A DIGITAL LATTICE FOR COCHLEAR PARAMETER IDENTIFICATION

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ABSTRACT

Here we present a digital filter cochlear model from one of the circuit models in the literature [Zwi.65], thus obtaining a lattice structure by rephrasing the model equations in terms of the incident and reflected waves [Góm.82]. The theory developed can be used in characterizing the auditory system, and should be of particular use in clinical applications for the hearing impaired. This would allow noninvasive measurements to be made via the Kemp-Echo [Kem.78] from which the lattice parameters could be determined allowing the design of hearing aids to fit a particular ear.

INTRODUCTION

The present work is aimed to model cochlear mechanics as a digital filter. For such we have chosen a model of the mechanical activity in the cochlea as a unidimensional transmission line [Zwi.65]. More refined models may be found in the literature [All.85] but they seem to be less adequate to the purposes above mentioned because of their complexity, both from a mathematical and a computational point of view. The model may be expressed by the differential equations:

$$\frac{\partial P}{\partial x} = - Z_s U \qquad (1)$$

$$\frac{\partial U}{\partial x} = - Y_p P \qquad (2)$$

P and U being the Laplace transform of the differential pressure (p) between both scales and of the volumetric velocity (u) in the vestibular scale:

$$P(x,s) = \mathcal{L}\{p(x,t)\} \qquad (3)$$

$$U(x,s) = \mathcal{L}\{u(x,t)\} \qquad (4)$$

Equation (1) stands for fluid mechanics at each scale, and (2) shows the influence between both scales through the partition membranes. We have defined:

$$Z_s = r + \ell s \qquad (5)$$

$$Y_p = [\mu s + \sigma + \varepsilon s^{-1}]^{-1} \qquad (6)$$

r and ℓ being the viscid and inertial parameters of both scales, μ (mass), σ (viscosyty) and ε (elasticity) being the parameters of the partition. (See Fig. 1).

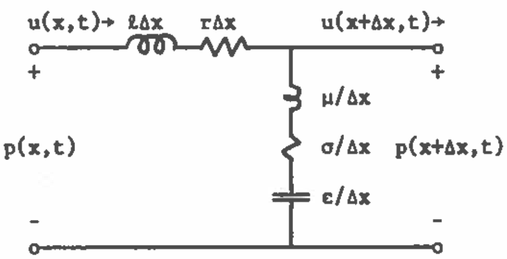


Fig. 1. Equivalent circuital model.

Splitting up P and U into an incident (F) and a reflected wave (G):

$$\begin{bmatrix} P \\ U \end{bmatrix} = \begin{bmatrix} Z_c & Z_c \\ +1 & -1 \end{bmatrix} \begin{bmatrix} F \\ G \end{bmatrix} \qquad (7)$$

Z_c being the "characteristic impedance":

$$Z_c^2 = Z_s/Y_p \qquad (8)$$

equations (1) and (2) may be written as:

$$\frac{\partial F}{\partial x} = (\rho - \gamma) F + \rho G \qquad (9)$$

$$\frac{\partial G}{\partial x} = \rho F + (\rho + \gamma) G \qquad (10)$$

the reflectivity (ρ) and the propagation functions (γ) being defined as:

$$\rho = \frac{1}{2Z_c} \frac{\partial Z_c}{\partial x} \qquad (11)$$

$$\gamma^2 = Z_s Y_p \qquad (12)$$

INCREMENTAL SOLUTIONS

To derive the desired digital filters, the inner ear model in Fig. 2, has been divided into sections of length Δx.

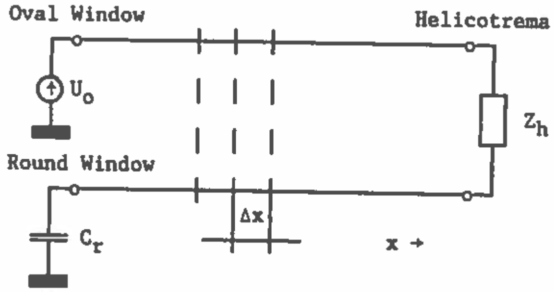


Fig. 2. Incremental division of the line.

Within each section we will assume that Z_c is constant, and as such $\rho=0$. We will allow changes in Z_c to take place at the boundary between two sections. Then the effects of propagation and dispersion may be separated. For $\rho=0$ equations (9) and (10) show a simple solution as follows:

$$F_k(x) = F_k(0)\, e^{-\gamma(x_k)x} \qquad (13)$$

$$G_k(x) = G_k(0)\, e^{+\gamma(x_k)x} \qquad (14)$$

Now assuming that $\gamma=0$, equations (9) and (10) may be integrated by means of the trapezoidal rule [Hil.74] yielding:

$$\begin{bmatrix} F_k(0) \\ G_k(0) \end{bmatrix} = \frac{1}{\tau_k} \begin{bmatrix} 1 & \rho_k \\ \rho_k & 1 \end{bmatrix} \begin{bmatrix} F_{k-1}(\Delta x) \\ G_{k-1}(\Delta x) \end{bmatrix} \qquad (15)$$

for which we have taken:

$$\rho_k = \rho(x_k)\,\Delta x \qquad (16)$$

$$\tau_k = 1 - \rho_k \qquad (17)$$

The effects of both integrations may be represented in the flow graph of Fig. 3.

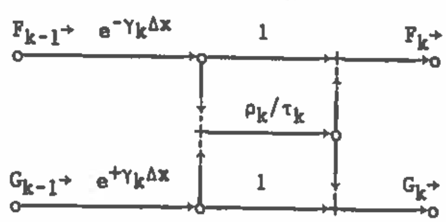


Fig. 3. Filter equivalent of a section.

SYNTHESIS OF PARTIAL TRANSFER FUNCTIONS

We will first synthesize the propagation functions $e^{-\gamma_k \Delta x}$ and $e^{+\gamma_k \Delta x}$ taking:

$$e^{-\gamma_k \Delta x} \approx \frac{2 - \gamma_k \Delta x}{2 + \gamma_k \Delta x} \qquad (18)$$

using the "bilinear transformation" [Opp.75]:

$$s = 2 f_m \frac{1 - z^{-1}}{1 + z^{-1}} \qquad (19)$$

with f_m the sampling frequency. From (12):

$$\gamma_k = b_{\gamma k0} \frac{1 - z^{-1}}{[1-z^{-1}z_{\gamma k1}]^{\frac{1}{2}}[1-z^{-1}z_{\gamma k2}]^{\frac{1}{2}}} \qquad (20)$$

with:

$$b_{\gamma k0} = \{\ell_k/(\mu_k + \sigma_k + \varepsilon_k)\}^{\frac{1}{2}} \qquad (21)$$

$$z_{\gamma k1} = \frac{1}{2 f_m} \frac{1 + s_{\gamma k1}}{1 - s_{\gamma k1}} \qquad (22)$$

$$z_{\gamma k2} = \frac{1}{2 f_m} \frac{1 + s_{\gamma k2}}{1 - s_{\gamma k2}} \qquad (23)$$

$$s_{\gamma k1,2} = \frac{-\sigma_k \pm [\sigma_k^2 - 4\,\mu_k\,\varepsilon_k]^{\frac{1}{2}}}{2\,\mu_k} \qquad (24)$$

for which we have assumed no losses at the scales ($r=0$). Now (20) may be implemented as a series expansion [Opp.75, pp. 55-56]:

$$H_{k1}(z) = [1-z^{-1}z_{\gamma k1}]^{-\frac{1}{2}} = \sum_{i=0}^{\infty} h_{k1i}\, z^{-i} \qquad (25)$$

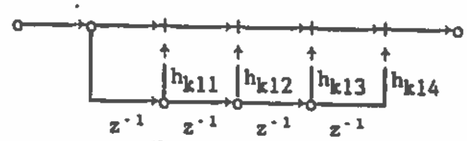
$$H_{k2}(z) = [1-z^{-1}z_{\gamma k2}]^{-\frac{1}{2}} = \sum_{i=0}^{\infty} h_{k2i}\, z^{-i} \qquad (26)$$

with:

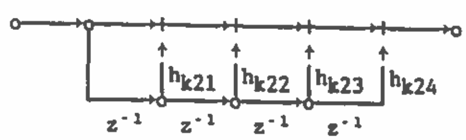
$$h_{k1i} = (z_{\gamma k1}/4)^i \frac{(2i)!}{(i!)^i} \qquad (27)$$

$$h_{k2i} = (z_{\gamma k2}/4)^i \frac{(2i)!}{(i!)^i} \qquad (28)$$

A brief examination of the impulse responses shows that with four terms in (25) and (26) a good description of both transfer functions is obtained [Góm.82], as shown in Fig. 4. The synthesis of the propagation transfer functions is shown in Fig. 5.

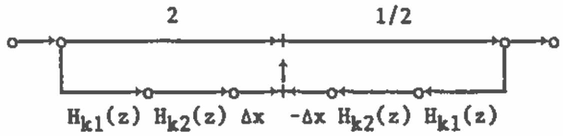


a) Realization of $H_{k1}(z)$.

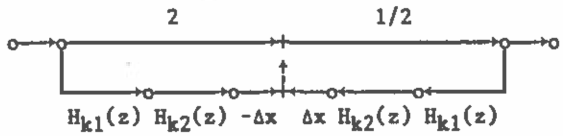


b) Realization of $H_{k2}(z)$.

Fig. 4. Synthesis of $H_{k1}(z)$ and $H_{k2}(z)$.



a) Realization of $e^{-\gamma_k \Delta x}$.



b) Realization of $e^{+\gamma_k \Delta x}$.

Fig. 5. Propagation transfer functions.

Now, from (5), (6), (8), (11), (16) and (17) ρ_k/τ_k may be obtained (See Fig. 6):

$$\frac{\rho_k}{\tau_k} = \frac{c_{k0}\,(1 + z^{-1})^2}{(a_{k0}-c_{k0})+(a_{k1}-2c_{k0})z^{-1}+(a_{k2}-c_{k0})z^{-2}} \tag{29}$$

having taken [All.77]:

$$\mu(x) = \mu_0 \tag{30}$$

$$\sigma(x) = \sigma_0 \tag{31}$$

$$\varepsilon(x) = \varepsilon_0\, e^{-ax} \tag{32}$$

$$c_{k0} = -\,1/4\; a\, \varepsilon_k\, \Delta x \tag{33}$$

$$a_{k0} = 4\,\mu_k\, f_m^2 + 2\,\sigma_k\, f_m + \varepsilon_k \tag{34}$$

$$a_{k1} = 2\,\varepsilon_k - 8\,\mu_k\, f_m^2 \tag{35}$$

$$a_{k2} = 4\,\mu_k\, f_m^2 - 2\,\sigma_k\, f_m + \varepsilon_k \tag{36}$$

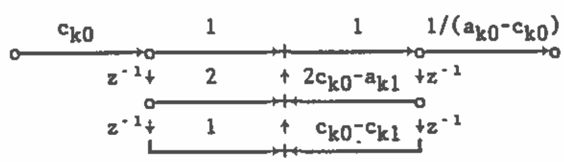


Fig. 6. Realization of ρ_k/τ_k.

We may obtain the patterns of movement (υ) of the partition membranes (See Fig.7), D being a proportionality parameter as:

$$\dot{\upsilon} = 1/(D\Delta x)\;(F_k - G_k - F_{k-1} + G_{k-1}) \tag{37}$$

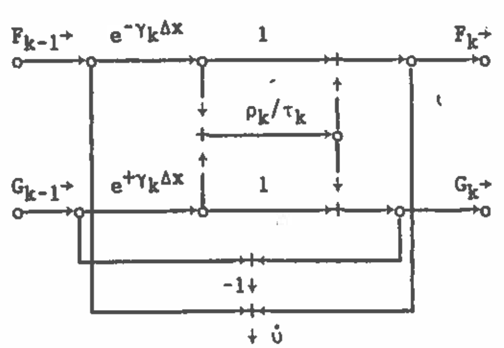


Fig. 7. Derivation of membrane dynamics.

LINE INVERSION

In order to get estimates of the model parameters we will proceed as in the case of the acoustic tube in speech processing [Rab.78]. The main difference in our case is based in the structure of the model equations, as derived from (13), (14) and (15):

$$F_k(z) = F'_{k-1}(z) + \rho_k(z)\, G'_{k-1}(z) \tag{38}$$

$$G_k(z) = \rho_k(z)\, F'_{k-1}(z) + G'_{k-1}(z) \tag{39}$$

having defined:

$$F'_{k-1}(z) = \frac{e^{-\gamma_k \Delta x}}{1 - \rho_k}\, F_{k-1}(z) \tag{40}$$

$$G'_{k-1}(z) = \frac{e^{+\gamma_k \Delta x}}{1 - \rho_k}\, G_{k-1}(z) \tag{41}$$

with which the time counterparts of (38) and (39) may be expressed as:

$$f_k(n) = f'_{k-1}(n) + \rho_k(n) * g'_{k-1}(n) \tag{42}$$

$$g_k(n) = \rho_k(n) * g'_{k-1}(n) + g'_{k-1}(n) \tag{43}$$

where (*) stands for the discrete convolution. The process of estimating $\rho_k(n)$ is the result of minimizing the energy of the waves $f_k(n)$ or $g_k(n)$ with respect to $\rho_k(n)$, this being causal and of infinite length:

$$E_{fk} = \sum_n \{f_k(n)\}^2 = \sum_n \{f'_{k-1}(n)\}^2 + \sum_n \{\sum_i \rho_{ki}\, g'_{k-1}(n-i)\}^2 + 2 \sum_n f'_{k-1}(n)\, \{\sum_i \rho_{ki}\, g'_{k-1}(n-i)\} \tag{44}$$

the minimization process implying:

$$\frac{\partial E_{fk}}{\partial \rho_{km}} = 0; \qquad 0 \le m \le \infty \tag{45}$$

and from this:

$$\sum_{i=0}^{\infty} \rho_{ki}\, r(i-m) = -\, x(m) \qquad 0 \le m \le \infty \tag{46}$$

having defined:

$$r(i-m) = \sum_n g'_{k-1}(n-i)\, g'_{k-1}(n-m) \tag{47}$$

$$x(m) = \sum_n f'_{k-1}(n)\, g'_{k-1}(n-m) \tag{48}$$

Equations (46) will allow us to determine the impulse response of the reflectivity function, and from this, the parameters of the model may be recovered. It will not be necessary to solve infinite equations as stated in (46), because ρ_{ki} decays with i increasing as seen in Fig. 8, in which a typical trace is shown for a given value of k [Rod.83], and because only the first three values of ρ_{ki} would be necessary to recover the model parameters.

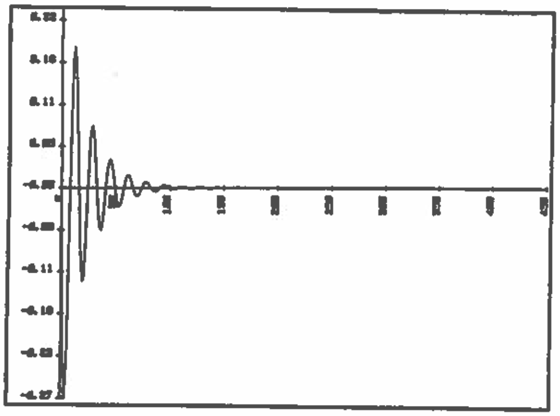


Fig. 8. Typical ρ_{ki} trace.

In fact, it may be seen that:

$$\rho_k(z) = \frac{c_{k0}\,(1 + z^{-1})^2}{a_{k0} + a_{k1}\, z^{-1} + a_{k2}\, z^{-2}} =$$

$$= \sum_{i=0}^{\infty} \rho_{ki}\, z^{-i} \qquad (49)$$

and from this last relation it may be checked that:

$$\rho_{k0}\, a_{k0} = c_{k0} \qquad (50)$$

$$\rho_{k1}\, a_{k0} + \rho_{k0}\, a_{k1} = 2\, c_{k0} \qquad (51)$$

$$\rho_{k2}\, a_{k0} + \rho_{k1}\, a_{k1} + \rho_{k0}\, a_{k2} = c_{k0} \qquad (52)$$

these being a set of linear equations from which a_{k0}, a_{k1} and a_{k2} may be inferred. From them and (33-36) the parameters of the model may be obtained.

DISCUSSION

The digital filters shown may be useful in building new signal processing devices to help the severe deaf people through the implementation of cochlear implants [Loe.83]. The model shown may be especially useful for these purposes, because it is based on the mechanical activity of the inner ear, rather than in selective bandpassing. Another advantage is based on the possibility of characterizing the parameters of the model directly from external measurements. This fact may help in developing new tools for processing the otoacoustic emissions, which would be of great help in understanding the activity of the inner auditory system. Some problems may appear when applying these techniques due to the assumed nonlinear behavior of the auditory system. For such, the structures shown are under test, their implementation by computer and consequent refinement being on the way.

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