# MA Lattice Coefficients with Application to ECG Signal Processing

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Abstract:

The theory is developed for MA (Moving Average) lattice design applicable to PARCOR (PARtial CORrelation) techniques of ECG (Electro-Cardiogram) ARMA signal analysis and synthesis. Discussion is given on the determination and impulse response specification of the MA portion of such filters in lattice form. A means is given for obtanining the reflection coefficients iteratively from this impulse response. Results are given from programs carrying out the developed algorithm for several ECG waveforms.

## 1. Introduction.

In some applications, such ECG simulation, it is of interest to obtain digital transfer functions whose unit pulse response is a given waveform. That is, from a given set of samples it is desired to obtain a digital filter whose unit pulse response is those samples. If the number of samples is equal to the order of the digital filter then this is a simple matter and can be accomplished in several ways, for example by an AR (All pole) filter. However it is usually desirable to use a lower filter degree than the number of samples in which case one is faced with the problem of estimation. This problem has been nicely solved using lattices in the case of estimation of power spectra by the Levinson, Durbin, or Burg theories, [1,2,3]. Because the determination of cascade structures is so nice, using these theories one would like to be able to apply their results to the design of filters to yield given unit responses. However, in attempting to do this one finds that a number of problems are met. Of most importance is that a number of different unit pulse responses may yield the same power spectra. Some of the possible transfer functions may include finite zeros whereas the lattice structures considered are AR in which case all zeros are at infinity. Otherwise put, a given unit pulse response may come from an ARMA filter whereas there is an AR lattice which has the same power spectra; design of a filter using the later may give a unit pulse response which is nowhere close to the data samples desired to be matched. Besides this problem a number of other situations may occur; the process yielding the data to be matched may be time-varying (as in the case of the ECG) or even nonlinear.

## 2. MA filter determination.

Given a set of L (for length) data points h(I), I = 1,...L, one can set up an L-degree MA filter of z-transform transfer function $T_{MA}(z)$ which has its unit pulse response at time i exactly h(i). This is:

$$T_{MA}(z) = \sum_{i=1}^{M} a(i)\, z^{-i} \qquad where\; a(i)=h(i);\; M=L \qquad (1)$$

Likewise, one can solve a set of L linear equations and find an AR filter whose transfer function $T_{AR}(z)$ also yields exactly h(i):

$$T_{AR}(z) = \frac{1}{\sum_{i=1}^{N} d(i)\, z^{-i}}\,; \qquad N = L \qquad (2)$$

If an MA or an AR filter of degree less than L is desired then one may attempt estimation of the signal by either of the filter types. One method of estimating the MA transfer function is to work with the inverse of it, which is an AR transfer function, and use AR estimation techniques on the "inversed data". In order to find the inversed data, call it $h_I(i)$, one notes that:

$$\left\{\sum_{i=1}^{L} h(i)\, z^{-i}\right\}\left\{\sum_{j=0}^{\infty} h_I(j)\, z^{-j}\right\} = 1 \qquad (3)$$

This gives a recursion relation which allows calculation of the $h_I(j)$ from the given h(i). Because of the upper limit on the sum of infinity, one notes that generally more than L terms of $h_I(j)$ are needed to yield the h(i) as inversed data from the $h_I(j)$; typically we have found that 3L terms will suffice but in any event more data samples are often needed that are originally given.

Now we note that if we do an estimation by any means of the data samples via an AR transfer function then we can multiply it by a correcting MA transfer function to improve the response. In other words if we have an N-degree AR filter we can cascade it with an M-degree MA filter and if we properly choose the MA filter we should be able to much improve the situation. Indeed if we choose M = L-N then we should be able to exactly achieve the given data samples as the unit pulse response of the cascade. Indeed such is possible. One method suggested in [4] makes use of the Steiglitz-McBride recursion, which essentially consists in performing an ARMA synthesis by the minimization of the total mean square error between the desired and the synthesized data series. For that here we assume that we have a lattice filter for the AR structure, which we synthesize following the traditional techniques of linear predictive filtering as exposed for example in [5]. As such we will try to obtain a lattice modeling filter also for the MA section of the whole structure. A cascade of the two would of course yield a filter of degree M + N, but as we discuss below, this can be reduced to a cascade filter of lower degree in many cases.

To achieve our goal we revert to the above equations, assuming that an AR transfer function of degree N is given in the form of (2) and that an overall system of unif pulse response h(i) is desired via cascade with a correcting MA filter of the form of (1). Then we desire:

$$\frac{\sum_{i=1}^{M} a(i)\, z^{-i}}{\sum_{j=1}^{N} d(j)\, z^{-j}} = \sum_{k=1}^{L} h(k)\, z^{-k} \tag{4}$$

for an exact fit. For an estimation L is replaced by:

$$L' \langle L$$

These equations are linear in the a(i) and are readily solved.

## 3. MA Lattice.

For the m-th stage of a Prediction Error Lattice, as shown in Fig. 1 we find that the input and output forward signal z-transforms are related by:

$$F_m(z) = F_{m-1}(z) - k_m z^{-m} F_{m-1}(z^{-1}) \tag{5}$$

with:

$$F_0(z) = 1$$

$$F_M(z) = T_{MA}(z)$$

and:

$$B_m(z) = z^{-m} F_m(z^{-1})$$

Here $k_m$ is the reflection coefficient of the lattice section, $B_m(z)$ is the reverse signal z-transform at the output of the m-th lattice section, and $T_{MA}(z)$ is the transfer function of an M stage filter that is the cascade of M lattice sections. Equation (5) allows us to use recursion in finding the lattice sections comprising an M stage lattice MA filter.

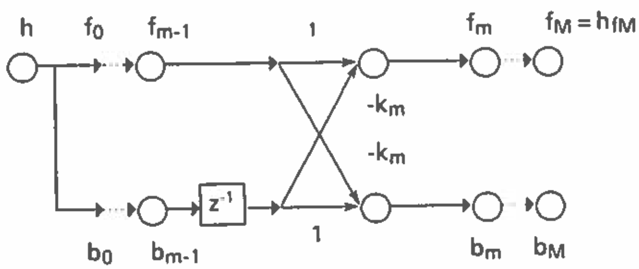


Fig. 1a). Prediction Error Lattice

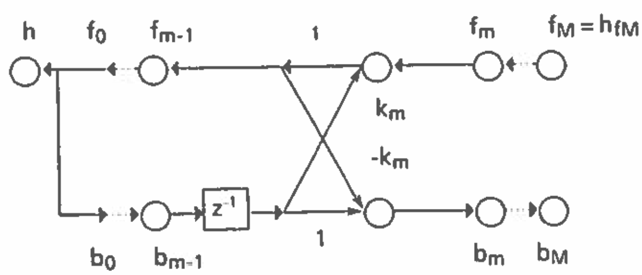


Fig. 1b). Synthesis Lattice

First we note that if $T_{MA}(z)$ is of degree M in $z^{-1}$ then the coefficient of the M-th power of $z^{-1}$ is the negative of the M-th stage reflection coefficient, that is $a_N = -k_M$. Assuming that we have found the reflection coefficients and input forward wave signals for the m-th to M-th stages, then eq. (5) allows us to go to the m-1st stage. Thus, on replacing z by $z^{-1}$ in (5), multiplying by $k_m z^{-m}$, and adding this to (5), we find:

$$F_{m-1}(z) = \frac{F_m(z) + k_m z^{-m} F_m(z^{-1})}{1 - k_m^2} \tag{6}$$

with $k_{m-1}$ being the negative coefficient of the highest power of $z^{-1}$ in $F_{m-1}(z)$. We note that (6) is only valid for reflection coefficients which are not unity in absolute value. If $k_m^2 = 1$ occurs then some symmetry conditions hold for the structure to be realizable in lattice form. In fact, when this singularity condition appears, we can get a special expression for (5) as shown:

$$F_m(z) = 1 - \sum_{j=1}^{m-1} \left[ k_j^{m-1} - k_{m-j}^{m-1} \right] z^{-j} - z^{-m} \tag{7}$$

and this can be interpreted in the sense that:

$$k_j^m = k_j^{m-1} - k_{m-k}^{m-1} \; ; \; with \;\; 1 \le j \le m-1 \tag{8}$$

or symmetricaly:

$$k_{m-j}^m = k_{m-j}^{m-1} - k_j^{m-1} \tag{9}$$

Then if at any step $k_m = \pm 1$ the realizability constraint:

$$k_j^m = \pm k_{m-j}^m \quad ; \quad 1 \le j \le m-1 \tag{10}$$

must hold at that step, and if not, presumably no lattice exists, and we can choose:

$$k_j^{p-1} = \frac{1}{2} k_j^p \; ; \; 1 \le j \le p-1 \; ; \; 1 \le p \le m-1 \tag{11}$$

If these symmetry conditions do not hold then no lattice exists for the given data, though often simple modifications in the data do allow for a lattice to result.

## 4. Some results.

In Fig. 2a) we can see a fragment of a typical ECG waveform and its associated power spectrum can be seen in Fig. 2b).

This signal, after an AR lattice filtering is reduced to the Residual Error Signal (RES) which can be seen in Fig. 3a). The associated power spectrum for the RES is shown in Fig. 3b).

From the direct comparison between both we notice the whitening properties of AR lattice filters, and also it can be easily seen that whilst the original ECG signal showed a mixed pole-zero behavior in its spectrum, the main characteristics of the RES are those of an all-zero spectrum, showing several important lags in it, as those pointed out with a circle. The coefficient series for the whitening filter is also presented in Fig. 4.

Now, when we build the synthesis filter following (6), and introduce the RES signal through it, we are able of reconstructing the original EGC signal, as seen in Fig. 5. But for this we need to use L filter coefficients and also L RES data, which mean 2L data points, and this is not the best choice. Instead that, some optimization criteria can be used to reduce the length of the inverse filter coefficients, and the RES data needed. On one hand, an optimization criterion on the evolution of the energy associated with the RES data can be used to establish the order of N, as suggested for example in [5, pp. 41-42]. The simplest method to be implemented relies on the ratio of the RES energy after each iteration in the AR lattice filter.

This energy can be defined as:

$$E_m = \sum_{i=1}^{L} \left| h_{fm}^2(i) \right| \tag{12}$$

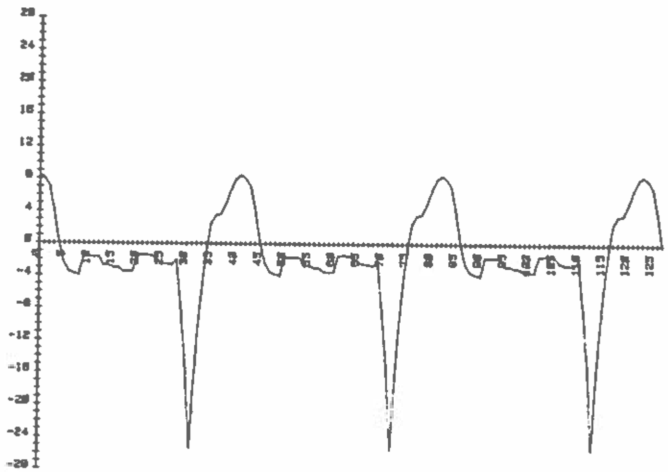

Fig. 2a) Original ECG Signal (Arbitrary Units.)

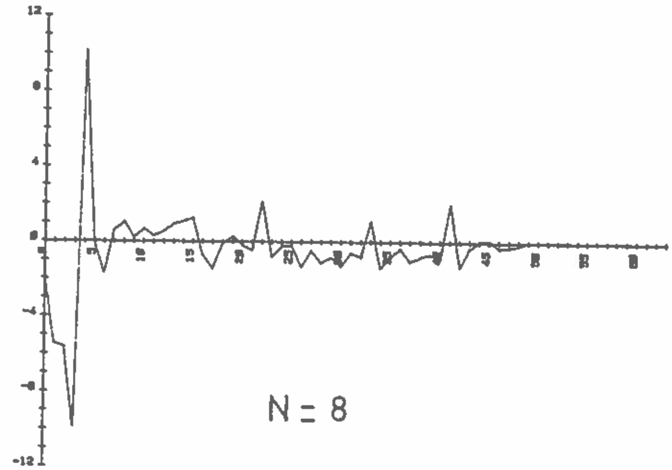


Fig. 3a) RES time data

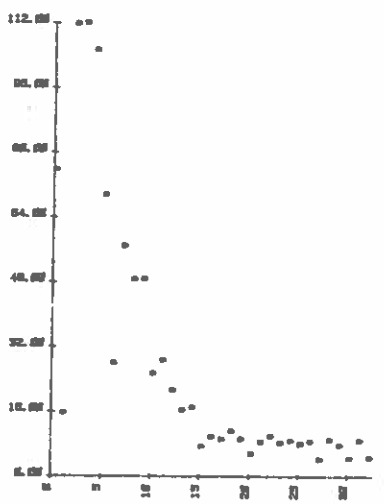

Fig. 2b) Associated ECG Spectrum.

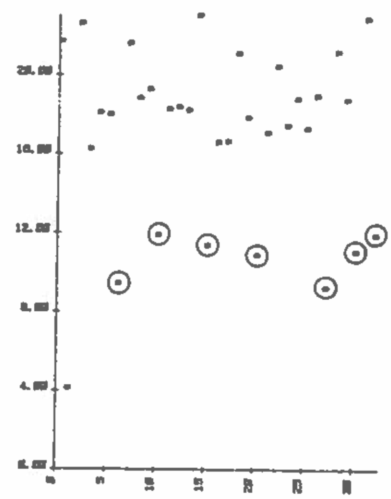

Fig. 3b) RES power spectrum

The ratio between two succesive energy calculations can be shown to yield [5]:

$$\frac{E_m}{E_{m-1}} = 1 - k^2_{m-1} \qquad (13)$$

When this ratio approaches the unity, that is when $k_m$ approaches to zero for several iterations we can conclude that the process has reached to an end, and as such we stop the calculations and assume the correct value of m for the order of the AR filter. In any circumstance, we have to assume that in putting such a criterion into practice we are introducing a certain degree of truncation in the whitening series according with the evolution of the energy in the RES after each step of calculation.

On the other hand, to determine the order of the MA filter a similar criterion can be used. A truncation procedure of certain degree must be introduced in the RES in order for it to serve as an optimal representation of the MA filter impulse response, having in mind that the optimal approach in the true sense should be to take the full length equal to L for both series. In this case, we can try with a truncated version for both the whitening series and the RES data. As it should be expected, when operating the synthesis filter in Fig. 1 with the truncated versions for d(i) (or equivalently $k_m$), and of $h_{fm}$ (i) we will obtain a reconstruction for the ECG signal, which will differ to certain degree with the original data as given in Fig. 5. Calling this reconstructed signal $h_r(i)$ we will be able of calculating the mean square error in the reconstruction, or in other words the energy in the error, which can be evaluated as:

$$P^r_{MN} = \sum_{i=1}^{L} \left| h(i) - h_r(i) \right|^2 \qquad (14)$$

An useful measure for the error involved in the truncations can be given by:

$$SN_{rel} = 10\ lg_{10} \frac{P^r_{MN}}{P_s} \qquad (15)$$

where $P_s$ is the associated energy of the original ECG signal. In this sense the measurement obtained through the application of (15) can be seen as the natural signal-to-noise relationship in the process. In any way the shown criterion has mainly "a posteriori" utility in evaluating the degree of fitness in the methods shown.

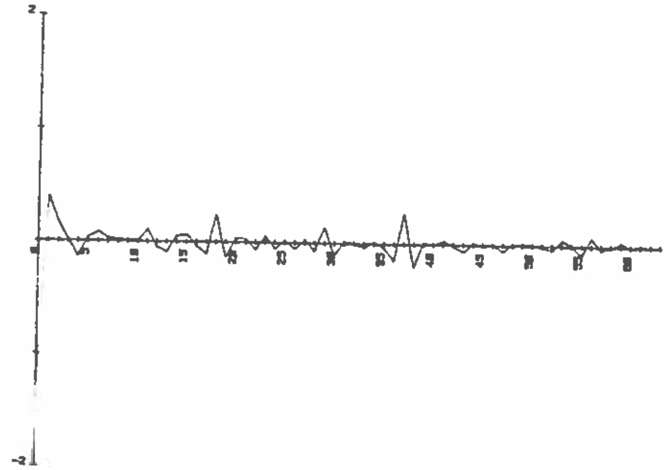

Fig. 4. Whitening series.

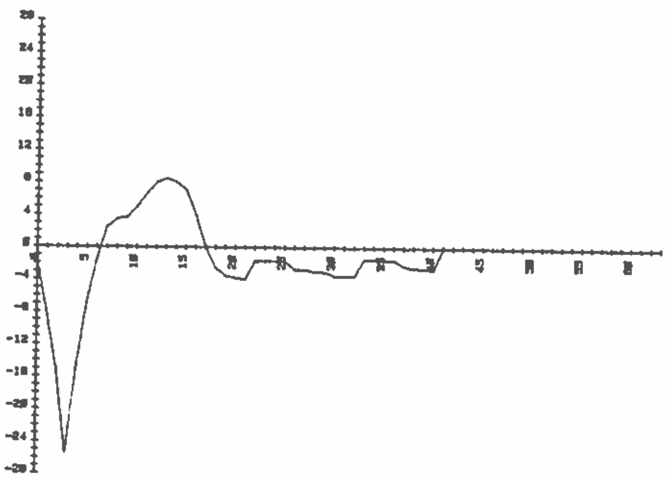

Fig. 5. Reconstructed ECG Series.

## 5. Discussion.

Some things have to be said on the potential usefulness of the former methods for ECG signal processing, storing and identification. First of all the method can reconstruct a given signal from its decomposition with an acceptable degree of accuracy. This can be easily established and calculated at the time ECG signals are converted into the AR and MA coefficients for the model. Practical decompositions show that data compresion ratios of 1:5 can be easily achieved, resulting in great savings in storage and transmission requirements when these signals have to be handled. This can be a very important factor when clinical records have to be kept or when mobile life-support systems have to be linked to main data collecting and analyzing facilities, such those in central administrations or big hospitals. A second problem regarding the meaning of the coefficients in the AR and MA sections remains still unresolved. The next steps should be given in this direction in order to properly identify common ECG features [6] and posibly use the resuts in automatic diagnosis assistance helps or in portable early-warning systems for people with high heart dissease risk. A realization in cascade form of minimum degree has not been discused here in depth although it seems to be one of the most important problems in the caracterization of impulsive responses through ARMA models. In this sense the ideas shown theoreticaly by Dewilde et al. [7] seem to offer a good method to afford this task. On the other hand there are no guarantees in any sense which can assure us the stablility of the synthesized filters, although, in many instances, when only a finite ammount of time is considered this may no be of great concern for the digital case approach.


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