"LATTICE FILTER ORDER DETERMINATION USING ARMA SPECTRA OF ECG SIGNALS"

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ABSTRACT.

The modeling of signals by means of Auto-Regressive-Moving-Average (ARMA) systems is being widely used for the decomposition, identification and characterization of systems in a great variety of applications. One of the problems in such modelization arises when the best number of coefficients for both the AR and MA parts has to be decided in terms of a given optimization criterion. In the present work a simple yet sensitive criterion is given and the methods related are briefly discussed on the basis of their application to a given Electro-Cardiographic (ECG) Signal. Results are presented and reviewed and some possible applications are pointed out.

INTRODUCTION

The key ideas in the decomposition techniques which are suggested in the present work can be stated as follows:

- To use only Linear Prediction methods [1] for the characterization of the AR and MA section orders in the spectrum.
- To determine the orders of the AR and MA portions in the spectrum according with the behavior of the Residual Error Signal (RES) Energy when plotted against the increasing number of lattice sections in the Linear Predictive Filters used for the Spectral Inversion [2].

The idea of using Linear Prediction Methods to separate ARMA spectra into their AR and MA portions is well known [3-9], and may become more and more popular with the development of computational tools devoted to the solution of simple identification and characterization problems in an inexpensive way, rather than using more sophisticated and less cost-effective techniques. The aim of the present work is related mainly with suggesting practical methods for determining the orders of the AR and MA portions of the spectrum rather than with developing new algorithms of decomposition. The approach is related with the ideas shown in [7], [8], and especially [9].

PROPOSED DECOMPOSITION TECHNIQUES

As it is well known, the ARMA spectrum of a given signal $x(n)$ can be expressed as a ratio of polynomials in z on the unit circle:

$$X(z) = \frac{B(z)}{A(z)} \qquad (1)$$

$X(z)$ being the z-transform of the time series $x(n)$; and $A(z)$, $B(z)$, polynomials in z accounting for the AR and MA portions of the given spectrum:

$$A(z) = 1 - \sum_{i=1}^{M} a_i z^{-i} \qquad (2)$$

$$B(z) = \sum_{i=0}^{L} b_i z^{-i} \qquad (3)$$

in which $\{a_i\}$ and $\{b_i\}$ are the respective coefficients and M, L, the orders of both portions. The model implied in (1), (2) and (3) can easily be interpreted by means of

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Fig. 1 as a signal processing structure of IIR filter type.

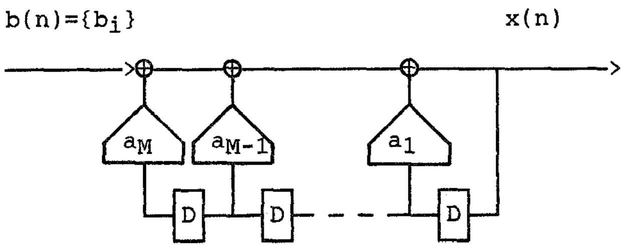

Fig. 1. Model for the production of $x(n)$.

In all what follows we will implicitly assume that $x(n)$ is of causal, stable and invertible nature. This last assumption states that if we represent the inverse series $y(n)$ as the inverse z-transform of $X(z)$:

$$y(n) = \mathcal{Z}^{-1}\{\frac{B(z)}{A(z)}\} \qquad (4)$$

where $\mathcal{Z}^{-1}\{*\}$ stands for the inverse z-transform of a given series, then $y(n)$ should also be a causal, stable and invertible signal. We will not go further in deep on the particular conditions which $A(z)$ and $B(z)$ have to fulfill to meet the above mentioned assumptions, these being stated for example in [4]. In Fig. 1 a symbolic representation of a production model for $x(n)$ is shown, the model consisting basically in an Autoregressive Structure of order M, which is fed with a Finite Impulse Response series $b(n)=\{b_i\}$ which in a general sense do not need to be of white noise nature, and whose samples are those same coefficients for the MA portion we are looking for. Of course, when we are given only $x(n)$ we know neither $\{a_i\}$ nor $\{b_i\}$, M and L being also unknown. The solutions to this problem may not be unique, because of the many different choices of $\{a_i\}$, $\{b_i\}$, M and L, which can be done for a particular case. The problem then have to be solved under some optimization criterion, which in most of the classical approaches is of Least Squares nature. That is to say, it is first assumed that a solution can be anticipated as a set of estimates $\{a_{ki}\}$, $\{b_{ki}\}$, M_e and L_e, and a certain estimate for the original series is built from the model shown in Fig. 2 as $x_k(n)$, with k integer being the index of the estimating stage:

$$x_k(n) = \sum_{i=1}^{k} a_{ki}\, x(n-i) \qquad (5)$$

and then forming the Residual Error Signal (RES) of order k given by:

$$\varepsilon_k(n) = x(n) - x_k(n) \qquad (6)$$

The next step should be the minimization of the energy in the RES by Least Squares methods. In our approach we used Burg's Algorithm [11] to get the RES with increasing k. An original ECG signal, its RES(k=4) and the set of Reflection Coefficients α_k, $1 \le k \le 64$, associated with the extraction process can be seen in Fig. 3, a, b and c.

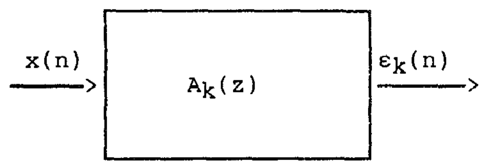

Fig. 2. All-Zero Predictor of order k.

Having then (5) and (6) in mind we would be able of making the following identification:

$$\varepsilon_k(n) = x(n) - \sum_{i=1}^{k} a_{ki}\, x(n-i) =$$

$$= \sum_{i=0}^{L_e} b_{ki}\, \delta(n-i) \qquad (7)$$

and from this it can be easily seen:

$$\varepsilon_k(n) = \begin{cases} b_{ki}; & 0 \le n \le L_e \\ 0; & |n| > L_e \end{cases} \qquad (8)$$

Now the problem of estimating M can be seen as that of choosing the best value of k which meet a given optimization criterion. The criterion we will use is to choose the lowest k which reduces the energy of the RES(k) to its asymptotic tendency. In fact, it is well known that we can reduce this energy E_k to a constant value independent of k when $k \to \infty$. In the case that the original signal being processed $x(n)$ is of ARMA nature (see [2]), being E_k the energy for $\varepsilon_k(n)$ given as:

$$E_k = \sum_{n} \{\varepsilon_k(n)\}^2 \qquad (9)$$

as stated in [2], we should expect that:

$$\lim_{k \to \infty} E_k = E_\infty = \sigma^2 \qquad (10)$$

and from the fact that:

$$E_k = E_{k-1}\,(1-\alpha_k^2) \qquad (11)$$

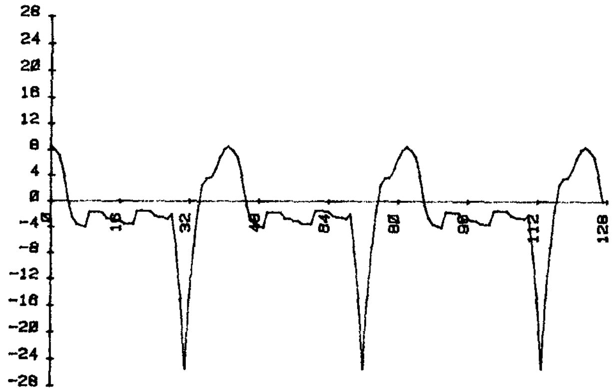

Fig. 3 a. Original Sampled ECG Signal.

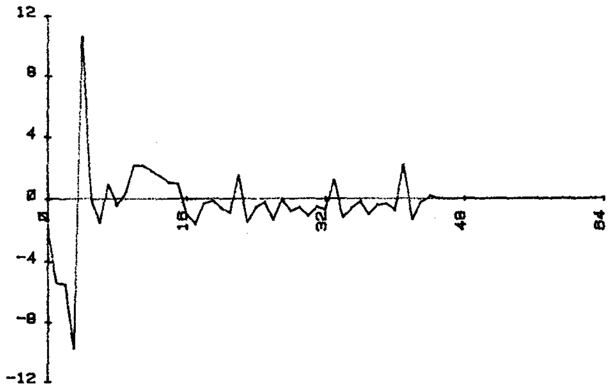

Fig. 3 b. Residual Error Signal (RES) for k = 4.

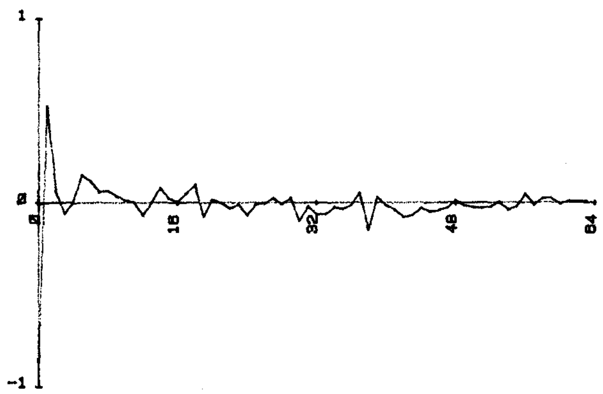

Fig. 3 c. Set of Reflection Coefficients for $1 \le k \le 64$.

it is easy to see that being α_k a real number such that $|\alpha_k|<1$, the evolution of E_k will be monotonically decreasing, depending upon the consecutive values of α_k, as seen for example in Fig. 3. c. This evolution can be represented in its tendency by the Cumulative Energy Factor (CEF):

$$P_k = \prod_{i=1}^{k} (1-\alpha_i^2) \qquad (12)$$

which for the example under study is plotted vs. k in Fig. 4. As it was suggested before, we will determine the optimum value for k just after the bending in the plot of Fig. 4. This value for k (in this case k=4) will be the optimum in the sense that it assures us a near-to-constant value for E_k with the lowest value for k.

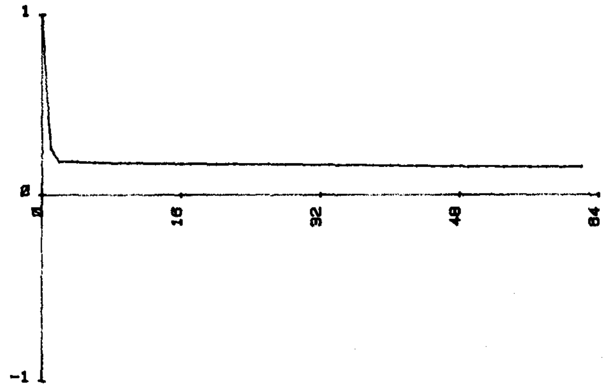

Fig. 4. Evolution of the RES energy with increasing k.

Once we have decided on this basis which should be the best and most economical value for M_e (M_e=k=4) we will be interested in knowing the spectral behavior of the equivalent MA signal, which on this basis should be RES(k=4), or equivalently b_{4i}. This signal was shown in Fig. 3 b., and in Fig. 5 we represent its Discrete Fourier Transform modulus vs. frequency.

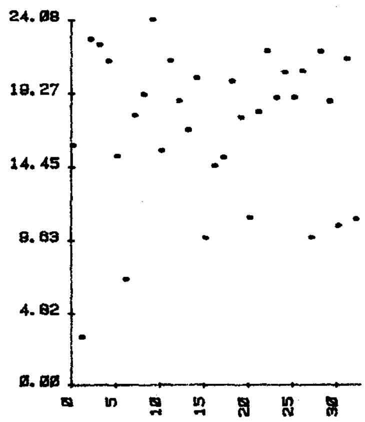

Fig. 5. Modulus of the DFT for RES(k=4).

It can be easily seen how the behavior of the RES4 spectrum is that of an MA signal, having most of the amplitudes around a given level (should be about 19.27 arbitrary units in our case). Well below that mean level up to 7 amplitude minima can be distinguished around samples numbered 1, 6, 15, 20, 27, 30 and 32, revealing the presence of certain zeros in the RES spectrum. If we proceed further on increasing the number of k, these 7 minima become 8 for k=8, and are clearly distinguishable in the RES(k) well up to k≈32, showing the MA nature of RES(k). For higher values of k the spectra become flatter and flatter, and as k reaches 64 (the number of samples in the processed version of x(n)) most of the transmission minima have vanished out and the spectrum can be considered as reasonably flat. On this basis, a new criterion for the determination of L_e can be developed. Let's define $a_{\infty i}$ as:

$$a_{\infty i} = \lim_{k \to \infty} a_{ki} \qquad (13)$$

We should expect this set of coefficients as being the specific series which reduce x(n) to a white signal, that is to say, when substituted in the Spiking Filter of Fig. 2, it should be expected that its output be a given signal RES(∞), with white spectrum. For periodic signals we should expect RES(∞) to be, except for a gain factor G, a "delta function", δ(n), or symbolically:

$$x(n) * a_{\infty}(n) = G\, \delta(n) \qquad (14)$$

where (*) stands for the operation of Discrete Convolution. In such a case $a_{\infty}(n)$ is known as the equivalent "Wiener Filter" [2], or consequently the "Inverse Associated Series" (IAS) of the given signal x(n). Let's define one such IAS for the RES(4), that is, for the error signal in Fig. 3 b. coming out from the fourth order lattice filter whose input is x(n). For practical reasons we will proceed with k=64 filtering steps to obtain an estimate of the IAS of RES(4). In Fig. 6 we represent the DFT modulus for such a signal. It can be seen that there is some degree of correspondence between the amplitude maxima in Fig. 6 and the minima in Fig. 4, showing the process of inversion produced by an exhaustive application of Linear Predictive Methods on RES(4). The main disagreements between both figures seem to be due to sampling windows which do not exactly match the size of the ensambles and to low frequency resolution. Some other facts which have not been taken into account are the influence of truncations in the updated forward and backward error series in the implementation of Burg's Algorithm, and other practical reasons

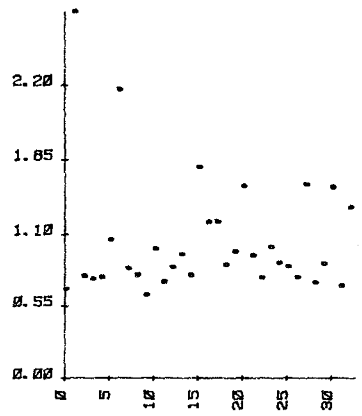

Fig. 6. IAS Spectrum modulus for RES(4) with a_0 = 1.

which play a secondary but no negligible role. Ignoring these second order facts we will concentrate upon the characterization of this IAS(4) as if it were a true AR signal, and as such we will treat it by Linear Predictive methods again. The most important objective in our case is to determine the minimum number of poles which will render the Energy of the IAS(4) to its asymptotic tendency. For such we represent the Cumulative Energy Factor (CEF) associated with the IAS(4) vs. increasing k as we did in Fig. 4 for the RES(4). This plot is shown in Fig. 7.

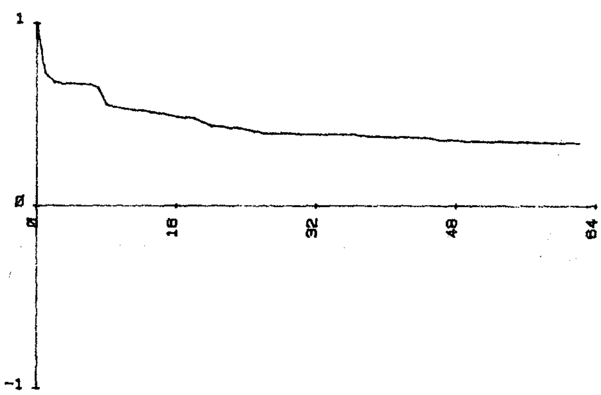

Fig. 7. Cumulative Energy Factor for IAS(4) without a_0 = 1.

The analysis of the IAS(4) evolution with the increasing number of filter sections shows to be quite interesting. A first bending in the CEF plot is detected after 3 filtering sections. This could represent a coarse approach for L_e, but after 8 sections a new and remarkable decrement in the IAS energy shows up. A progressive monotonic but slower decrease still takes place showing little descents after 20 and 26 filtering stages. In the limit the IAS energy seems to reach a stable asymptotic tendency toward a given constant value, as it should be expected. These facts seem to be of crucial importance in the characterization of an ARMA signal by the number of coefficients in its AR and MA sections under a spectral point of view. In our case it should be reasonable to expect that 4 coefficients for the AR part and 8 coefficients for the MA part should be enough to represent the signal with a given accuracy, and in this sense new efforts are being done.

DISCUSSION AND APPLICATIONS

A first remarkable fact in the methods shown which must be pointed out dealing with the relative number of coefficients neccesary to represent the AR and MA parts, is that the number of coefficients to represent each part shows to be independent of the necessary number for the other part. This fact may offer a more general point of view than that in well known approaches [10, 11], in which the number of poles must be higher than the number of zeros (at least one more). Other important facts to be remarked are the use of Linear Predictive methods only, which mean lower processing time and power, well known theories, former developments to benefit from and better understandability. The descriptors for the AR and MA parts could be the respective Direct and Inverse Reflection Coefficients (DRC and IRC) for both the AR and MA parts, and as such a similar lattice structure could be used as a joint model for both sections [13]. In a general sense the algorithms used for the decomposition and the determination of filter orders are economical in their time and memory requirements, although their accuracy and performance in this kind of processing has not yet been evaluated in deep. The suggested approach works better with periodical or quasi-periodical signals, because a preliminary detection and isolation of single-period frames helps greatly in determining the size of the windows to be used. The accuracy of the method depends also upon the spiking properties of the Linear Prediction algorithms in use, being it convenient to reduce the RES to their whitest versions before determining the IAS. This point, although not critical for the determination of filter orders may be crucial in the process of reconstructing the original signals from the compressed coefficients. The methods are primarily designed for their use with stationary or quasi-stationary signals, and more work have to be done to extend them for treating non-stationary cases in an adaptive way. Applications of the current methods should be data compression for storing, transmitting and retrieving large records of ECG signals, like in big hospitals or sanitary departments, and even for the distant tracking of patients with heart diseases. Their application to the recognition of abnormalities in the Electro-Cardiogram and to the design of Diagnosis helps in Vector-Cardiography could be afforded via the multichannel version of Linear Prediction algorithms [14]. The methods in their current version are very useful for the isolation of cardiac rhythms, because of their spiking properties. Their application to other biological signals like Electro-Myograms (EMG) and Electro-Encephalograms (EEG) is also being explored, and the preliminary prospectives look fairly good.

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