,[4] the system is initially relaxed or in what is sometimes called the "zero-state." Thus the result given here applies only to a test of "zero-state" stability. The above results are stated for single input systems. For multiple-input systems one easily sees that each entry of the impulse response matrix must satisfy the conditions of the theorem.

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Audio-Frequency Characterization of Radar-Absorbing Material*

Several electromagnetic methods of detecting or utilizing anisotropic conditions in solids are well-known. For example, photoelastic stress analysis using polarized light has received attention since the 1930's.[1] At somewhat lower frequencies, a number of microwave techniques have been developed in the past few years.[2,3] At still lower frequencies, anisotropy in conductors has been detected using eddy current techniques.[4]

This communication describes an approach to the problem of characterizing radar-absorbing material (RAM) using audio frequency (af) capacitance measurements which correlate with the orientation and distribution of conductors in a non-conducting matrix.

Composite materials consisting of conductors embedded in a plastic or elastomeric matrix have gained attention recently through applications such as RF gasketing[5] and reduction of radar cross section of shaped and attitude-controlled re-entry vehicles.[6] In the latter application, when the cross section approaches 1 cm², even localized imperfections in RAM vitiate an otherwise acceptable vehicle design.

Specifications such as vehicle reflectivity as a function of aspect angle and frequency appear necessary, but not sufficient to characterize RAM-coated vehicles. Repair and process control procedures require information describing local departures from specifications. It has been found that af dielectric measurements provide some of the required information through correlation with RAM composition and structure, and with radar response. Therefore, prediction of microwave performance is possible based on audio measurements. From inspection and other viewpoints the audio measurements, as a complementary technique, offer distinct advantages over microwave measurements alone.

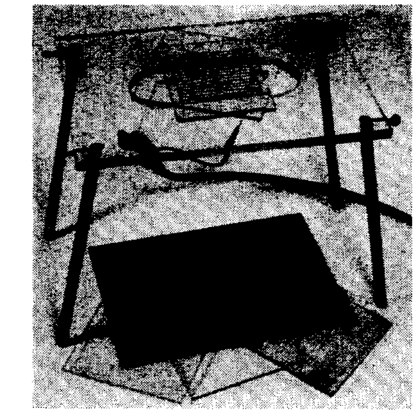

Fig. 1—Idealized RAM and low-frequency, two-terminal, direction-sensitive probe.

TABLE I
Correlation of Microwave and AF Measurements on Idealized RAM

Spacing Between 1 Mil Mo Wires		X-Band I.L., db		10 Kc Probe Capacity, pf	
Inches	X-Band Wavelengths	Wires $\parallel E$	Wires $\perp E$	Wires $\parallel E$	Wires $\perp E$
0.000	0	∞	∞	20.0	20.0
0.092	1/8	4.7	0.8	15.3	10.5
0.183	1/4	1.5	0.8	14.4	10.3
0.275	3/8	1.3	0.8	13.1	10.2
0.367	1/2	1.0	0.8	12.6	10.0
∞	∞	0.8	0.8	9.4	9.4

Af and microwave experiments were conducted initially on "idealized" RAM. Here, idealized RAM consisted of acrylic panels in which 1 mil molybdenum wires were embedded at spacings corresponding to X-band $\lambda/2$, $3\lambda/8$, $\lambda/4$ and $\lambda/8$ intervals (Fig. 1).

Measurement of X-band insertion loss[7] showed that wires perpendicular to the electric field did not interact with the field, whereas for wires parallel to the field, the insertion loss increased rapidly as the spacing became less than $\lambda/4$ (Table I).

The af measurement was simply the 10-kc capacity of the two-terminal comblike probe of Fig. 1. Analogous to the X-band result, probe capacity similarly depended on wire spacing and orientation (Table I).

Based on the simplicity of this 10 kc–10 Gc correlation for idealized RAM, experiments were extended to more practical RAM, such as an ablative-type epoxy (Avcoat) and low-density polyurethane foams.

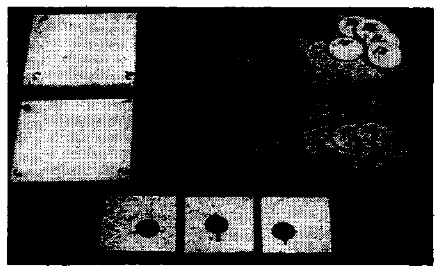

Fig. 2—Polyurethane foams contain oriented conductive fiber inclusions.

TABLE II
K vs. Loading Factor for Approximately 6 PCF Foam

Loading Factor, mg fiber/cc foam	Dielectric Constant at 1 kc	
	Fiber $\parallel E$	Fiber $\perp E$
0.7	7	5
2.5	26	15

Consider the foam samples shown in Fig. 2. Pore morphology indicates foaming direction, which determines fiber orientation. For example, panels designated 76A, B and D have fibers parallel to their faces, while for 76C, fibers are normal to the face. The effect of fiber orientation and concentration on af dielectric constant K is shown in Table II.

Care must be exercised in expressing an af-microwave frequency correlation, because dielectric measurements, particularly at low frequencies, are somewhat sensitive to undesirable variables such as matrix composition, density, cure and absorbed moisture. Nevertheless, for a number of specified epoxy formulations, af determination of fiber distribution and orientation has been correlated with microwave behavior.

It is suggested that systematic materials evaluation can indicate the degree of corre-

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