A ROBOT ARM DYNAMICAL MODEL IN TERMS OF RESONANT TRANSFER FUNCTIONS

V. Rodellar(*), P. Gómez(*), M. A. Pascual(*) and R. W. Newcomb(*#)

(*) Grupo de Sistemas PARCOR
Fac. de Informática
Univ. Politécnica de Madrid
Ctra. de Valencia, Km. 7,00
28031 Madrid, ESPAÑA

(#) Microsystems Laboratory
Electrical Eng. Dept.
University of Maryland
College Park, MD, 20742
U.S.A.

ABSTRACT

The present paper gives a description of a typical Robot Arm Dynamical System in terms of an electromechanical analogy for small movements (those implying small angular displacements and velocities). The model may be expressed as a set of linear equations involving the inertial, frictional and elastic parameters of the system. These may be estimated through Linear Prediction Techniques commonly used in the characterization of pole-zero systems, with the advantage in our case that the system may be regarded as mainly deterministic. Practical applications of the methods shown are related with the modeling of small movements in robot arms, such as in handling and assembling small pieces, prosthesis and orthosis control, decomposing and reproducing cursive writing, and in general in the characterization and control of undesired vibrations in a given pattern of movement. The paper exposes the main hints in the approach for a small educational robot arm, and a discussion is presented on the posibilities of the methods own.

INTRODUCTION

As it is well known [Bra.83], Robot Arm Dynamics has been increasingly studied for different purposes and from quite different points of view. As such many formulations have arosen during the past years, each of which is intended to cover a different perspective towards a different application. Most of them are based on Lagrangian Formulation [Hea.85] and give a description of long range movements, as those implied in moving parts from one place to another in relatively fast and non complex movements [Pau.81]. Of course, this is by no means a common situation in many applications of robotics, but there are some other cases in which robot arm dynamics is characterized by sudden and rapidly changing movements, implying small velocities and displacements, but relatively large accelerations in complex patterns, such as in small parts selection, handling and assembly, and in some others. In these cases, second order factors, such as inertial, frictional and elastic components may reduce the ability of Lagrangian Formulation to give an accurate yet easy to handle description of the fast and complex patterns of movement, and as such a reduction in the control capability of the models being used may arise. Our aim is to show the way for deriving simple models which may be helpful in developing algorithms for fast and small movement control by means of the adequate assumptions which allow for a linearization of the Lagrangian Formulation involved. In such a way many benefits may be expected, as the posibility of independently determining the model parameters in real time for the different sections of a typical robot arm. This may represent a second indirect benefit from the fact that individual microcontrollers may in real time keep track of the arm section under their responsibility, thus locally controling the dynamics of the whole system. One such case in which fast yet small amplitude movements may be found is in cursive writing decomposition [Yas.83]. We may use the model under development both in natural and artificial reproduction of cursive writing, both for recognition and identification purposes together with the aim of helping handicapped people by means of robotic or prosthetic electromechanical aids [Phi.85]. With this objective in mind we have redone the Lagrangian Formulation for a simple arm structure composed of only two joints (let's say "shoulder" J_1 and "elbow" J_2), as it can be seen in Fig. 1, assuming this structure being able of moving only in the vertical plane (x-y axis).

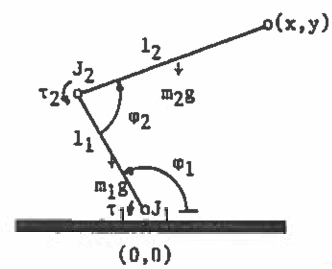

Fig. 1. Simple Robot Arm Structure.

In this rather simplified model we assume that joints are point-like structures, and as such, all the elastic and frictional torques may be regarded as being concentrated at the joints. Both sections will behave as rigid bars, l_1 and l_2 being their lengths, and m_1 and m_2 their respective masses. The origin of coordinates is taken at joint J_1, and the coordinates (x,y) describe the arm end effector position. The torques applied to both joints are τ_1 and τ_2, and may be regarded as the inputs to the dynamical system, whilst the angles φ_1 and φ_2 will be the outputs. Nevertheless, in many cases it will be more interesting to trace the coordinates x, y, or both, instead of φ_1 or φ_2 as we will see. This simple model is a good approach for many situations in which a more complicated structure with n joints is involved in small movements, only two joints being operative at a time. These situations often arise when modeling cursive writing and in the manipulation of small parts in a plane, as in assembly lines.

α_1 and α_2 having the meaning of small oscilations around the resting considered values for φ_1 and φ_2. Obviously the following relations will hold:

$$\dot{\varphi}_1 = \dot{\alpha}_1; \qquad \ddot{\varphi}_1 = \ddot{\alpha}_1 \tag{17}$$

$$\dot{\varphi}_2 = \dot{\alpha}_2; \qquad \ddot{\varphi}_2 = \ddot{\alpha}_2 \tag{18}$$

and with the assumption of small amplitude movements, that is, with $\alpha_1 \ll \pi/2$ and $\alpha_2 \ll \pi/2$, it will be possible to substitute $\cos\varphi_1$, $\cos\varphi_2$, $\sin\varphi_1$ and $\sin\varphi_2$ by the first two terms in their series expansions as follows:

$$\cos \varphi_1 \approx - \alpha_1 \tag{19}$$

$$\cos \varphi_2 \approx - \alpha_2 \tag{20}$$

$$\sin \varphi_1 \approx 1 \tag{21}$$

$$\sin \varphi_2 \approx 1 \tag{22}$$

Substituting now the above aproximations in equations (6) and (7) these may be rewritten as:

$$\tau_1 = \mu_{11}\, \ddot{\alpha}_1 + \mu_{12}\, \ddot{\alpha}_2 + \theta_1\, \dot{\alpha}_1\, \dot{\alpha}_2 + \theta_2\, \dot{\alpha}_2^2 - \varepsilon_1\, \alpha_1 + \tau_a \tag{23}$$

$$\tau_2 = \mu_{21}\, \ddot{\alpha}_1 + \mu_{22}\, \ddot{\alpha}_2 + \theta_3\, \dot{\alpha}_1^2 + \tau_a \tag{24}$$

for which we have taken:

$$\cos(\varphi_1 + \varphi_2) \approx -1 \tag{25}$$

and the equation parameters have been redefined as:

$$\mu_{11} = \frac{m_1\, l_1^2}{3} + m_2\, l_1^2 + \frac{m_2\, l_2^2}{3} + m_2\, l_1\, l_2\, \alpha_2 \tag{26}$$

$$\mu_{12} = \mu_{21} = \left[\frac{m_2\, l_2^2}{3} + \frac{m_2\, l_1\, l_2\, \alpha_2}{2}\right] \tag{27}$$

$$\mu_{22} = \frac{m_2\, l_2^2}{3} \tag{28}$$

$$\theta_1 = m_2\, l_1\, l_2 \tag{29}$$

$$\theta_2 = - \theta_3 = \frac{m_2\, l_1\, l_2}{2} \tag{30}$$

$$\varepsilon_1 = \left[\frac{m_1}{2} + m_2\right] g\, l_1 \tag{31}$$

$$\tau_a = \frac{m_2\, g\, l_2}{2} \tag{32}$$

Equations (23) and (24) are greatly simplified, although they are not yet linear, due in part to the parameters μ_{11} and μ_{12}, and in part, to the presence of the terms in $\dot{\alpha}_1^2$, $\dot{\alpha}_2^2$ and $\dot{\alpha}_1\, \dot{\alpha}_2$. The parameters μ_{11} and μ_{12} may be approximated as follows unless the mass and length of section 2 would be much smaller than that of section 1:

$$\mu_{11} \approx \frac{m_1\, l_1^2}{3} + m_2\, l_1^2 + \frac{m_2\, l_2^2}{3} \tag{33}$$

$$\mu_{12} = \mu_{21} \approx \left[\frac{m_2\, l_2^2}{3}\right] \tag{34}$$

for which we have considered that $\alpha_2 \ll 1$. The assumption of low angular velocities mentioned before may be carried out in rather a similar way. Just comparing θ_1 and θ_2 with the remnant parameters in equations (23) and (24), and assuming low angular velocities:

$$|\dot{\alpha}_1| \ll |\ddot{\alpha}_1| \tag{35}$$

$$|\dot{\alpha}_2| \ll |\ddot{\alpha}_2| \tag{36}$$

a new version of (24) and (25) may be written:

$$\tau_1 = \mu_{11}\, \ddot{\alpha}_1 + \mu_{12}\, \ddot{\alpha}_2 - \varepsilon_1\, \alpha_1 + \tau_a \tag{37}$$

$$\tau_2 = \mu_{21}\, \ddot{\alpha}_1 + \mu_{22}\, \ddot{\alpha}_2 + \tau_a \tag{38}$$

At this point it seems reasonable to explore the nature of the parameter τ_a which appears in the above expressions. It is not difficult to see that this parameter must have the meaning of a torque with constant value, depending on the force of gravity. So, it must be caused by the weight of the second section. τ_1 and τ_2 must compensate this torque in order to prevent gravity from distorting or collapsing the positioning of the arm. Usually this static torque due to gravity is compensated by the static brake system of joint motors. Then we should decompose each torque τ_1 and τ_2 in a static torque τ_{1s} and τ_{2s} contributing to compensate τ_a and a dynamic torque τ_{1d} and τ_{2d} contributing to create movement as follows:

$$\tau_1 = \tau_a + \tau_{1d} \tag{39}$$

$$\tau_2 = \tau_a + \tau_{2d} \tag{40}$$

and the equations (37) and (38) may be rewritten as:

$$\tau_{1d} = \mu_{11}\, \ddot{\alpha}_1 + \mu_{12}\, \ddot{\alpha}_2 - \varepsilon_1\, \alpha_1 \tag{41}$$

$$\tau_{2d} = \mu_{21}\, \ddot{\alpha}_1 + \mu_{22}\, \ddot{\alpha}_2 \tag{42}$$

Now it is easy to see that the system given by equations (41) and (42) is of linear nature, as was our purpose. We may be interested in considering frictional effects although we did not introduced them at the beginning. For such we will assume ideal frictional torques taking place and being directly related with the angular velocities $\dot{\alpha}_1$ and $\dot{\alpha}_2$ by means of coefficients σ_{11} and σ_{22}. We will also introduce the elasticity inherent in the material constituting the sections of the arm by means of the elasticity parameters ε_{11} and ε_{22}, in such a way that the elastic torque will be proportional to the angular oscillations α_1 and α_2. ε_{11} accounting for both the gravitational (ε_1) and elastic (ε_a) effects:

$$\varepsilon_{11} = \varepsilon_a + \varepsilon_1 \tag{43}$$

With this in mind equations (41) and (42) must be redone to include these last terms:

$$\tau_{1d} = \mu_{11}\, \ddot{\alpha}_1 + \mu_{12}\, \ddot{\alpha}_2 + \sigma_{11}\, \dot{\alpha}_1 + \varepsilon_{11}\, \alpha_1 \tag{44}$$

$$\tau_{2d} = \mu_{21}\, \ddot{\alpha}_1 + \mu_{22}\, \ddot{\alpha}_2 + \sigma_{22}\, \dot{\alpha}_2 + \varepsilon_{22}\, \alpha_2 \tag{45}$$

These two last equations together with the definitions of the different parameters involved constitute the basic model we are going to use for the purposes before mentioned of characterizing and controlling small movements in robot arms. To accomplish these objectives we will search in the next sections for different methods leading to the inversion of the model, which will allow us to determine their parameters from a given set of measurements.

EQUIVALENT CIRCUITAL MODELS

It is very interesting to consider the above expressions as these obtained in a circuital model as the one shown in Fig. 3., in which we have defined v_1 and v_2 as the electric potentials and u_1 and u_2 as the currents at the input and output of this two-port circuit.

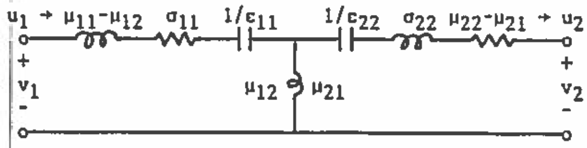

Fig. 3. Electromechanical equivalent model.

As it may be easily seen, v_1 and v_2 will be the angular moments associated with the dynamical torques τ_{1d} and τ_{2d} due to the joint motors and defined as:

$$v_1(t) = \int_{-\infty}^{t} \tau_{1d}(\zeta)\, d\zeta \tag{46}$$

$$v_2(t) = \int_{-\infty}^{t} \tau_{2d}(\zeta)\, d\zeta \tag{47}$$

in which expressions t stands for the continuous time variable, u_1 and u_2 being identified with the angular velocities $\dot{\alpha}_1$ and $\dot{\alpha}_2$ in the system. With these definitions equations (44) and (45) may be expressed as:

$$v_1 = \mu_{11}\, \dot{u}_1 + \mu_{12}\, \dot{u}_2 + \sigma_{11}\, u_1 + \varepsilon_{11} \int_{-\infty}^{t} u_1(\zeta)\, d\zeta \tag{48}$$

$$v_2 = \mu_{21}\, \dot{u}_1 + \mu_{22}\, \dot{u}_2 + \sigma_{22}\, u_2 + \varepsilon_{22} \int_{-\infty}^{t} u_2(\zeta)\, d\zeta \tag{49}$$

With the adequate definitions both expressions may be written in the complex domain of Laplace frequencies as follows:

$$V_1 = (\mu_{11}\, s + \sigma_{11} + \varepsilon_{11}\, s^{-1})\, U_1 + \mu_{12}\, s\, U_2 \tag{50}$$

$$V_2 = \mu_{21}\, s\, U_1 + (\mu_{22}\, s + \sigma_{22} + \varepsilon_{22}\, s^{-1})\, U_2 \tag{51}$$

where V_1, V_2, U_1 and U_2 stand for the Laplace transform $\mathcal{L}\{\cdot\}$ of v_1, v_2, u_1 and u_2 respectively:

$$V_1(s) = \mathcal{L}\{v_1(t)\} \tag{52}$$

$$V_2(s) = \mathcal{L}\{v_2(t)\} \tag{53}$$

$$U_1(s) = \mathcal{L}\{u_1(t)\} \tag{54}$$

$$U_2(s) = \mathcal{L}\{u_2(t)\} \tag{55}$$

which may be reformulated in matrix form as follows:

$$\begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} Z_{11}(s) & Z_{12}(s) \\ Z_{21}(s) & Z_{22}(s) \end{bmatrix} \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix} \tag{56}$$

the parameters of the impedance matrix given by:

$$Z_{11}(s) = \mu_{11}\, s + \sigma_{11} + \varepsilon_{11}\, s^{-1} \tag{57}$$

$$Z_{12}(s) = \mu_{12}\, s \tag{58}$$

$$Z_{21}(s) = \mu_{21}\, s \tag{59}$$

$$Z_{22}(s) = \mu_{22}\, s + \sigma_{22} + \varepsilon_{22}\, s^{-1} \tag{60}$$

This set of expressions will be of great interest later in designing the inversion algorithms, as it will be shown in brief. We will deal now with the relationship between the dynamical variables α_1 and α_2 and the observed or desired variables on the handling or writing plane (x-y), as before stated.

X-Y DECOMPOSITION

Let's review again our main objective, which is as before said, to control the small movements of a robot arm end effector in an x-y plane as accurately as possible by appropriately actuating on the torques τ_1 and τ_2 of the motors at the joints of a robot arm. Then we should consider τ_1 and τ_2 as the inputs to a dynamical system, being the angular displacements α_1 and α_2 the outputs of the system. But for many purposes it will be desirable to measure the output in terms of the linear displacements of the end effector on a plane, let's say the x-y plane in Fig. 4.

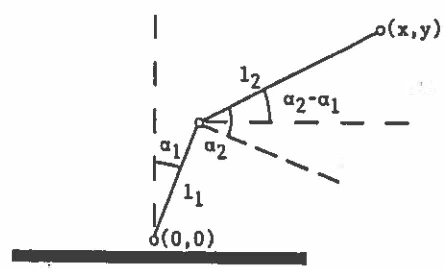

Fig. 4. Relations among x, y, α_1 and α_2.

As it will be easily inferred from the diagram in Fig. 4., the following relations will hold:

$$x = l_1 \sin \alpha_1 + l_2 \cos (\alpha_2 - \alpha_1) \tag{61}$$

$$y = l_1 \cos \alpha_1 + l_2 \sin (\alpha_2 - \alpha_1) \tag{62}$$

and having in mind the restrictions for small movements imposed on α_1 and α_2, (61) and (62) may be rewritten as:

$$x \approx l_1\, \alpha_1 + l_2 \tag{63}$$

$$y \approx l_1 + l_2\, (\alpha_2 - \alpha_1) \tag{64}$$

In a practical case it will be easier to measure small displacements in x, rather that absolute values, and as such we will be more interested in evaluating such displacements as the difference between the resting value for x or y, let's say x_0 and y_0, and the real values. Then we will find out by simple inspection of Fig. 4. that:

$$x_0 = l_2 \tag{65}$$

$$y_0 = l_1 \tag{66}$$

and the incremental displacements Δx and Δy will then be approximated as:

$$\Delta x \approx l_1 \alpha_1 \tag{67}$$

$$\Delta y \approx l_2 (\alpha_2 - \alpha_1) \tag{68}$$

these relations showing a linear dependence between the angular and linear displacements. They may be used to determine the parameters of the model in which we would call the solution of the "inverse problem". We will give some hints on how to proceed to estimate these parameters practically.

PROPOSED INVERSION METHODS

Now we want to estimate the model parameters through practical measurements. Several variables could be measured in order to extract these parameters, some of them being:

Measurements of the angular displacements α_1 and α_2 may be obtained from the robot controller, because they are used for control purposes. This case may be cheap and easy to implement, but will not be very accurate in general.

Coordinates x and y may be easily traced on a screen, using special pens and paper, as shown in Fig. 5., but the digitizing process may be more complicated.

An accelerometer may be used to determine accelerations in both x and y axis. This method may be quite accurate, althoug it requires more complex instrumentation, such as A/D converters. In Fig. 6. a general schematic for such an experiment under way is shown.

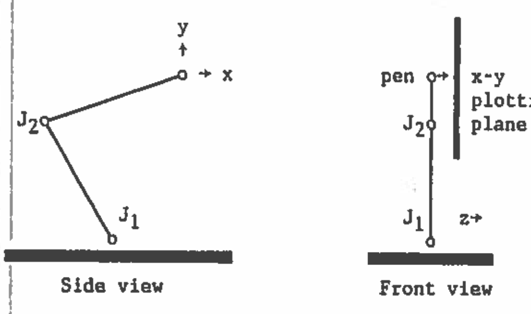

Fig. 5. Registering x-y coordinates.

These, of course will be some of the possibilities, many others existing. In general, the method consists in introducing step-like or impulse-like actions on joint motors, measuring the response according with the methods above mentioned, and applying the techniques we are going to mention in the present section.

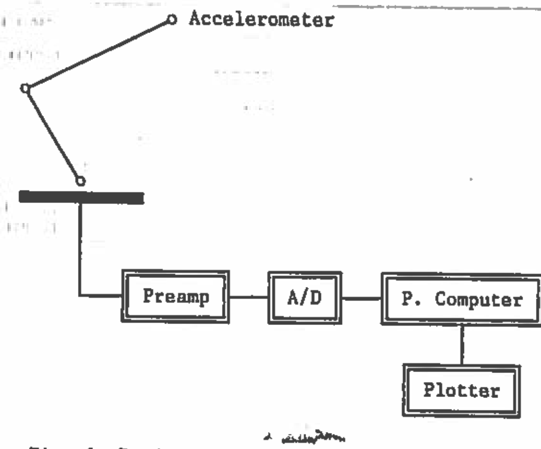

Fig. 6. Registering accelerations.

To show the algorithms which will be used in estimating the parameters of the model, we will start with equations (56), obtaining from them the values of $U_1(s)$ and $U_2(s)$ by inverting the matrix in impedances:

$$\begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix} = \begin{bmatrix} Y_{11}(s) & Y_{12}(s) \\ Y_{21}(s) & Y_{22}(s) \end{bmatrix} \begin{bmatrix} V_1(s) \\ U_2(s) \end{bmatrix} \tag{69}$$

having then as it is well known:

$$Y_{11} = \frac{Z_{22}}{\Delta} \tag{70}$$

$$Y_{12} = -\frac{Z_{12}}{\Delta} \tag{71}$$

$$Y_{21} = -\frac{Z_{21}}{\Delta} \tag{72}$$

$$Y_{22} = \frac{Z_{11}}{\Delta} \tag{73}$$

$$\Delta = Z_{11} Z_{22} - Z_{12} Z_{21} \tag{74}$$

Now several possibilities for determining the parameters of the model, namely μ_{11}, μ_{12}, μ_{22}, σ_{11}, σ_{22}, ε_{11} and ε_{22}, are open, depending upon the output variables to be measured. To illustrate on this point, we will develop the mathematics for a given case a little bit more. Let's suppose we are able of measuring small displacements in the y-axis, these being stated as Δy. Then taking Laplace transforms in (68) the following should hold (small movements are implied):

$$\Delta Y(s) = l_2 s^{-1} [U_2(s) - U_1(s)] \tag{75}$$

and, from (69) it will be easy to see that:

$$\Delta Y(s) = l_2 s^{-1} \{ [Y_{21} - Y_{11}] V_1(s) + [Y_{22} - Y_{12}] V_2(s) \} \tag{76}$$

This last expression means that $\Delta Y(s)$ may be seen as a linear combination of motor actions taken at joints J_1 and J_2, and as such the superposition principle may be applied, depending on whether J_1 or J_2 or both at a time will be activated.

We will explore the first two possibilities:

a) J_1 active, J_2 inactive. This would yield an incremental motion at the end effector given by:

$$\Delta Y_1(s) = l_2\, s^{-1}\, [Y_{21} - Y_{11}]\, V_1(s) \tag{77}$$

b) J_1 inactive, J_2 active. This would yield an incremental motion at the end effector given by:

$$\Delta Y_2(s) = l_2\, s^{-1}\, [Y_{22} - Y_{12}]\, V_2(s) \tag{78}$$

We will go further in deep assuming that an ideal impulse-like action should be applied at any joint motor (this according to (46) or (47) would mean applying two short stroke-like torques of opposite sign one immediately after the other to the given joint motor). Then we should expect that:

$$V_1(s) \approx 1 \quad \text{or} \quad V_2(s) \approx 1 \tag{79}$$

and the impulse response in terms of frequency for this action being exerted at J_1 would be given by:

$$\Delta Y_1(s) = - l_2\, s^{-1} \frac{Z_{21} + Z_{22}}{Z_{11}\, Z_{22} - Z_{12}\, Z_{21}} \tag{80}$$

for which we have used (77), (72), (73) and (74). This last expression may be further developed taking into account (57 - 60) to yield:

$$\Delta Y_1(s) = - l_2 \frac{\gamma_{10} + \gamma_{11}\, s^{-1} + \gamma_{12}\, s^{-2}}{\beta_{10}\, s^2 + \beta_{11}\, s + \beta_{12} + \beta_{13}\, s^{-1} + \beta_{14}\, s^{-2}} \tag{81}$$

with the definitions:

$$\gamma_{10} = \mu_{12} + \mu_{22} \tag{82}$$

$$\gamma_{11} = \sigma_{22} \tag{83}$$

$$\gamma_{12} = \epsilon_{22} \tag{84}$$

$$\beta_{10} = \mu_{11}\, \mu_{22} - \mu_{12}\, \mu_{21} \tag{85}$$

$$\beta_{11} = \sigma_{22}\, \mu_{11} + \mu_{22}\, \sigma_{11} \tag{86}$$

$$\beta_{12} = \mu_{11}\, \epsilon_{22} + \epsilon_{11}\, \mu_{22} + \sigma_{11}\, \sigma_{22} \tag{87}$$

$$\beta_{13} = \sigma_{11}\, \epsilon_{22} + \epsilon_{11}\, \sigma_{22} \tag{88}$$

$$\beta_{14} = \epsilon_{11}\, \epsilon_{22} \tag{89}$$

Then, from the measurement of the impulse response in the time domain $\Delta y_1(t)$ we would be able to determine the coefficients of both the numerator and denominator in (81), which would render the desired model parameters. In order for this to be done by computer, we would have to assume discrete time quantization, and time-dependent variables should have to be measured at fixed time intervals of τ seconds, n being the discrete time index:

$$t = n\, \tau \tag{90}$$

With this in mind, we will translate (81) to its discrete time counterpart in terms of z-transforms by means of bilinear transformation [Opp.75, pp. 206-211]:

$$s = \frac{2}{\tau} \frac{1 - z^{-1}}{1 + z^{-1}} \tag{91}$$

and introducing this transformation into (81) the following expression will hold in terms of z-transforms for the incremental impulse response to the J_1 motor on the y-axis:

$$\Delta Y_1(z) = A_0 \frac{\sum_{i=0}^{4} b_{4i}\, z^{-i}}{\sum_{i=0}^{4} a_{4i}\, z^{-i}} \tag{92}$$

having redefined after some calculations:

$$b_{40} = \gamma_{10} + \gamma_{11} + \gamma_{12} \tag{93}$$

$$b_{41} = 2\, \gamma_{11} + 4\, \gamma_{12} \tag{94}$$

$$b_{42} = - 2\, \gamma_{10} + 6\, \gamma_{12} \tag{95}$$

$$b_{43} = - 2\, \gamma_{11} + 4\, \gamma_{12} \tag{96}$$

$$b_{44} = \gamma_{10} - \gamma_{11} + \gamma_{12} \tag{97}$$

$$a_{40} = \beta_{10} + \beta_{11} + \beta_{12} + \beta_{13} + \beta_{14} \tag{98}$$

$$a_{41} = - 4\, \beta_{10} - 2\, \beta_{11} + 2\, \beta_{13} + 4\, \beta_{14} \tag{99}$$

$$a_{42} = 6\, \beta_{10} - 2\, \beta_{12} + 6\, \beta_{14} \tag{100}$$

$$a_{43} = - 4\, \beta_{10} + 2\, \beta_{11} - 2\, \beta_{13} + 4\, \beta_{14} \tag{101}$$

$$a_{44} = \beta_{10} - \beta_{11} + \beta_{12} - \beta_{13} + \beta_{14} \tag{102}$$

We have by now reached our main objective, as far as the transfer function expressed in (92) shows a pole-zero structure. For this kind of structures much research have been done during last years in order to get consistent estimates of their parameters when driven by random signals (white noise) as well as by impulsive inputs. We propose here a very interesting and powerful method [Tsa.84] which may be implemented using Linear Predictive Filtering (LPF) [Hon.84]. Other methods based also in LPF could also be used [Rod.85], extracting first the poles of the Transfer Function, inverting after the spectrum of the residual error traces, and extracting again from the inverted versions the zeros in the Transfer Function. We will not go further in deep in explaining how these methods work, because it would take us out of the scope of the present paper. On the other hand they are very well documented in the specialized literature. References to the most interesting ones may be found for example in [Por.83] and [Ros.85].

. DISCUSION AND APPLICATIONS

In the present section we will first point out as a summary the main steps of the method being proposed. These will be as follows:

Using a spectral estimation technique of the kind before mentioned, determine first the parameters a_{ki} and b_{ki} in the transfer function given in (92).

By means of the linear relations in (93) to (107) determine both γ_{1i} and β_{1i}. It must be mentioned that some of these relations are redundant, and as such some inconsistencies might arise, this factor having to be tested experimentally.

Relations (82) to (89) will allow us to determine the mechanical parameters of the model. Some redundancy may also be observed in this set of equations, and as such a special care should be taken in testing consistency.

Finally, physically meaning coefficients, such as masses, elasticities, and frictions could be inferred from equations (31), (33), (34) and (43). It must be pointed out that these parameters do not have to be in full agreement with the real ones which could be directly obtained by static measurements, but would rather represent the global behavior of the model. Nevertheless, they will be a good representation of the real structure as a whole.

One of the main advantages of using such a model will be the predictability of the behavior of the real system when stimulated by given torques. It may be easily implemented by computer, and even simulated as a digital filter. The model may well be used in several different applications as will be mentioned below with very simple means. Among the disadvantages of the model we should mention the restrictions imposed to linearize the structure, thus limiting the field of applications to low range movements. Consistency of estimates is another concern, because of redundancy in some of the relations. Consistency may be tested using alternate measurements, as for example activating J_2 instead of J_1, and comparing results. Incremental measurements on the x-axis could also be used to compare results with those from the y-axis. One factor which has not been taken into account until now is the mechanical response of a real motor to an electrical excitation. In general this response will introduce its own inertial an frictional components in the model. Two possible strategies could be adopted concerning this fact, and possibly others, one being to determine first this response and after deconvolve it from the measurements, and the other being to assume it globalized to the inertial and frictional parameters of the whole structure. In this last case a disparity will be more clearly appreciated between real and estimated effector parameters.

All along the former sections a linear model has been built to characterize small movements in robot end effectors. This model may be of great interest in the following applications:

Determine the parameters of a given effector to be used in further correcting undesired behavior, such as trembling motion, and possibly others.

Adjust the actual trajectory to a desired one, by means of some error criteria, as for example, Least Squares.

Store and reproduce some characteristic trajectories, as those in cursive writing. This kind of methods could greatly help to understand the biomechanics of wrist and elbow in human writing [Yas.83].

Design better motor drivers and controllers to achieve a finer distributed joint robot motion control.

Study the degree of controllability of several robot effectors by means of human electromyographic signals, to be used in the design of helping devices for severely handicaped people.

In order to elaborate further in deep on this line of research, some experiments of the kind shown in Fig. 5 and 6 are being prepared on a Rhino Mark III educational robot arm, and a further revision of the model is on the way. Factors such as different resting positions at non right angles, three dimensional movements, extended number of joints, and some others are also under study.

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