Nonlinear Description of an MOS Universal Circuit

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Abstract:
In this paper the nonlinear state-space equations are set up for an MOS universal circuit.

I. Introduction

A universal circuit is a second order circuit which by a change of parameters allows it to generate stable & unstable nodes, stable & unstable foci, and saddle points. Such a circuit with a linear analysis was introduced using vacuum tubes in [1, p.194] and redone using MOS transistors in [2]. Since the actual operation of such a circuit must be nonlinear, here we inititate a study of the nonlinear behavior by setting up the nonlinear state-space equations which describe the universal circuit.

Since nodes, foci, and saddle points are the most frequently met critical points of second order linear and nonlinear differential equations, a universal circuit can be used as a basic building block for the design of systems from their differential equations. Essentially one only need have it on hand, along with connecting nonlinearities, to construct almost arbitrary systems in electronic form.

II. The Universal Circuit

Figure 1 shows the universal circuit to be considered. In Fig. 1 r_1 and r_2 are the circuit elements to be changed to adjust the behavior of the circuit, that is by a proper choice of these two elements with suitable choices for the other circuit elements any type of node, focus, or saddle point may be obtained. Toward this a graphical representation of design values is available [1, p.196]. Since there are two independent capacitors, the system is second order and, as is customary, one may choose the capacitor voltages as the state variables. However, it is more convenient to use the currents, i_1 and i_2, in the two design parameter resistors; so we choose them as the two state variables. In discovering this convenience lies the creativeness of Chaikin [1]. Proper biasing of the circuit is also required and we assume that this occurs, as is set up in [2], such that the circuit when at rest is biased in the current source (saturation) region of transistor operation.

We assume that the MOS devices are described by the functional laws

$$i_g = 0 \tag{1a}$$
$$i_d = f(v_{gs}, v_{ds}) \tag{1b}$$

where i_g is the gate current, i_d the drain current, v_{gs} the gate to source voltage, and v_{ds} the drain to source voltage; $f(.,.)$ is a two variable function giving the transistor curves. Since there are two transistors we use subscripts 1 and 2 on f, and some other parameters, to refer to the respective (left and right) transistors.

III. The State Equations

A. Review of Linear Case

As backround for the nonlinear case we review here the known results for the linear case. For this we assume that the signal component of the drain current is given by

$$\dot{i}_d = g_m \cdot v_{gs} \tag{1c}$$

In essence, (1c) replaces (1b) in the linear case. For the signal components the state equations are then [2]

$$\dot{i}_1 = \frac{1}{(R_{L2}+r_1\alpha-\beta)}\cdot\frac{1}{C_1C_2}\cdot$$
$$\{-\alpha C_2 i_1+[(C_1+C_2)\alpha-C_1]i_2\} \tag{2a}$$

$$\dot{i}_2 = \frac{1}{r_2}\cdot\frac{1}{C_1C_2}\cdot\{C_2 i_1-[C_1+C_2]i_2\} \tag{2b}$$

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where the super dot denotes time differentiation and

$$\alpha = 1 - (R_{L1}g_{m1})\cdot(R_{L2}g_{m2}) \quad (2c)$$
$$\beta = 0 \quad (2d)$$

Here β is included to allow generalization, as given below, to the almost identical nonlinear equations. These state equations are easily rewritten in matrix form as di/dt=Ai from which the characteristic polynomial is found using $P(s)=\det(sI_2-A)$ with I_2 the 2X2 identity matrix. Thus,

$$P(s) = s^2 + a_1 s + a_0 \quad (3a)$$

with

$$a_1 = \frac{1}{C_1}\cdot\left[\frac{\alpha}{(R_{L2}+\alpha r_1)} + \frac{C_1+C_2}{C_2 r_2}\right] \quad (3b)$$

$$a_0 = \frac{1}{r_2 C_1 C_2 (R_{L2}+\alpha r_1)} \quad (3c)$$

Since α will normally be chosen negative and since r_1 and r_2 enter independently in these two coefficients, circuit elements may be adjusted to obtain the two roots of P(s) anywhere in the finite complex plane. As these roots are the natural frequencies of the linearized circuit, we see that the linearized universal circuit achieves its purpose of realizing any two (finite) natural frequencies in the complex plane.

B. Nonlinear Treatment

A straightforward circuit analysis of Fig. 1 shows that the state variable equations, (2a) & (2b) above, obtained in the linear case carry over identically to the nonlinear case with the only change being a modification to the definitions of α and β and consideration of i_1 and i_2 as total currents (rather than just signal). Indeed we find

$$\alpha(i_1,i_2) = 1 - \alpha_1\cdot\alpha_2 \quad (4a)$$

$$\alpha_1 = \frac{R_{L1}\cdot f_{1x}(v_1,v_2)}{1 + R_{L1}\cdot f_{1y}(v_1,v_2)} \quad (4b)$$

$$\alpha_2 = \frac{R_{L2}\cdot f_{2x}(v_2,v_3)}{1 + R_{L2}\cdot f_{2y}(v_2,v_3)} \quad (4c)$$

$$\beta(i_1,i_2) = \frac{R_{L2}\cdot f_{2y}(v_2,v_3)}{1 + R_{L2}\cdot f_{2y}(v_2,v_3)} \quad (4d)$$

In these f_x & f_y represent partial derivatives with respect to the first and second variables of $f(x,y)$ and v_1 and v_2 are the gate to source voltages of the left and the right transistors, respectively, while v_3 is the drain to source voltage of the right transistor. These voltages are then to be expressed in terms of the state variables i_1 and i_2, which can be done as follows. We have by KVL & KCL on Fig. 1

$$v_1 = r_1 i_1 + r_2 i_2 + V_B \quad (5a)$$
$$v_2 = -R_{L1}f_1(v_1,v_2) + V_D + V_B \quad (5b)$$
$$v_3 = -R_{L2}[i_1 + f_2(v_2,v_3)] + V_D + V_B \quad (5c)$$

Equation (5a) is to be substituted into (5b) and then (5b) solved for v_2. Assuming such a solution exists, which will almost always be the case for MOS device functions, we can find a function $g_2(.,.)$ to write

$$v_2 = g_2(i_1,i_2) \quad (6a)$$

The resulting v_2 is then substitued into (5c) which in turn is solved for v_3 for which we write

$$v_3 = g_3(i_1,i_2) \quad (6b)$$

Having these solutions in hand, the equations (4) for α and β are expressed completely in terms of the two current state variables.

As a reasonable example, if we assume that the transistors are identical and always in the square law (saturation) region with

$$F(x) = f(x,y) = K[x-V_T]^2 1(x-V_T) \quad (7)$$

where K & V_T are real constants and $1(\cdot)$ is the unit step function, then analytic studies can proceed. In this case we obtain

$$\alpha = 1 - (R_{L1}F'(v_1))\cdot(R_{L2}F'(v_2)) \quad (8a)$$
$$\beta = 0 \quad (8b)$$

where

$$F'(x)=dF(x)/dx=2K(x-V_T)1(x-V_T) \quad (8c)$$

and v_1 and v_2 are expressed directly in terms of i_1 and i_2 via (5a,b) without the need to solve feedback equations (the impulse resulting from differentiation of the unit step function disappears since it gets multiplied by zero due to $x=V_T$).

By linearizing about the bias point the previous results, summarized in A above, are obtained since a Taylor series expansion gives

$$g_{m1} = F'(V_1),\ g_{m2} = F'(V_2) \quad (9)$$

where V_1 and V_2 are the zero signal (bias) values of the gate to source voltages v_1 and v_2. Substitution of (9) in (8c) yields (2c) as desired.

IV. Discussion

Here the nonlinear state variable equations have been presented for an MOS universal circuit. These are seen to be identical to those for the linearization about the bias point, except that the parameters α and β become nonlinear functions of the state variables and total rather than just signal components are used. In fact the parameter β is absent in the linearized case and only present in the nonlinear situation when the second transistor's drain current depends upon the drain to source voltage causing feedback that alters the description; this is basically only in the ohmic region of transistor operation but also allows consideration of nonzero slope of the constant current curves. Although the parasitic capacitors of the transistors are ignored, the results are otherwise very general, and, hence, will hold for self generated signals at low enough frequencies where the transistor parasitic capacitances are negligible.

In summary, the nonlinear state-variable equations at low frequencies for this universal circuit are given by eqs. (2a,b), with α and β evaluated using (4) and (6).

Since analytic expressions for MOS transistors exist [3, p.51], it remains to evaluate the state variable equations presented here in terms of them. Because of the square-law nature of MOS transistors in their saturation regions it does appear that Volterra analysis of this universal circuit may yield some interesting results toward its design. Likewise it remains to run CAD curves in the nonlinear case, though some results that show the feasibility of the circuit have been obtained in the linear case [4].

In order to obtain the desired self-generated responses it is necessary to set initial conditions. A study also needs to be made to determine circuit modifications needed to conveniently make these settings in possible applications.

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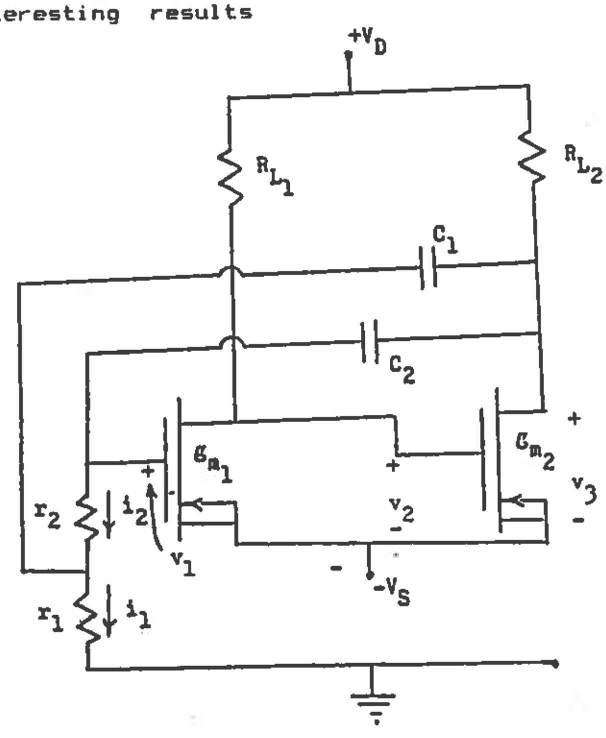

Figure 1
An MOS Universal Circuit

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