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An Example of the Continuation Method of Solving Semistate Equations

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Abstract

The continuation method of solving semistate equations is applied to the example of a solid-state diode rectifier circuit. The example points out a number of features of the continuation method and particularly its ease of use for the solution of practical circuits via a PC.

I. Introduction

The continuation method is well recognized to be a useful tool in solving nonlinear nondynamic equations, see for example [1] and [2]. Because semistate equations will generally have nondynamic variables mixed in with dynamic ones, we previously introduced the continuation method as a possible way to solve semistate equations [3]. Here we present a practical solid-state diode example to show that the method really works and that it is simple enough to easily implement on a personal computer.

In section II we review the method and in section III we introduce the example with resulting continuation method equations. In section IV we present the results and the program used to solve the circuit.

II. Review of the Method

We assume that the canonical semistate equations

$$\mathcal{A}\dot{x} + \mathcal{B}(x,t) = \mathcal{B}u \qquad (1a)$$
$$y = \mathcal{C}u \qquad (1b)$$

have the non-output equations brought into the form

$$\dot{x}_1 + \mathcal{B}_1(x,t) = \mathcal{B}_1 u \qquad (2a)$$
$$\mathcal{B}_2(x,t) = \mathcal{B}_2 u \qquad (2b)$$

Here the n-vector x is the semistate and is partitioned into two subvectors, an m-vector x_1 and an (n-m)-vector x_2 (u is the input and y is the output); we assume that $n>m>0$ for convenience in the treatment.

Introducing a parameter λ and constant matrices B_{11}, B_{21}, and B_{22}, the latter being nonsingular, we write our equations in the following continuation form

$$\dot{x}_1 + \lambda[\mathcal{B}_1(x,t) - B_{11}x_1] + B_{11}x_1 = \mathcal{B}_1 u \qquad (3a)$$

$$\lambda[\mathcal{B}_2(x,t)-B_{21}x_1-B_{22}x_2]+B_{22}x_2 = \mathcal{B}_2 u-B_{21}x_1 \qquad (3b)$$

For $\lambda = 1$ eqs. (3) reduce to (2) while for $\lambda = 0$ these are readily solved; we extend the solution $x(t,\lambda)$ of (3) from $x(t,0)$ to $x(t,1)$ while making convenient choices for the B_{ij} to simplify as much as possible the work involved. At $\lambda = 0$ we solve $\dot{x}_1+B_{11}x_1= \mathcal{B}_1 u$ over a small interval of time, $t-t_o$, subject to the initial conditions $x_1(t_o)$. Then, using this solution, $x_1(t,0)$, (3b) is solved by

$$x_2(t,0) = B_{22}^{-1}[\mathcal{B}_2 u - B_{21}x_1(t,0)] \qquad (4)$$

Via the normal continuation method we next solve (3b) over a small increment Δ in λ, $\lambda\in[0,\Delta]$, for x_1 fixed at $x_1(t,0)$. This gives a value for $x_2(t,\Delta)$ which we next use in (3a) with $\lambda=\Delta$ to solve for x_1 by any standard technique, thus giving a value for $x_1(t,\Delta)$. This cycle is repeated, the next cycle giving $x_2(t,2\Delta)$ by the standard continuation method on (3b) over $\lambda\in[\Delta,2\Delta]$ for x_1 chosen as the previously found $x_1(t,\Delta)$; $x_1(t,2\Delta)$ is then found from (3a) with $\lambda=2\Delta$ and $x_2=x_2(t,2\Delta)$, etc.. Choosing $\Delta=1/N$ for N a large enough integer, this cycle is repeated N times such that the final iteration has $\lambda=1=N\Delta$, in which case the solution approximates that of (2) giving the desired semistate.

For the continuation method we solve the nondynamic equation

$$C(x_2,t,\lambda) = 0 \qquad (5a)$$

where C is the "continuation" function

$$C(x_2,t,\lambda) = \lambda[\mathcal{B}_2(\begin{bmatrix}x_1\\x_2\end{bmatrix},t)-B_{21}x_1-B_{22}x_2]+B_{22}x_2-\mathcal{B}_2 u+B_{21}x_1 \qquad (5b)$$

Equation (5a) is to be solved at the ith step over $[i-1]\Delta \leq\lambda\leq i\Delta$ subject to $x(t,[i-1]\Delta)$ given while x_1 is held fixed at $x_1(t,[i-1]\Delta)$; this is to be done for $i=1,\ldots,N=1/\Delta$ with $x_1(0)$ given [$x_1(t,[i-1]\Delta)$ being found from (3a) as described above]. The continuation method forms $dC/d\lambda = 0$, by forming partial derivaties and assuming that t and λ, are independent to give

$$\frac{\partial x_2}{\partial\lambda} = -\left[\frac{\partial C}{\partial x_2}\right]^{-1}\cdot\frac{\partial C}{\partial\lambda} =$$

$$-\left[\lambda\frac{\partial\mathcal{B}_2}{\partial x_2} + (1-\lambda)B_{22}\right]^{-1}\cdot[\mathcal{B}_2-B_{21}x_1-B_{22}x_2] \qquad (6)$$

Equation (6) is the differential equation in λ to be solved for x_2. As for the ODE for x_1, (6) can be solved for x_2 by any standard technique over the intervals $[i-1]\Delta \leq\lambda\leq i\Delta$ with given $x_1(t,[i-1]\Delta)$ and initial $x_2(t,[i-1]\Delta)$ to yield $x_2(t,\lambda)$ over $[i-1]\Delta \leq\lambda\leq i\Delta$ from which $x_2(t,i\Delta)$ is found. The whole process is then repeated by incrementing in t, the previous time now serving as the new t_o.

Although this procedure looks rather complicated it is rather easy to carry out. Nevertheless, some simplifications can be carried out to make it even more tractable. For one, if the increments in t are made small enough so that x_1 does not significantly vary during the increment in t then $\lambda=1$ can be used in (3a) with the only incrementation of λ occuring in (3b). If again small enough increments are made in λ, that is if Δ is small enough, then (6) can be assumed rational in λ since the x terms on the right can be taken constant in λ over the interval; in many such cases integral tables can then be used to get accurate solutions for x_2 via integration at (6) (again such occurs in our example

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below).

III. Diode Rectifier Circuit

Consider the solid-state-diode rectifier circuit of Fig. 1a) with the circuit graph chosen as in Fig. 1b) with branches 1 and 2 taken to form the tree. The solid-state diode is taken to be described by

$$i_2 = I_s[\exp(v_2/V_T) - 1] \tag{7}$$

where I_s is the reverse saturation current and V_T is the thermal voltage (25 millivolts at room temperature). For our normalizations we note that if the input has a maximum of its absolute value of I_I and if the input were a step function of this value then the output will eventually reach a maximum of RI_I. Therefore we normalize the system by choosing

$$x_1 = v_1/(RI_I) \tag{8a}$$
$$x_2 = v_2/(RI_I) \tag{8b}$$
$$x_3 = i_3/I_I \tag{8c}$$
$$f(x_2) = I_s[\exp(RI_Ix_2/V_T) - 1]/I_I \tag{8d}$$
$$t = (\text{true time})/(RC) \tag{8e}$$
$$u = i_I/I_I \tag{8f}$$
$$y = x_1 \tag{8g}$$

The canonical normalized semistate equations are then

$$\begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix}\dot{x} + \begin{bmatrix}0&0&-1\\0&0&-1\\1&1&1\end{bmatrix}x + \begin{bmatrix}0\\f(x_2)\\0\end{bmatrix} = \begin{bmatrix}0\\0\\1\end{bmatrix}u \tag{9a}$$

$$y = [1,\ 0,\ 0]x \tag{9b}$$

To move into the continuation form we choose the B_{ij} to give as many zeroes as possible. Thus, we choose

$$B_{11} = 0,\quad B_{21} = \begin{bmatrix}0\\1\end{bmatrix},\quad B_{22} = \begin{bmatrix}0&-1\\1&1\end{bmatrix} \tag{10}$$

The continuation form of the semistate equations then becomes

$$\dot{x}_1 + \lambda[-x_3] = 0 \tag{11a}$$

$$\lambda\begin{bmatrix}f(x_2)\\0\end{bmatrix} + \begin{bmatrix}0&-1\\1&1\end{bmatrix}\begin{bmatrix}x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\1\end{bmatrix}u + \begin{bmatrix}0\\-1\end{bmatrix}x_1 \tag{11b}$$

At $\lambda = 0$ this is

$$\dot{x}_1 = 0 \tag{12a}$$

$$\begin{bmatrix}x_2\\x_3\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix}u + \begin{bmatrix}-1\\0\end{bmatrix}x_1 \tag{12b}$$

Upon defining at initial time t_o (which is reset after each pass through the continuation cycle; our first value for t_o was generally taken as zero)

$$K_o = x_1(t_o,0) = x_1(t_o,1) = x_1(t_o) \tag{13a}$$

we have for $x(t,\lambda)$ at $\lambda = 0$

$$\begin{bmatrix}x_1(t,0)\\x_2(t,0)\\x_3(t,0)\end{bmatrix} = \begin{bmatrix}K_o\\u(t)-K_o\\0\end{bmatrix} \tag{13b}$$

For equation (6) we find, on setting

$$a = RI_I/V_T,\quad b \equiv I_s/I_I \tag{14a}$$

$$\frac{\partial}{\partial\lambda}\begin{bmatrix}x_2(t,\lambda)\\x_3(t,\lambda)\end{bmatrix} = b\{[1-\exp(ax_2)]/[1+\lambda ab\exp(ax_2)]\}\begin{bmatrix}1\\-1\end{bmatrix} \tag{14b}$$

Assuming Δ is chosen small enough that x_2 is constant for λ in any interval of length Δ, (14) integrates to

$$\begin{bmatrix}x_2(t,i\Delta)\\x_3(t,i\Delta)\end{bmatrix} = \begin{bmatrix}x_2(t,[i-1]\Delta)\\x_3(t,[i-1]\Delta)\end{bmatrix} + L\begin{bmatrix}1\\-1\end{bmatrix} \tag{15a}$$

where, with

$$G(x_2) = \exp(-ax_2) \tag{15b}$$

we set

$$L = (1/a)[1-G(x_2)]\ln\{[(i-1)\Delta ab+G(x_2)]/[i\Delta ab+G(x_2)]\} \tag{15c}$$

In some cases the logarithm will underflow on the computer (underflow occurs whenever $[i\Delta ab+G]>10^{+8}$ in the IBM PC BASIC used in our situation) in which cases an expansion for small arguments of the logarithm was used in the programs at (15c) [that is $\ln(1+x) \approx x$]. Overflow can also occur in which case the circuit should be checked to see if the circuit components have reasonable values to keep the diode current below burnout range.

For $x_1(t,\lambda)$ we need to solve the differential equation (11a). Using the trapezoidal rule this gives

$$x_1(t,i\Delta) = x_1(t_o,i\Delta) + i\Delta[x_3(t,i\Delta)+x_3(t_o,i\Delta)](t-t_o)/2 \tag{16a}$$

In solving this equation one needs to store both $x_1(t_o,i\Delta)$ and $x_3(t_o,i\Delta)$ for all i at the previous time, t_o; in one of our programs we did this via dimensioning arrays. However, practically we find that changes only occur in the fourth significant digit if we choose small increments in t, that is, $t-t_o$ small enough, set $\lambda=1$ and integrate x_3 directly. That is, using the trapezoidal rule again, we can replace (16a) by the much simpler equation

$$x_1(t) = x_1(t_o) + [x_3(t,1)+x_3(t_o)](t-t_o)/2 \tag{16b}$$

IV. Program and Results

In Fig. 2 we give a flow chart of the algorithm used to implement the continuation technique presented above for the circuit of Fig. 1. This was implemented in the BASIC routine, given in Fig. 3, and run with various inputs and choices of parameters. Figure 4 shows results for the choice R=10 kilohms, C=10 microfarads, I_s=1 microamp, V_T= 25 millivolts, and the input a sine wave of peak I_I=0.1 milliamperes and period 0.1 second. The rectifying nature of the circuit is clearly shown by the change in "time constant" which occurs at the peak of the input (which forces a back bias on the diode). It should be noted that R was purposely chosen large such that some decay in the output could be seen on the printout during the times when the diode is back biased. If R=10 is chosen, with I_I raised to keep RI_I to one volt, then the decay between cycles shows up in the calculations but is too small to show up on the plot. It should also be noted that N=10 was used in the program; using N=100 only improved calculations in the fourth to seventh significant digit and slows down the program by a factor of ten. However, only when N is below 5 do errors of any consequence seem to appear.

By eliminating x_1 and x_3 in the semistate equations a state-variable, first order differential, equation can be obtained, this being

$$\dot{x}_2 = b[G(x_2)-1]/[G(x_2)+ab] \tag{17}$$

where G and a & b are as above. Initial conditions must be found for this, for example in the step function case by solving $x_2-1-(I_I/I_s)+\exp(RI_sx_2/V_T)=0$, and then standard routines can be attempted to solve (17). In doing this we found for example that the fourth order Runge-Kutta method failed completely for a number of element value choices for which the continuation method proceeded without hitch. Since the continuation method given here rapidly yields results on a standard IBM PC, we feel this example shows that the method can be of considerable importance for practical nonlinear circuit analysis.

References

[1] H. Wacker, Editor, "Continuation Methods," Academic Press, NY, 1978. See particularly J. C. Alexander, "The Topological Theory of an Embedding Model," pp. 37-68.

[2] K.-S. Chao and R. Saeks, "Continuation Methods in Circuit Analysis," Proceedings of the IEEE, Vol. 65, No. 8, August 1977, pp. 1187 - 1194.

[3] B. Dziurla and R. W. Newcomb, "A Continuation-Type Method for Solving Semistate Equations," MTNS, Israel, June 1983.

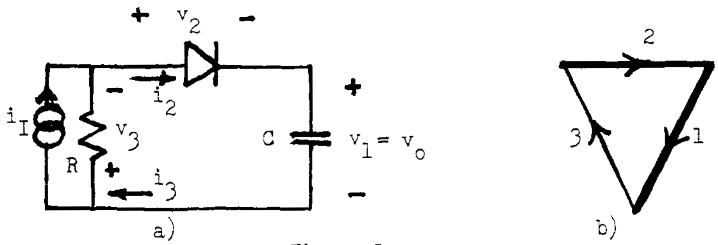

Figure 1
Circuit a) and Graph b)

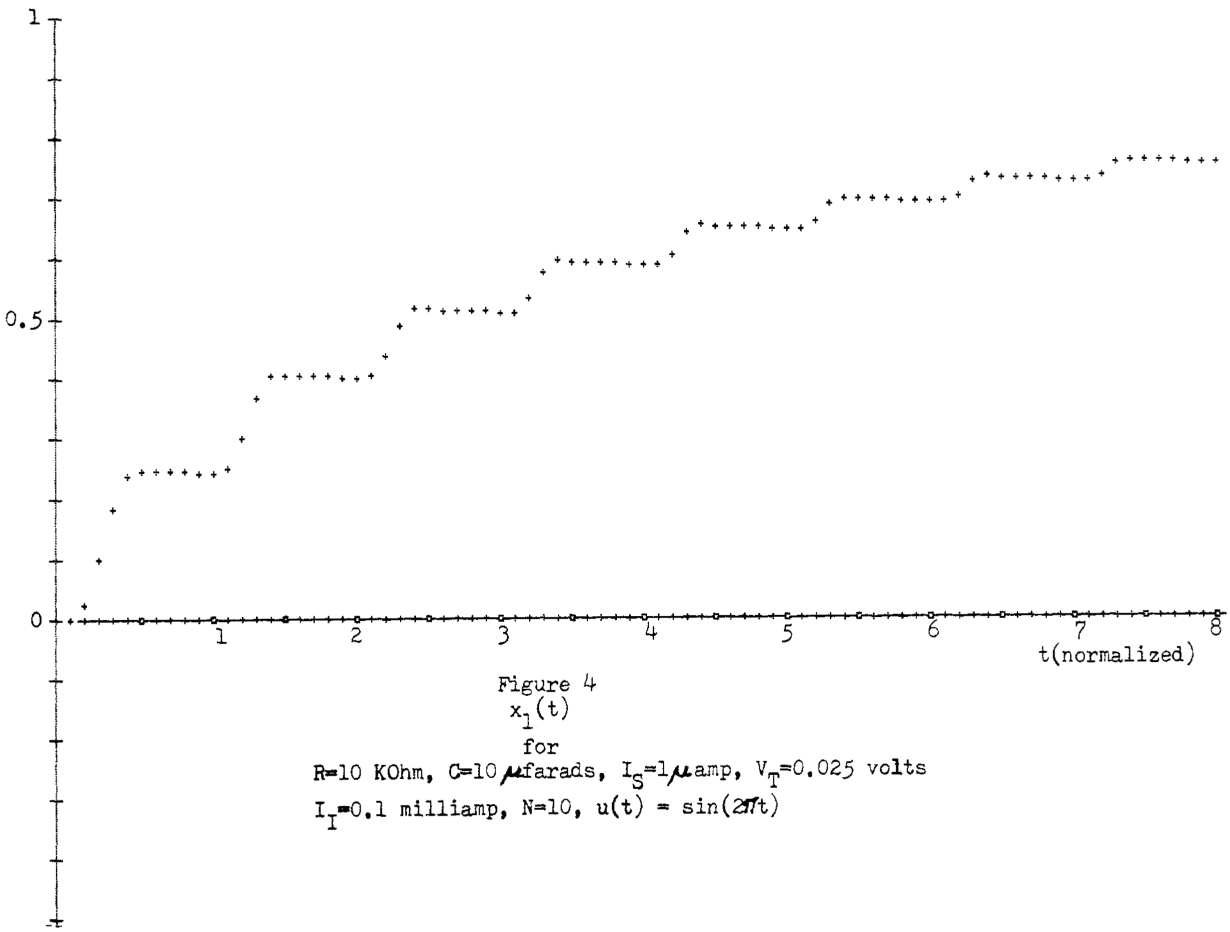

Figure 4
$x_1(t)$
for
R=10 KOhm, C=10 μfarads, I_S=1 μamp, V_T=0.025 volts
I_I=0.1 milliamp, N=10, u(t) = sin(2πt)

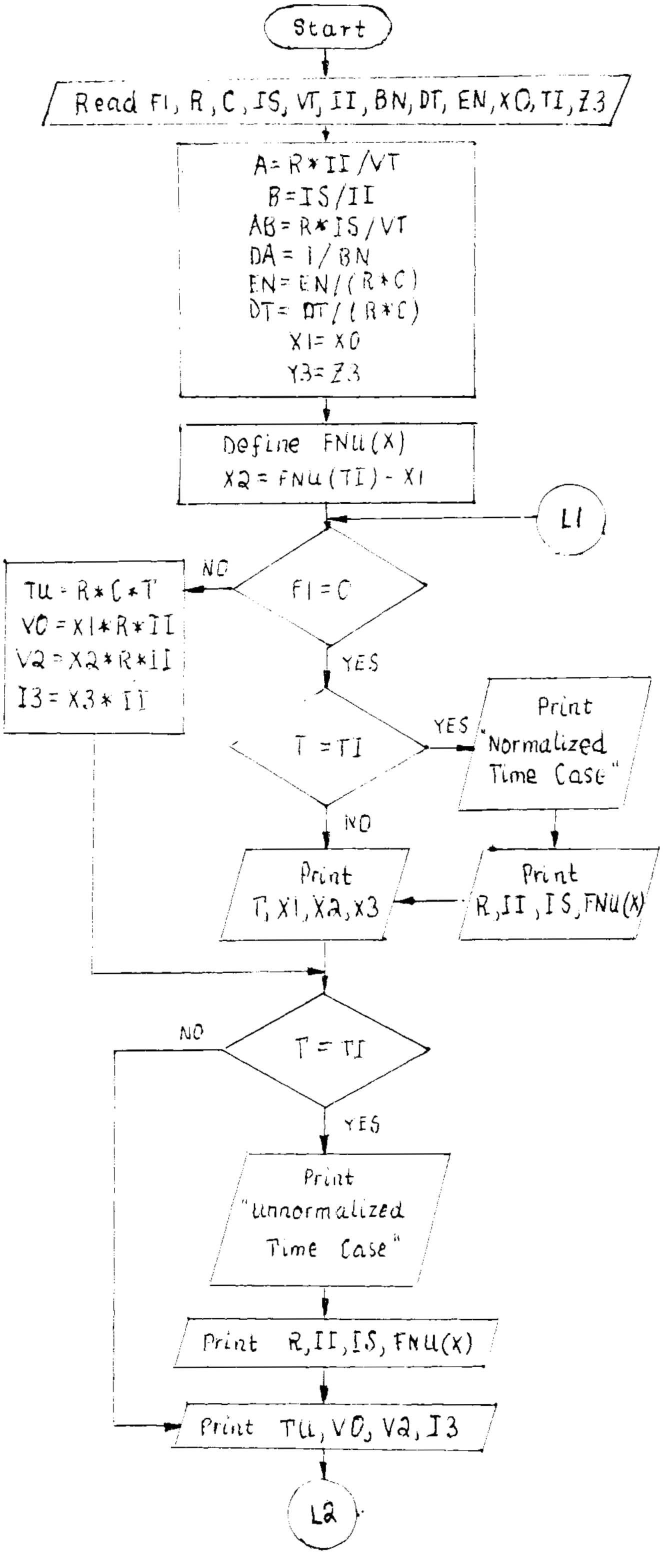

Figure 2

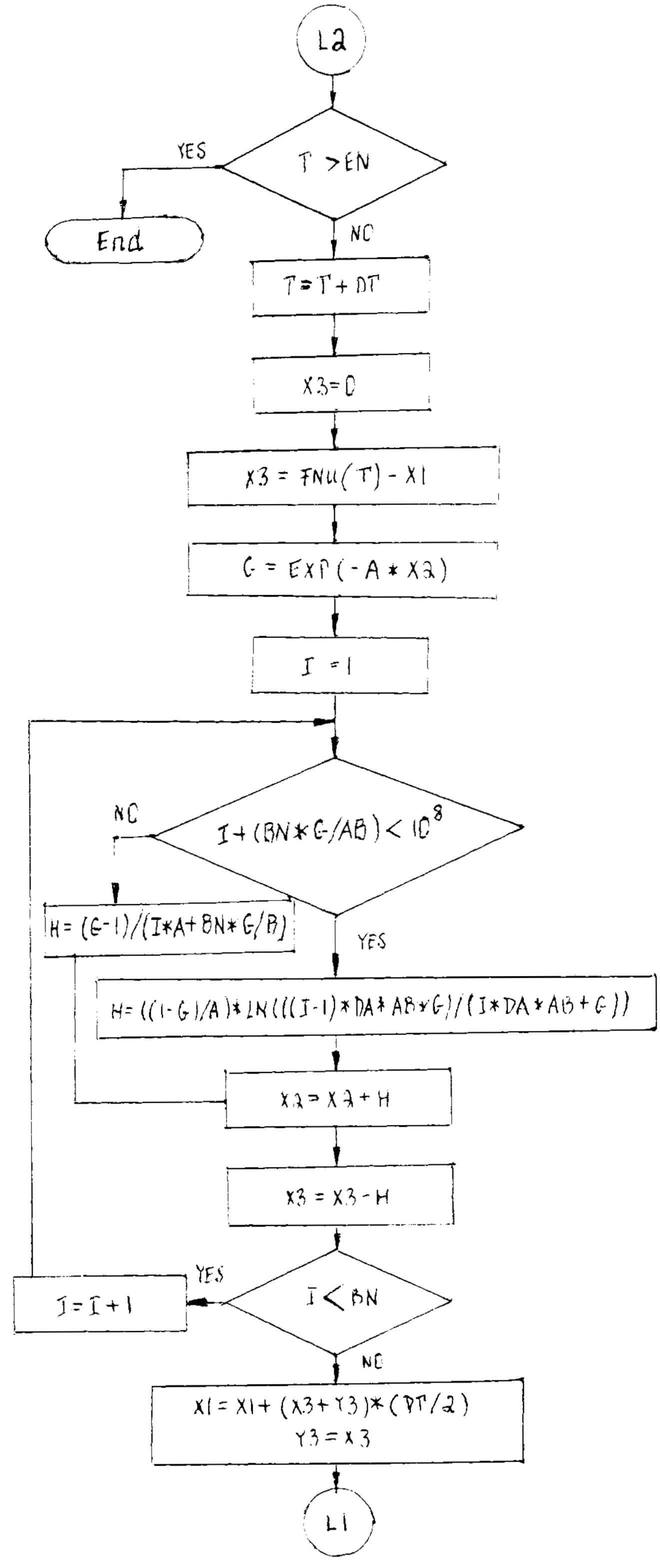

Figure 2, Continued

Figure 3

```
100 REM SEMICONT.CAC   8/19/84B
105 REM PROGRAM TO CALCULATE OUTPUT FOR DIODE CIRCUIT USING CONTINUATION METHOD
        ON THE SEMISTATE EQUATIONS
106 REM F1=1=UNNORMALIZED TIME
107 REM BN=NUMBER OF LAMBDA VARIATIONS, DT=INCREMENT IN TIME (UNNORMALIZED AT
        READ, NORMALIZED AT 160)
108 REM X1, X2, X3 = NORMALIZED SEMISTATES, X0=X1(0), Z3=X3(0)
109 REM TI=INITIAL NORMALIZED TIME, TU=UNNORMALIZED TIME, II=INPUT MAGNITUDE,
        EN=END TIME (UNNORMALIZED AT READ, NORMALIZED AT 150)
110 READ F1, R, C, IS, VT, II, BN, DT, EN, X0, TI, Z3
120 DATA 0, 1E+4, 1E-5, 1E-6, 0.025, 1E-4, 10, 1E-3, 1E-0, 0, 0, 0
130 A=R*II/VT: B=IS/II: AB=R*IS/VT 'CONSTANTS, AB=A*B
140 DA=1/BN 'DELTA = CHANGE IN LAMBDA
150 EN=EN/(R*C) 'NORMALIZATION
160 DT=DT/(R*C) 'NORMALIZATION
170 X1=X0
180 Y3=Z3
185 M=0 'COUNTER FOR NUMBER OF PASSES THROUGH CALCULATION FOR PRINTING
190 PI=3.141593: DEF FNU(X)=SIN(2*PI*X): LET F$ = "SIN(2PIX)"
195 X2=FNU(TI)-X1
198 GOTO 340
200 X3=0
210 LET X2=FNU(T)-X1
220 G=EXP(-A*X2)
230 FOR I=1 TO BN
250    IF (I+(BN*G/AB))<1E+08 THEN LET
          H=((1-G)/A)*LOG(((I-1)*DA*AB+G)/(I*DA*AB+G)) ELSE
            LET H=(G-1)/(I*A+(BN*G/B))
260     X2=X2+H
270     X3=X3-H
310 NEXT I
320 X1=X1+(X3+Y3)*(DT/2)
330 Y3=X3
340 IF F1=0 THEN GOTO 435 'TO PRINT FOR NORMALIZED TIME
350 TU=R*C*T 'DENORMALIZATION OF TIME
360 V0=X1*R*II 'DENORMALIZATION OF FIRST SEMISTATE
370 V2=X2*R*II 'DENORMALIZATION OF SECOND SEMISTATE
380 I3=X3*II 'DENORMALIZATION OF THIRD SEMISTATE
384 IF T=TI THEN PRINT "UNNORMALIZED TIME CASE" 'PRINT LABEL ON FIRST PASS
385 IF T=TI THEN LPRINT "UNNORMALIZED TIME CASE"
386 IF T=TI THEN PRINT "R=";R, "II=";II, "IS=";IS, "FN(X) = ";F$
387 IF T=TI THEN LPRINT "R=";R, "II=";II, "IS=";IS, "FN(X) = ";F$
390 PRINT "TIME=";TU, "V0=";V0, "V2=";V2, "I3=";I3
400 LPRINT "TIME=";TU, "V0=";V0, "V2=";V2, "I3=";I3
430 GOTO 500
435 S=10: Q=M MOD (S): M=M+1 'PRINT EVERY 10TH CALCULATION, UPDATE PASS COUNTER
436 IF Q=0 THEN LET F3=1 ELSE LET F3=0 'FLAG FOR GOING TO PRINT ROUTINE
444 IF T=TI THEN PRINT "NORMALIZED TIME CASE" 'PRINT LABEL ON FIRST PASS
445 IF T=TI THEN LPRINT "NORMALIZED TIME CASE"
446 IF T=TI THEN PRINT "R=";R, "II=";II, "IS=";IS, "FN(X) = ";F$, "BN=";BN
447 IF T=TI THEN LPRINT "R=";R, "II=";II, "IS=";IS, "FN(X) = ";F$, "BN=";BN
448 IF T=TI THEN PRINT
449 IF T=TI THEN LPRINT
450 PRINT "T=";T, "X1=";X1, "X2=";X2, "X3=";X3
460 IF F3=1 THEN LPRINT "T=";T, "X1=";X1, "X2=";X2, "X3=";X3
500 IF T>EN THEN GOTO 550 ELSE T=T+DT
520 GOTO 200
550 END
```

Program Output of Data Plotted in Figure 4

```
NORMALIZED TIME CASE
R= 10000       II= .0001     IS= .000001   FN(X) = SIN(2PIX)              BN= 10

T= 0           X1= 0         X2= 0         X3= 0
T= .1          X1= 2.755257E-02            X2= 2.290731E-02               X3=
 .5425142
T= .2          X1= 9.941237E-02            X2= 2.290731E-02               X3=
 .8370418
T= .3          X1= .1821362  X2= 2.290731E-02             X3= .7536768
T= .3999999    X1= .2385934  X2= 2.290749E-02             X3= .329852
T= .4999998    X1= .2477991  X2=-.237989   X3=-9.908839E-03
T= .5999998    X1= .2466887  X2=-.8245731  X3=-.01
T= .6999996    X1= .2456887  X2=-1.186844  X3=-.01
T= .7999996    X1= .2446887  X2=-1.185846  X3=-.01
T= .8999994    X1= .2436887  X2=-.8215765  X3=-.01
T= .9999994    X1= .242601   X2=-.2325378  X3=-1.017716E-02
T= 1.099999    X1= .2517405  X2= .0229076  X3= .3160463
T= 1.199999    X1= .3020598  X2= 2.290731E-02             X3= .6323352
T= 1.299999    X1= .3653129  X2= 2.290725E-02             X3= .5686421
T= 1.399999    X1= .4041701  X2= 2.296728E-02             X3= .1625385
T= 1.499999    X1= .40597    X2=-.3961066  X3=-.01
T= 1.599999    X1= .4049698  X2=-.9828493  X3=-.01
T= 1.699999    X1= .4039697  X2=-1.345123  X3=-.01
T= 1.799999    X1= .4029695  X2=-1.344128  X3=-.01
T= 1.899998    X1= .4019693  X2=-.9798616  X3=-.01
T= 1.999998    X1= .4009692  X2=-.3910785  X3=-.01
T= 2.099998    X1= .4029374  X2= 2.296602E-02             X3= .16325
T= 2.199998    X1= .4387287  X2= 2.290731E-02             X3= .4942771
T= 2.299998    X1= .4888505  X2= 2.290728E-02             X3= .4438523
```

```
T= 2.399998    X1= .5158363   X2= 2.260231E-02                X3= 5.010827E-02
T= 2.499998    X1= .5152846   X2=-.505373    X3=-.01
T= 2.599998    X1= .5142844   X2=-1.092159   X3=-.01
T= 2.699998    X1= .5132842   X2=-1.454437   X3=-.01
T= 2.799998    X1= .5122841   X2=-1.453444   X3=-.01
T= 2.899998    X1= .5112839   X2=-1.08918    X3=-.01
T= 2.999998    X1= .5102837   X2=-.5003991   X3=-.01
T= 3.099997    X1= .5098185   X2= .0228486   X3= 5.544556E-02
T= 3.199997    X1= .5353376   X2= 2.290727E-02                X3= .3966861
T= 3.299997    X1= .5761771   X2= 2.290734E-02                X3= .3556404
T= 3.399997    X1= .5949484   X2=-4.895323E-03                X3=-2.144819E-03
T= 3.499997    X1= .5938921   X2=-.5839745   X3=-.01
T= 3.599997    X1= .592892    X2=-1.170763   X3=-.01
T= 3.699997    X1= .5918918   X2=-1.533042   X3=-.01
T= 3.799997    X1= .5908916   X2=-1.532054   X3=-.01
T= 3.899997    X1= .5898915   X2=-1.167791   X3=-.01
T= 3.999997    X1= .5888913   X2=-.5790117   X3=-.01
T= 4.099999    X1= .5878931   X2=-1.03853E-04                 X3=-5.248995E-05
T= 4.200001    X1= .6060416   X2= 2.290754E-02                X3= .3252707
T= 4.300003    X1= .6400875   X2= 2.290806E-02                X3= .2910684
T= 4.400006    X1= .6533518   X2=-5.651771E-02                X3=-9.145142E-03
T= 4.500008    X1= .6522652   X2=-.6424169   X3=-.01
T= 4.60001     X1= .651265    X2=-1.229203   X3=-.01
T= 4.700012    X1= .6502649   X2=-1.591446   X3=-.01
T= 4.800015    X1= .6492647   X2=-1.590392   X3=-.01
T= 4.900017    X1= .6482645   X2=-1.226062   X3=-.01
T= 5.000019    X1= .6472644   X2=-.6372411   X3=-.01
T= 5.100022    X1= .6462168   X2=-4.954961E-02                X3=-8.863117E-03
T= 5.200024    X1= .6592891   X2= .0229089   X3= .271526
T= 5.300026    X1= .6882188   X2= 2.291168E-02                X3= .2424
T= 5.400028    X1= .6977928   X2=-.1003937   X3=-9.854929E-03
T= 5.500031    X1= .6967916   X2=-.687089    X3=-.01
T= 5.600033    X1= .6957915   X2=-1.273845   X3=-.01
T= 5.700035    X1= .6947913   X2=-1.636017   X3=-.01
T= 5.800038    X1= .6937911   X2=-1.634873   X3=-.01
T= 5.90004     X1= .692791    X2=-1.270472   X3=-.01
T= 6.000042    X1= .6917908   X2=-.6816238   X3=-.01
T= 6.100045    X1= .690735    X2=-9.301545E-02                X3=-9.804601E-03
T= 6.200047    X1= .7003449   X2= 2.291403E-02                X3= .2300942
T= 6.300049    X1= .7253296   X2= 2.292301E-02                X3= .2048562
T= 6.400051    X1= .7323678   X2=-.1349781   X3=-9.96618E-03
T= 6.500054    X1= .7313145   X2=-.7217555   X3=-.01
T= 6.600056    X1= .7303143   X2=-1.308484   X3=-.01
T= 6.700058    X1= .7293141   X2=-1.670584   X3=-.01
T= 6.800061    X1= .728314    X2=-1.669352   X3=-.01
T= 6.900063    X1= .7273138   X2=-1.304876   X3=-.01
T= 7.000065    X1= .7263136   X2=-.7160029   X3=-.01
T= 7.100067    X1= .7252398   X2=-.1272584   X3=-9.950914E-03
T= 7.20007     X1= .732448    X2= 2.292714E-02                X3= .1976969
T= 7.300072    X1= .7543473   X2= 2.294768E-02                X3= .175474
T= 7.400074    X1= .7596063   X2=-.1623093   X3=-9.990711E-03
T= 7.500077    X1= .7585716   X2=-.7491563   X3=-.01
T= 7.600079    X1= .7575714   X2=-1.335861   X3=-.01
T= 7.700081    X1= .7565713   X2=-1.697885   X3=-.01
T= 7.800083    X1= .7555711   X2=-1.696564   X3=-.01
T= 7.900086    X1= .7545709   X2=-1.332017   X3=-.01
T= 8.000088    X1= .7535708   X2=-.7431164   X3=-.01
```