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The Lewis Semistate Decomposition for PARCOR Lattices*

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Abstract:
The semistate equations of multichannel PARCOR (PARtial CORrelation) prediction error and synthesis lattice sections are found. Then the Lewis' backward-forward decomposition is obtained for these lattices from which the relationship between the Lewis' backward-forward semistate variables and the PARCOR lattice backward-forward variables is found.

I. Introduction

Recently semistate (sometimes called singular, descriptor, or differential-algebraic) descriptions have been seen to be of interest for the theory of control systems [1]. This interest arises practically since semistate descriptions occur naturally without the need for reductions to eliminate variables, as in the case of estimation of two-point boundary value processes [2, p. 810], while also they generalize the state-variable descriptions. Consequently, efficient methods for decomposing and solving semistate equations, such as the forward-backward technique of [3] and [4], are also of importance to control system studies. Likewise the theory of PARCOR lattices, which have proven so valuable in the area of speech synthesis [5], have considerable application in the estimation of control systems [6] where a number of processes fit into the lattice form. Interestingly, these lattices also have their signals considered as forward and backward signals. However, the use of the words forward and backward in the two instances has arisen quite differently and independently, as discussion with Professor Lewis has borne out. Thus, one working between the two fields is led to wonder if there is not a tie between the forward-backward concepts as they are present in both lattice systems and the decomposition of semistate descriptions.

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Here we investigate the tie between these two uses of the words forward and backward to show that the Lewis analysis can be almost directly used on PARCOR lattices.

Toward this we note that in [3] Professor Lewis has conceived the decompostion of semistate described systems into backward and forward subsystems to allow one to work forward and backward in time to solve the semistate equations. On the other hand forward and backward waves are of particular interest to systems having descriptions formulated on a scattering basis where forward and backward signals flow through the system. This scattering kind of description is of considerable importance in the digital filtering area for the lattice filter class [7, p. 414]. To investigate the question raised above we here obtain the Lewis decomposition for multichannel digital filter lattices [8] by considering the semistate equations of multichannel lattice sections. By use of a simple transformation on the semistate the Lewis decomposition into backward and forward subsystems is obtained.

II. Review of the Lewis Decomposition

Here we quickly review semistate equations and their backward-forward decompostion. Thus, we recall the general canonical form for semistate equations of linear time-invariant systems, this being [9,p. 66]

$$\mathcal{A}zx + \mathcal{B}x = \mathcal{C}u \qquad (1a)$$
$$y = \mathcal{D}x \qquad (1b)$$

where the vector quantities u, x, and y are the input, semistate and output, respectively, and $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, and $\mathcal{D}$ are constant matrices of appropriate size and here assumed to be real; z is a system operator which we take to be unit time advance. We will assume that $z\mathcal{A}+\mathcal{B}$ is nonsingular, that is that $\mathcal{A}$ & $\mathcal{B}$ form a regular pencil (which means that the system has unique solutions). Then by performing certain linear transformations on these semistate equations [10, p. 35], as illustrated in the next section and discussed further in the discussion section IV, we can bring $\mathcal{A}$ and $\mathcal{B}$ to a particular form which brings into focus the Lewis decomposition. Thus, there are nonsingular matrices P and Q such that

$$\mathcal{A}' = P\mathcal{A}Q = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & \mathcal{A}_N \end{bmatrix}, \quad \mathcal{B}' = P\mathcal{B}Q = \begin{bmatrix} \mathcal{B}_N & 0 & 0 \\ 0 & \mathcal{B}_1 & 0 \\ 0 & 0 & I \end{bmatrix} \qquad (2a,b)$$

$$= I \dotplus I \dotplus \mathcal{A}_N, \qquad\qquad = \mathcal{B}_N \dotplus \mathcal{B}_1 \dotplus I$$

where I and 0 denote the identity and zero matrices, respectively, of appropriate sizes, $\dotplus$ denotes the matrix direct sum, $\mathcal{B}_1$ is a nonsingular matrix and $\mathcal{A}_N$ & $\mathcal{B}_N$ are nilpotent matrices (meaning some power of them is zero) which can be written as the direct sum of matrices of the form

$$\begin{bmatrix} 0 & \vline & \\ . & \vline & I \\ . & & \\ 0 & \ldots & 0 \end{bmatrix} \quad (2c)$$

where the zeros are scalar zeros. In eqs. (2a-c) any of the rows and columns may be absent if the corresponding portion of the decomposition is not present. For example, if $\mathcal{B}$ has rank zero then only $\mathcal{B}_N$ is present and it is the direct sum of zeros.

The transformation in (2a-c) places a transformation on the semistate and the input and output matrices $\mathcal{B}$ & $\mathcal{C}$. Thus

$$\mathcal{B}' = P\mathcal{B}, \qquad \mathcal{C}' = \mathcal{C}Q \quad (2d,e)$$

$$x' = Q^{-1}x = \begin{bmatrix} x^{f\prime} \\ x^{fb\prime} \\ x^{b\prime} \end{bmatrix} \quad (2f)$$

where the rows of x' are partitioned as those of $\mathcal{Q}'$ with the superscripts f, b, and fb, denoting forward, backward, and forward-or-backward. Here $x^{fb\prime}$ has the choice of being allocated between the forward and backward subsystems; if it is all assigned to the forward subsystem then a "maximal forward" decomposition results whereas assignment to the backward subsystem gives a "maximal backward" decomposition [3, p. 169]. In the case that the forward-or-backward subsystem is absent the system is both maximal forward and maximal backward; such will be the case for the PARCOR lattices. The semistate equations are now eqs. (1) but with primes on all terms except the input and output which are unchanged by the transformations performed. In this form of the semistate equations broken into forward and backward subsystems one can solve them by iterating the forward subequations forward in time and the backward subequations backward in time as discussed by Lewis [3]. Similarly, for two point boundary value problems where a closed form solution exists [4, p. 126].

III. Lattice Semistate Equations

Here we obtain the semistate equations decomposed into forward and backward subsystems for both the synthesis lattice and the analysis (prediction error) lattice. In both cases we will use the lattice terminal variables as the original semistate variables and then transform these into new (primed) semistate variables that give the Lewis forward-backward decomposition.

Consider first the n-channel synthesis lattice section with the signal-flow graph of Fig. 1 where 1/z is unit delay and I designates the nxn identity matrix denoting n unit transmittances. The equations describing Fig. 1 can be written by inspection and are

$$f_{-1}(t) = f(t) + A(1/z)b_{-1}(t) \quad (3a)$$
$$b(t) = -\underline{A}f_{-1}(t) + (1/z)b_{-1}(t) \quad (3b)$$

where $f(.)$ & $b(.)$ are the forward and backward (n-vector) signals on the left (input) side and $f_{-1}(.)$ & $b_{-1}(.)$ are the same for the right (output) side of the n-channel lattice. Consequently, we take the input $u(.)$ and the output $y(.)$ (n-vectors) as

$$u(t) = \begin{bmatrix} f(t) \\ b_{-1}(t) \end{bmatrix}, \qquad y(t) = \begin{bmatrix} f_{-1}(t) \\ b(t) \end{bmatrix} \tag{3c}$$

Here t can be either continuous or descrete time and the input output choice yields a transfer function that is of the transfer scattering matrix form.

We use the forward and backward lattice variables as the semistate variables and group these in forward and backward pairs for comparison with the Lewis partition of equation (2f). Thus we choose the lattice semistate $x(.)$ 4n-vector as

$$x = \begin{bmatrix} f_{-1} \\ f \\ b_{-1} \\ b \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \tag{4}$$

from which we write the following as a set of semistate equations

$$\begin{bmatrix} 0 & -I & 0 & 0 \\ \underline{A} & 0 & 0 & I \\ \underline{0} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \mathcal{D}x + \begin{bmatrix} 0 & 0 & -A & 0 \\ 0 & 0 & -I & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \end{bmatrix} x = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ I & 0 \\ 0 & I \end{bmatrix} u \tag{5a}$$

$$y = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0 & 0 & I \end{bmatrix} x \tag{5b}$$

Here 0, as I, is an nxn matrix.

Next we transform these canonical semistate equations into the actual Lewis' forward-backward form by performing the following elementary column operations on the matrices multiplying x in eqs. (5a):

1. add $-\underline{A}$ times the fourth column to the first then
2. add the first column to the second then
3. interchange columns two and four

and follow these up by the following elementary row operations on eq. (5a)

4. add the fourth row to the second then
5. add A times the fourth row to the first then
6. interchange the third and fourth rows.

These operations give the following equivalent set of canonical semistate equations, now in the Lewis forward-backward form

$$\begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} z x' + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} x' = \begin{bmatrix} 0 & A \\ 0 & I \\ 0 & 0 \\ I & 0 \end{bmatrix} u \qquad (6a)$$

$$y = \begin{bmatrix} I & 0 & 0 & I \\ -\underline{A} & I & 0 & -A \end{bmatrix} x' \qquad (6b)$$

with

$$x' = \begin{bmatrix} f_{-1} - f \\ \underline{A}f_{-1} + b \\ \bar{b}_{-1} \\ f \end{bmatrix} = \begin{bmatrix} I & -I & 0 & 0 \\ \underline{A} & 0 & 0 & I \\ 0 & 0 & I & 0 \\ 0 & I & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = Q^{-1}x \qquad (6c)$$

As seen by comparing with eq. (2a,b), eqs. (6) give a decomposition of the semistate equations into a forward part, comprised of x_1' & x_2' and a backward part comprised of x_3' & x_4', and the decomposition is into a maximal forward maximal backward system.

We conclude that a forward-backward Lewis decompostion for the n-channel lattice has the forward subsystem semistate, $x^{f'}$, and the backward subsystem semistate, $x^{b'}$, of dimension 2n and that these comprise linear combinations of the forward and backward lattice variables; a possible combination is given in (6c). For completeness we exhibit the matrices P and Q, which are products of elementary matrices according to the rules above eq. (6).

$$P = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & I \\ 0 & 0 & I & 0 \end{bmatrix} \begin{bmatrix} I & 0 & 0 & A \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & I \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} = \begin{bmatrix} I & 0 & 0 & A \\ 0 & I & 0 & I \\ 0 & 0 & 0 & I \\ 0 & 0 & I & 0 \end{bmatrix} \qquad (7a)$$

$$Q = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ -\underline{A} & 0 & 0 & I \end{bmatrix} \begin{bmatrix} I & I & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0 & 0 & I \\ 0 & 0 & I & 0 \\ 0 & I & 0 & 0 \end{bmatrix} = \begin{bmatrix} I & 0 & 0 & I \\ 0 & 0 & 0 & I \\ 0 & 0 & I & 0 \\ -\underline{A} & I & 0 & -\underline{A} \end{bmatrix} \qquad (7b)$$

or

$$Q^{-1} = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0 & 0 & I \\ 0 & 0 & I & 0 \\ 0 & I & 0 & 0 \end{bmatrix} \begin{bmatrix} I & -I & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ \underline{A} & 0 & 0 & I \end{bmatrix} = \begin{bmatrix} I & -I & 0 & 0 \\ \underline{A} & 0 & 0 & I \\ 0 & 0 & I & 0 \\ 0 & I & 0 & 0 \end{bmatrix} \qquad (7c)$$

Next we consider the analysis (or what is also called the prediction error) filter of Fig. 2. Here eqs. (3a,b) are rewritten as

$$f(t) = f_{-1}(t) - A(1/z)b_{-1}(t) \qquad (8a)$$
$$b(t) = -\underline{A}f_{-1}(t) + (1/z)b_{-1}(t) \qquad (8b)$$

and we have

$$u(t) = \begin{bmatrix} f_{-1}(t) \\ b_{-1}(t) \end{bmatrix}, \qquad y(t) = \begin{bmatrix} f(t) \\ b(t) \end{bmatrix} \qquad (8c)$$

We choose the same semistate as in eq. (4) and find by inspection of (8)

$$\begin{bmatrix} I & -I & 0 & 0 \\ \underline{A} & 0 & 0 & I \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} zx + \begin{bmatrix} 0 & 0 & -A & 0 \\ 0 & 0 & -I & 0 \\ I & 0 & 0 & 0 \\ 0 & 0 & I & 0 \end{bmatrix} x = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ I & 0 \\ 0 & I \end{bmatrix} u \qquad (9a)$$

$$y = \begin{bmatrix} 0 & I & 0 & 0 \\ 0 & 0 & 0 & I \end{bmatrix} x \qquad (9b)$$

To put these canonical semistate equations into the forward-backward Lewis' form we add the first column to the second, subtract the second column from the first, subtract A times the fourth column from the second, interchange the second and fourth columns, add the fourth row to the second, add A times the fourth row to the first, and finally exchange the third and fourth rows. These operations are performed via

$$P = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & I \\ 0 & 0 & I & 0 \end{bmatrix} \begin{bmatrix} I & 0 & 0 & A \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & I \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} = \begin{bmatrix} I & 0 & 0 & A \\ 0 & I & 0 & I \\ 0 & 0 & 0 & I \\ 0 & 0 & I & 0 \end{bmatrix} \qquad (10a)$$

$$Q = \begin{bmatrix} I & I & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \begin{bmatrix} I & 0 & 0 & 0 \\ -I & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & -\underline{A} & 0 & I \end{bmatrix} \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0 & 0 & I \\ 0 & 0 & I & 0 \\ 0 & I & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & I \\ -I & 0 & 0 & I \\ 0 & 0 & I & 0 \\ 0 & I & 0 & -\underline{A} \end{bmatrix} \qquad (10b)$$

for which

$$Q^{-1} = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0 & 0 & I \\ 0 & 0 & I & 0 \\ 0 & I & 0 & 0 \end{bmatrix} \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & \underline{A} & 0 & I \end{bmatrix} \begin{bmatrix} I & 0 & 0 & 0 \\ I & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \begin{bmatrix} I & -I & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & I & 0 & 0 \end{bmatrix} = \begin{bmatrix} I & -I & 0 & 0 \\ \underline{A} & 0 & 0 & I \\ 0 & 0 & I & 0 \\ I & 0 & 0 & 0 \end{bmatrix} \qquad (10c)$$

It should be noted that the only difference between the Q^{-1} in the synthesis versus analysis cases is that the (4,1) and (4,2) entries are interchanged.

$$x' = Q^{-1}x = \begin{bmatrix} f_{-1} - f \\ Af_{-1} + b \\ b_{-1} \\ f_{-1} \end{bmatrix} \qquad (10d)$$

These operations give the forward-backward decomposed canonical semistate equations as

$$\begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} zx' + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} x' = \begin{bmatrix} 0 & A \\ 0 & I \\ 0 & I \\ I & 0 \end{bmatrix} u \qquad (11a)$$

$$y = \begin{bmatrix} -I & 0 & 0 & I \\ 0 & I & 0 & -A \end{bmatrix} x' \qquad (11b)$$

IV. Discussion

Here we have obtained forward-backward decompostions for the synthesis and the prediction error (analysis) multichannel lattice semistate descriptions. By suitable transformations these are brought into the forms of eq. (6a,b) and (11a,b) from which it is seen that the decompositions are canonical ones that are both maximal-forward and maximal-backward. The equations in the two cases, synthesis and analysis filters, are just about the same though there are slight differences in the input and output matrices and the transformed semistate itself. This is to be expected since in going from the synthesis to the analysis filter there is only a change in half of the input and output variables. What is interesting is that the forward and backward variables of lattice theory are somewhat different from those of the Lewis' decomposition but closely related through the transformations Q given above. Indeed one of the lattice forward variables, specifically f in the synthesis lattice and f_{-1} in the analysis lattice, is a Lewis backward variable. And from what we have seen here we believe that similar results may hold for more general systems when described via scattering variables.

As the lattice sections are of most interest when cascaded to form more complete systems, it is of future interest to develop the forward-backward decomposition for cascaded systems. Such should follow readily the results given here when combined with the results of [11].

The question of uniqueness of the decomposition is of interest. By considering new P's and Q's, as per (2), applied to (6a) & (11a), we see that in order to retain the identity matrices required by (2) in (6) and (11), the new P and Q must be direct sums, $P = [P_{11} \dotplus P_{22}]$, $Q = [Q_{11} \dotplus Q_{22}]$, with $P_{11}=Q_{11}^{-1}$ and $P_{22}=Q_{22}^{-1}$.

Thus, the forward and backward semistate subspaces are unique, that is invariant to further transformation. On the other hand, the semistate components within these two subspaces can be freely transformed among themselves (since the only constraint on these P_{11} and P_{22} is that they be nonsingular).

The results of this paper can be applied in various ways. For one, we see that, on transforming to its backward-forward decomposition given above, a lattice can be analyzed by applying the methods given by Lewis [3]; for example the very convenient double sweep method can be used for obtaining the signals in a lattice system.

As a second application of significance for system design, we note that if a system has a semistate description that can be transformed to the forms of (6) or (11), then it can be constructed as a lattice. Since we know that the lattice is transformable to these forms, we obtain necessary and sufficient conditions that a 2n-input 2n-output system be constructable by a lattice, these being that the system is maximal-forward and maximal-backward with the existence of a P and Q to bring $\mathfrak{B}$ & $\mathfrak{C}$ into the forms exhibited in (6) and (11). Although this reads as an existence theorem, the actual search for a suitable P and Q is not very difficult in practice due to the nice forms of the $\mathfrak{B}$ & $\mathfrak{C}$ of (6) and (11) where all the parameters occur in the A or $\underline{A}$ matrices.

When a given 2n-input 2n-output system is maximal-forward and maximal-backward, then the $\mathcal{A}$ & $\mathcal{B}$ matrices can always be transformed to the direct sum forms of (6a) and (11a). By further transformation of direct sum P & Q, as described two paragraphs up, various forms for $\mathfrak{B}$ and $\mathfrak{C}$ can be obtained. All of the systems so obtained are equivalent; of course for system design it is profitable to look for the forms most convenient for hardware or software realizations. As just discussed, if the system is equivalent to a lattice then (6), in the case of a synthesis lattice, or (11), in the case of an analysis lattice, can be obtained. But even if (6) or (11) can not be obtained, by bringing $\mathfrak{B}$ and $\mathfrak{C}$ as close as possible to those of (6) or (11), an equivalent system that somewhat resembles a lattice can be designed. For this one can take a lattice and insert branches to "correct" the lattice to yield the desired $\mathfrak{B}$ and $\mathfrak{C}$ matrices. In this sense the material of this paper shows that the lattice can be used as a basic building block for 2n-input 2n-output maximal- forward maximal-backward systems.

Finally we give a means to attain the decomposition of eqs. (2a-c). Since $z\mathcal{A}+\mathcal{B}$ is nonsingular, choose one of the real values of z, call it c, for which $z\mathcal{A}+\mathcal{B}$ has full rank. Using only real matrices P_i & Q_i, i=1,...,6, in the following, we first transform by $Q_1=I$ and $P_1=(c\mathcal{A}+\mathcal{B})^{-1}$ to get

$$P_1(z\mathcal{A}+\mathcal{B})Q_1 = P_1[(c\mathcal{A}+\mathcal{B})+(z-c)\mathcal{A}]Q_1 = I+(z-c)(c\mathcal{A}+\mathcal{B})^{-1} \qquad (12a)$$

Next transform by $Q_2=P_2^{-1}$ where P_2 is chosen to bring $(c\mathcal{A}+\mathcal{B})^{-1}$ into a direct sum of a nonsingular matrix J_1 and a nilpotent matrix J_N [10, p. 301-306]. Then

$$P_2P_1(z\mathcal{A}+\mathcal{B})Q_1Q_2 = [zJ_1+(I-cJ_1)]\dot{+}[zJ_N+(I-cJ_N)] \qquad (12b)$$

This we multiply by $P_3 = J_1^{-1} \dotplus (I-cJ_N)^{-1}$, with these inverses existing, and $Q_3 = I$. The matrix $J_1^{-1}(I-cJ_1)$ is further brought to a nilpotent, K_N, and nonsingular, K_1, direct sum by a matrix P_4 with $Q_4 = P_4^{-1}$ and the matrix $(I-cJ_N)^{-1}J_N$ is similarly brought to a direct sum $L_1 \dotplus L_N$, with L_1 nonsingular and L_N nilpotent, by P_5 and $Q_5 = P_5^{-1}$. At the end L_1 is brought to the identity by $P_6 = L_1^{-1}$, $Q_6 = I$. Thus

$$P(z\mathcal{Q}+\mathcal{R})Q = \{[P_4 \dotplus (P_6P_5)]P_3P_2P_1\}(z\mathcal{Q}+\mathcal{R})\{Q_1Q_2Q_3[Q_4 \dotplus (Q_5Q_6)]\} = \quad (12c)$$

$$(zI+K_N) \dotplus (zI+K_1) \dotplus (zI+L_1^{-1}) \dotplus (zL_N+I) = \quad (12d)$$

$$(zI+K_N) \dotplus (zI+[K_1 \dotplus (L_1^{-1})]) \dotplus (zL_N+I) \quad (12e)$$

The last is in the form of eqs. (2a,b), with $\mathcal{R}_1 = [K_1 \dotplus L_1^{-1}]$ and P & Q the matrices in braces. Equation (2c) follows if we choose the Jordan form for the nilpotent parts $K_N = \mathcal{R}_N$ and $L_N = \mathcal{Q}_N$, as is always possible ([10, p. 301] where we have zero eigenvalues for the nilpotent matrices). It should be noted that we can extend the form of eqs. (2a-c) to handle nonregular systems, that is systems for which $z\mathcal{Q}+\mathcal{R}$ is singular [10, pp. 35-48], essentially by inserting a direct sum of zero matrices, but in the lattice cases handled here nonregular systems are not really of interest.

Acknowledgments

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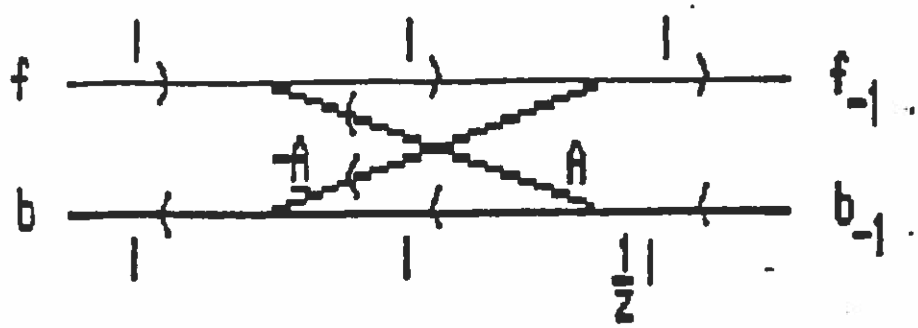
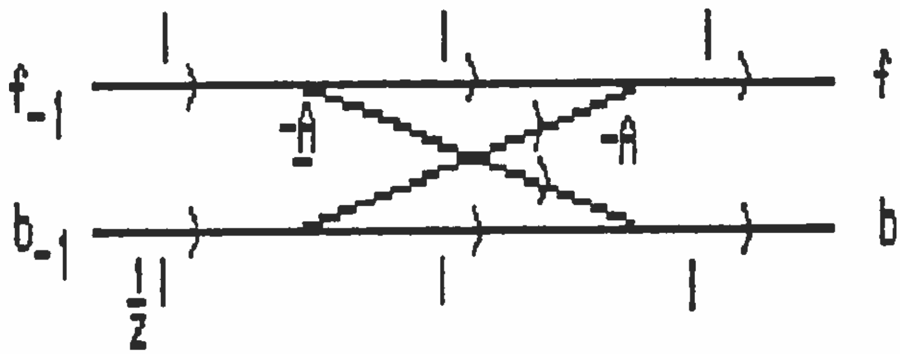
Figure Titles

1. Synthesis Lattice Section
2. Prediction Error (Analysis) Lattice Section



1/z = unit delay
I = identity

Figure 1
Synthesis Lattice



1/z = unit delay
I = identity

Figure 2
Analysis Lattice