Claim 2.1 asserts that

$$C_n \leqslant 1.21 \frac{b-\lambda-n+1}{n} C_{n-1}.$$

Rewriting, this is equivalent to

$$\frac{C_n pk-p+2}{C_{n-1} p-3} \leqslant 1.21.$$

As $k \to \infty$ and p is held fixed, the numerator above is dominated by the coefficient of k^{p-1}, which is

$$\frac{p^{p-1}}{(p-2)!} - p\binom{p}{2}.$$

Similarly, the denominator is dominated by the coefficient of k^{p-1}, which is

$$\frac{(p-3)p^{p-1}}{(p-1)!} - p(p-3).$$

When $5 \leqslant p \leqslant 12$, this ratio *exceeds* 1.21; in fact, for $p=5$, the limit as $k \to \infty$ is approximately 1.278. This is in contradiction to claim 2.1; hence, for sufficiently large k, $G_{p,k}$ forms a counterexample of the form desired, when $5 \leqslant p \leqslant 12$. ∎

The basic idea of exploiting graph-theoretical structure to obtain bounds is sound; in fact, it appears to be the most promising method of improving on the current best, the Ball–Provan bounds. However, these two theorems show that additional constraints must be identified before applying a technique such as that proposed by Leggett.

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References

[1] M. O. Ball and J. S. Provan, "Bounds on the reliability polynomial for shellable independence systems," *SIAM J. Alg. Disc. Meth.*, vol. 3, pp. 166–181, 1982.

[2] M. O. Ball and J. S. Provan, "Calculating bounds on reachability and connectedness in stochastic networks," *Networks*, vol. 13, pp. 253–278, 1983.

[3] R. Bixby, "The minimum number of edges and vertices in a graph with edge connectivity N and MN-bonds," *Networks*, vol. 5, 1975, pp. 259–298.

[4] I. M. Jacobs, "Connectivity in probabilistic graphs," Rep. 356, Res. Lab. Electronics, Massachusetts Institute of Technology, Cambridge, 1959.

[5] G. Katona, "A theorem on finite sets," in *Theory of Graphs*, (P. Erdos, G. Katona, Eds.), 1966, pp. 209–214.

[6] J. B. Kruskal, "The number of simplices in a complex," in *Mathematical Optimization Techniques* (R. Bellman, Ed.) Univ. California, 1963, pp. 251–278.

[7] J. D. Leggett, "Synthesis of reliable communication networks," Ph.D. dissertation, Moore School of Engineering, University of Pennsylvania, 1968.

[8] J. D. Leggett and S. D. Bedrosian, "Synthesis of reliable networks," *IEEE Trans. Circuit Theory*, vol. CT-16, pp. 384–385, 1969.

[9] J. S. Provan, "Polyhedral combinatorics and network reliability," Tech. Rep., Operations Research Center, University of North Carolina, Chapel Hill, 1983.

[10] J. S. Provan and M. O. Ball, "The complexity of counting cuts and of deciding whether a graph is connected," *SIAM J. Comput.*, vol. 12, pp. 777–788, 1983.

[11] R. M. Van Slyke, private communication, 1983.

[12] R. M. Van Slyke and H. Frank, "Network Reliability Analysis. Part I," *Networks*, vol. 1, pp. 279–290, 1972.

An Extended Version Frequency Transistorized Active-*R* Integrator

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***Abstract*—An all bipolar transistor active-*R* integrator is presented which has an extended integration frequency range, especially on the low-frequency end. The structure consists of a voltage-controlled current source feeding an *N*-Darlington stage and allows independent adjustment of dc gain and low-frequency cutoff frequency. A SPICE-II simulation shows the suitability for integrated circuit realization.**

I. Introduction

Active-*R* circuits are circuits which contain no individual reactive elements, the circuits being constructed solely from transistors (or transistor built devices) and resistors. In essence they use parasitics to obtain the system dynamics. Consequently active-*R* filters hold considerable promise for high frequency filtering. At present there are active-*R* circuits based upon active elements that are CMOS transistors [1], [2], op-amps [3]–[5], or bipolar transistors [6]. For completely integrated structures the CMOS and bipolar transistor circuits are the most natural since they use a reduced number of components though the op-amp ones are convenient for lumped designs where off-the-shelf components are used. In all cases it has been the pole in the op-amp or transistor gains which have been used to achieve the necessary dynamics for filters and these poles have been rather critically and sensitively dependent upon bias conditions, so much so that it appears that reproducibility has been rather difficult to achieve. Further, at low frequencies the active elements act as resistors and the approximation that a nonzero pole acts as a pole at zero no longer holds, invalidating the assumptions often made in carrying out active-*R* circuit designs.

In an effort to get around these two problems of limited low-frequency operation and sensitivity to bias conditions we have turned to consideration of appropriately designed and voltage adjustable bipolar transistor circuits for realization of active-*R* integrators. With this in mind we return to a previous study using bipolar transistors as the active-*R* dynamical element [6]. However, we greatly improve upon [6] by using the recently studied *N*-Darlington configuration [7] such that control on the transistor dynamics can be obtained. We also combine this *N*-Darlington transistor with a modified version of Fukahori's [8] voltage-controlled current source (VCCS) to obtain a much improved active-*R* integrator.

II. Basic Configuration and Motivating Idea

Fig. 1 shows the configuration for the active-*R* integrator we propose and study here. In Fig. 1 an *N*-Darlington transistor stage is fed by a high impedance source, the VCCS shown. The reason for feeding the *N*-Darlington with a current source is that a related study [7] shows that the source resistance R_s for the *N*-Darlington stage is a limiting factor with lower cut-off frequencies determined by higher R_s. The input VCCS in fig. 1 is that of

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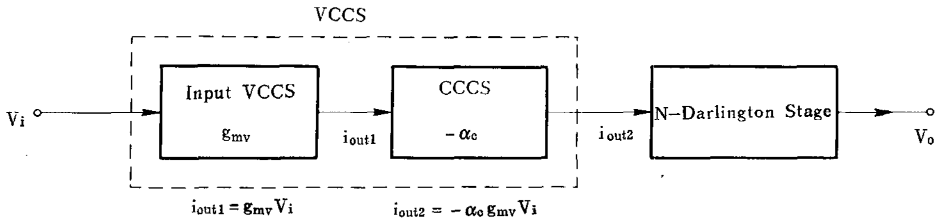

Fig. 1. Diagram of integrator.

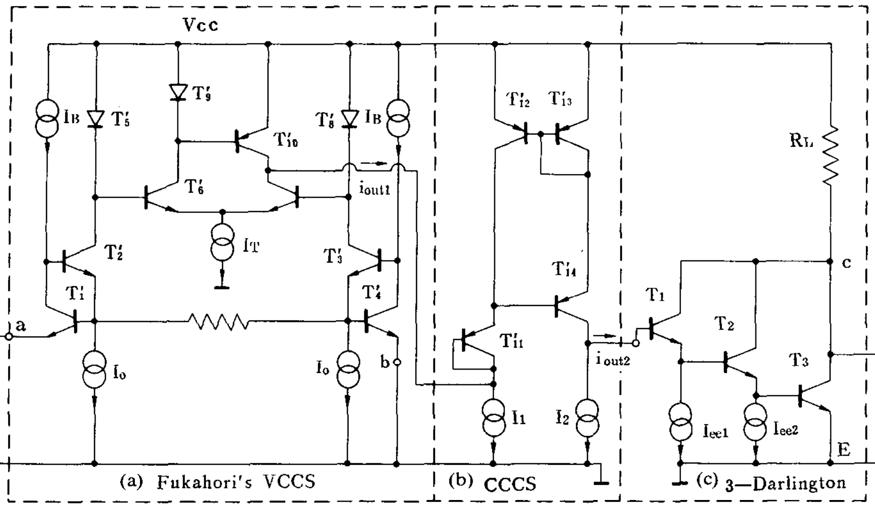

Fig. 2. Circuit and sections of integrator. (a) Fukahori's VCCS. (b) One scheme of CCCS. (c) 3-Darlington stage.

Fukahori [8], see Fig. 2, which uses only transistors and bias current sources. Hence the dc gain of the input VCCS holds over a wide frequency range. The current controlled current source, CCCS, of Fig. 1 is used to level shift the VCCS output to match the input of the N-Darlington stage. The CCCS is also shown in Fig. 2. The N-Darlington stage, shown for $N=3$ in Fig. 2, is used to introduce dynamics into the circuit, this arising primarily through the Miller effect on its equivalent circuit C_μ of Fig. 3. The N-Darlington stage consists of an N-Darlington transistor [7] with a resistor load in the common emitter configuration.

As we will see, (9d), over an extended frequency range the transfer function for Fig. 1 is to a good approximation

$$\frac{V_o}{V_i} \approx \frac{g_{mv}}{C_\mu} \cdot \frac{1}{s+\omega_c}, \qquad \omega_c = \frac{1}{r_\pi C_\mu [1 + g_m R_L]} \tag{1}$$

where the various parameters are defined through the Figs. 1–3. From (1) we see that the configuration of Fig. 1 acts as an integrator for $|s| > \omega_c$, that is for frequencies above the cutoff frequency. The cutoff frequency is seen to depend upon the Miller capacitance $C_\mu[1 + g_m R_L]$ and the hybrid-pi input resistance r_π. The N-Darlington configuration is designed to yield a large r_π as well as a large Miller capacitor so that ω_c can be made small.

By observing the circuits mentioned for realizing the subparts of Fig. 1 we see that the active-R integrator introduced here is easily realized by integrated circuit technology where typical numbers might be $r_\pi = 5$ MΩ, $g_m = 38$ mv, $R_L = 10$ KΩ, $C_\mu = 2$ pf giving $f_c = 42$ Hz.

III. Subcircuits

A. The N-Darlington Stage

The lower portion of the (c) section of Fig. 2 shows the N-Darlington transistor, for $N = 3$, this being the three terminal

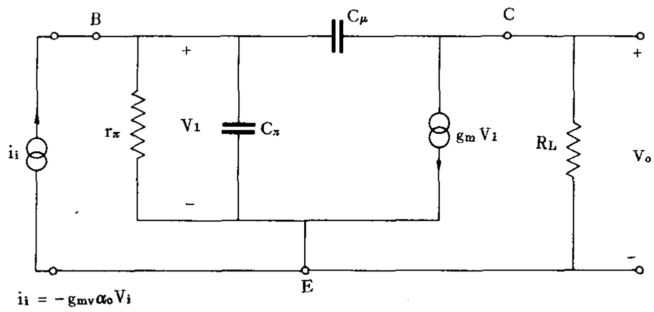

Fig. 3. Equivalent circuit of N-Darlington stage.

device inside nodes BCE. The extension to arbitrary N is made by inserting (or deleting) internal transistors such as T_2. The connection of R_L at point C to form a grounded emitter amplifier gives, again as in Fig. 2, what we call the N-Darlington stage.

Using all transistors to be identical, except possibly the left most (T_1 of Fig. 2) and by properly biasing we can obtain the small signal equivalent circuit of Fig. 3 for an N-Darlington stage where [7]

$$y_\pi = \frac{1}{\sum_{i=1}^{N} \left[\prod_{k=1}^{i-1} (1+\beta_k(s)) \right] z_{\pi_i}(s)} \approx g_\pi + sC_\pi. \tag{2a}$$

Here, i designates transistor T_i in the N-Darlington. For its individual transistors in terms of their hybrid-pi models we have

$$y_{\pi_i} = z_{\pi_i}^{-1} = g_{\pi_i} + sC_{\pi_i} = g_{\pi_i}(1+s\tau), \qquad \tau = C_{\pi_i}/g_{\pi_i} \tag{2b}$$

with τ the lifetime of base majority carriers, this being the same for all transistors (since all transistors are assumed to be on the

TABLE I
TRANSISTOR PARAMETERS FOR SPICE

Model	BF	BR	VPF V	IS A	RB Ω	RE Ω	RC Ω	TF ns	CJE pf	VJE V	MJE	CJC pf	VJC V	MJC	CJS pf	VJS V	MJS	Transistor
NPN	100	2	130	10^{-15}	200	2	200	0.35	1	0.7	0.33	0.3	0.55	0.5	3	0.52	0.5	T'_{1-8} T_{1-3}
PNP	100	4	50	10^{-14}	150	2	50	0.35	0.5	0.55	0.5	1	0.52	0.5	0	0	0	T'_{9-14}

same chip). Also

$$\beta_i(s) = g_{m_i}/y_{\pi_i}(s) = \frac{\beta_i(0)}{1+s\tau}. \tag{2c}$$

If we assume

$$\omega \ll (1/\tau) = \omega_m \tag{3}$$

then the approximation on the right side of (2a) holds with (here $r_{\pi_i} = 1/g_{\pi_i}$)

$$g_\pi = \frac{1}{\sum_{i=1}^{N}\left[\prod_{k=1}^{i-1}(1+\beta_k(0))\right] r_{\pi_i}}, \qquad C_\pi \approx 0. \tag{4}$$

Since typically $\omega_m \approx 10$ MHz, it is reasonable to make the assumption of (3) for most active-R filter applications, and we will so do.

If we also assume that, in the N-Darlington transistor, the upper transistor collector currents are dominated by those below them [7], as will be the case in a desirable design, then for Fig. 3

$$g_m \approx \frac{g_{m_N}}{\prod_{j=1}^{N-1}\left\{1+\frac{r_{\pi_{N-j}}}{\sum_{i=N+1-j}^{N}\left[\prod_{k=N-j}^{i-1}(1+\beta_k(0))\right] r_{\pi_i}}\right\}}. \tag{5}$$

With (5) all parameters for Fig. 3 are defined except for C_μ. Previously we found [7] all C_{μ_i} to have negligible effect except for $C_{\mu_1} = C_\mu$. Thus the upper left transistor in Fig. 2 should be designed to give the maximum possible C_μ in order, according to (1), to lower the cutoff frequency ω_c.

B. The Input VCCS

The input VCCS is shown as the a) section in Fig. 2 and is just the input portion of Fukahori's integrator [8]. The circuit has the advantage that it uses only paired transistors (including the diodes) and current sources, along with the one resistor R. Consequently is has a good frequency response so that its cutoff frequency is of the order of ω_m of (3) thus insuring that there is no interference with the integrating property.

Likewise this input VCCS is very simply implemented in integrated circuit form. Following Fukahori, the gain is [8]

$$\frac{i_{\text{out}\,1}}{V_i} = g_{mv} = \frac{1}{R}\frac{I_T}{I_o}. \tag{6}$$

In Fig. 2 we have used the noninverting input, point a. If we desire sign inversion from the input VCCS we can ground point a and introduce V_i between points b and ground, thus allowing for the opposite sign in (1).

C. The CCCS

The b) section in Fig. 2 shows the current mirror CCCS we use, this being a modified version of that in Herpy [9, p.107] (using n-p-n rather than p-n-p transistors and being biased at a relatively high current level in order to achieve a high f_T). As in Herpy we obtain the current gain as

TABLE II
OPERATING PARAMETERS AND CUT-OFF FREQUENCY
($V_{cc} = 24$ V, $R_L = 10$ KΩ)

N	I_O (μA)	I_T (μA)	B (μA)	I_1 (μA)	I_2 (μA)	I_{ee1} (μA)	I_{ee2} (μA)	f_c, C_μ (pf) 0	1	2	5
1	10.09	10	1.1	20	10			250KHZ	100KHZ	60KHZ	27 KHZ
2	10.09	10	1.1	10	9.81	0.1		3KHZ	1KHZ	600HZ	250HZ
3	10.09	10	1.1	10	9.81	9.1	91	1KHZ	100HZ	50HZ	25HZ

$$\frac{i_{\text{out}\,2}}{i_{\text{in}\,1}} = -\alpha_c = -\left[1 - \frac{2}{\beta^2+2\beta+2}\right] \approx -1 \tag{7}$$

where we assume all transistors have the same β. Because the transistors are paired the frequency response is good. The output impedance is also high, so that the N-Darlington following the CCCS is fed by a good current source, this being given by [9, p. 108]

$$R_{\text{out}} \approx \frac{r_e}{\mu}(1+\beta) \tag{8}$$

where $r_e = V_T/I_2$ is the emitter resistance of T_{14} and μ & V_T are the basewidth modulation factor and thermal voltage, respectively.

The important point about this CCCS is that it can be used to adequately shift the bias point at the output of the input VCCS down to the bias level needed for the input transistor of the N-Darlington section.

IV. THE ACTIVE-R INTEGRATOR GAIN

To obtain the overall gain for Fig. 2, we use the equivalent circuit of Fig. 3 getting

$$\frac{V_o}{V_i}(s) = g_{mv}\cdot(-\alpha_c) \cdot \frac{(-r_\pi R_L)(g_m - C_\mu s)}{R_L r_\pi C_\mu C_\pi s^2 + \left[C_\mu(r_\pi + R_L + g_m r_\pi R_L) + C_\pi r_\pi\right]s + 1}. \tag{9a}$$

This possesses two poles one of which is far from the other and can be dropped, the dropping of which is equivalent to $C_\pi \approx 0$ as

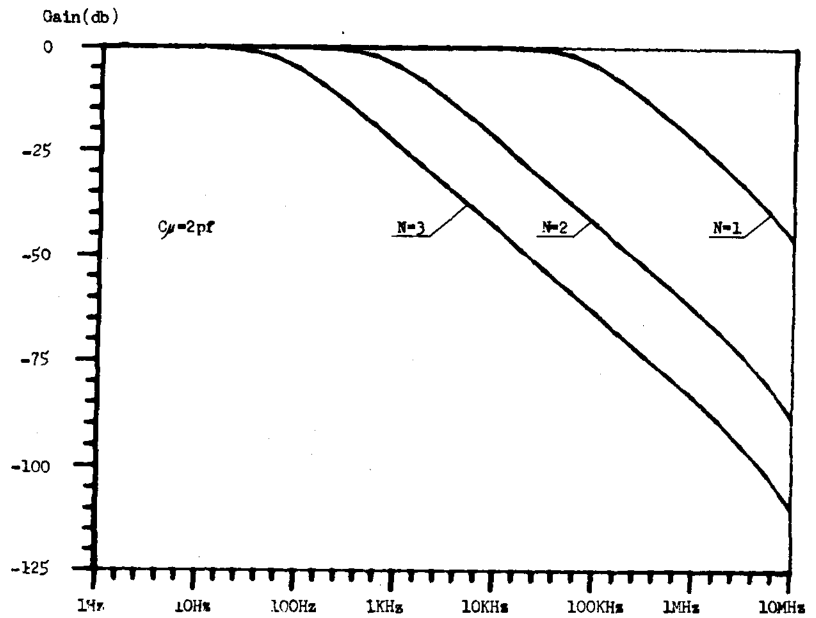

Fig. 4. Frequency responses of integrator with different N under $C_\mu = 2$ pF.

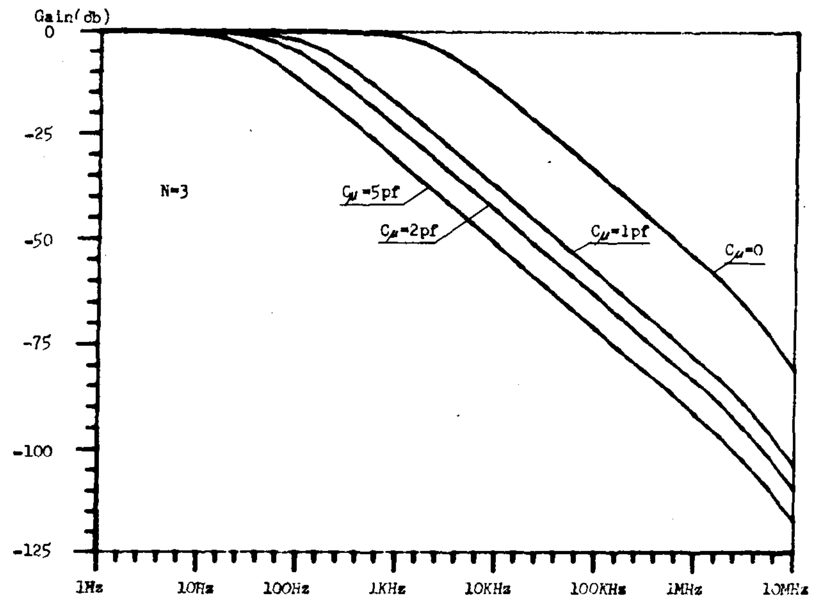

Fig. 5. Frequency response of integrator with different C_μ under $N = 3$.

per (4). Thus

$$\frac{V_o}{V_i}(s) \approx \alpha_c g_{mv} r_\pi R_L g_m \cdot \frac{\left(1 - s\frac{C_\mu}{g_m}\right)}{\left(1 + C_\mu\left[r_\pi(1 + g_m R_L) + R_L\right]s\right)}. \tag{9b}$$

But, by design

$$\frac{g_m}{C_\mu} \gg \omega_m, \qquad R_L \ll r_\pi \tag{9c}$$

and the transfer function becomes

$$\frac{V_o}{V_i} \approx \frac{\alpha_c g_{mv} r_\pi R_L g_m}{r_\pi C_\mu[1 + g_m R_L]} \cdot \frac{1}{s + \dfrac{1}{r_\pi C_\mu[1 + g_m R_L]}} \tag{9d}$$

which, with $\alpha_c \approx 1$ is (1). We see that (9d) is valid for $\omega \ll \omega_m$ and, hence, we have obtained an almost ideal integrator over the frequency range

$$\omega_c < \omega \ll \omega_m. \tag{10}$$

By example in the next section we give a feeling for the practical nature of these results.

V. Examples

In order to verify the theory, several examples were simulated on SPICE II [10].

Table I lists the different sets of transistor parameters we used on SPICE II for the n-p-n and p-n-p transistors. In using these, we always assumed that the bias source was $V_{cc} = 24$ V, the load resistor was $R_L = 10$ KΩ and the bias currents I_o, I_T, I_1, and I_2 were all fixed beforehand at the values given in Table II. Table II lists the various operating parameters for the integrator with different values for N and C_μ, these being $N = 1,2,3$ and $C_\mu = 0,1,2$ and 5 pF.

Fig. 4 shows a set of frequency responses with different N when $C_\mu = 2$ pF. Fig. 5 shows a set of frequency responses of the integrator with different C_μ when $N = 3$. The frequency responses of the input VCCS and the CCCS were also obtained as a check. These are almost identical with g_{mv} shown in Fig. 6 (the plot for α_a only differs by some slight wiggles at 2, 11 and 500 Hz).

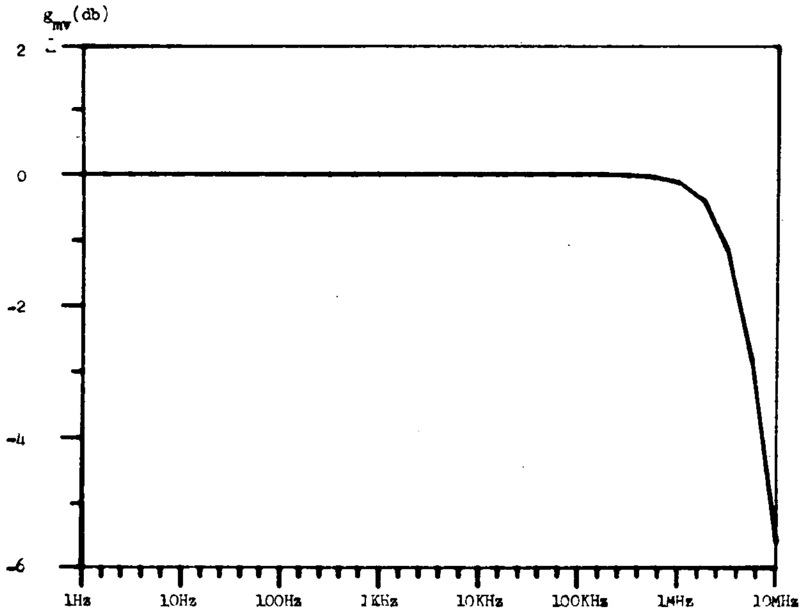

Fig. 6. Frequency response of the input VCCS.

VI. Discussion

Here we have presented an improved version of an all bipolar transistor–resistor integrator useful for integrated circuit realization of active-R systems. The circuit extends the frequency range of transistorized active-R integrators, and hence filters, to a lower cutoff frequency of tens of hertz (compared to previously reported results of 500 Hz for CMOS structures [1]) and to an upper cutoff frequency of low megahertz (comparing favorably with our previous results on CMOS structures [2] and the 250 kHz reported for op-amp configurations [11, p. 252]). Although it is true some previous op-amp structures had low end cut-off frequencies on the order of tens of hertz [3, p. 430] internal capacitors were apparently already incorporated in the compensation. The key component, which allows us to overcome the shortcomings mentioned in [5, p. 89], is the N-Darlington transistor with the key technique being that of feeding it with a wide-band current source. Fukahori's circuit acts to allow us to make the integrator a voltage-to-voltage one but if other types of transfer functions are desired, such as current-to-voltage, other input stages can be used (for example, simply omitting the input VCCS in the current-to-voltage case). The SPICE simulation results of Section V show that the integrator can be practically designed for realization as an integrated circuit. We do comment that *the gain and the low frequency cutoff frequency can independently be adjusted* in our integrator, something which does not happen in the previous CMOS and op-amp circuits. Thus the dc gain is, from (9b), $K_o \approx g_{mv} r_\pi R_L g_m$. By adjusting g_{mv}, the input VCCS transconductance, the dc gain can be varied while $\omega_c \approx \{r_\pi C_\mu [1 + g_m R_L]\}^{-1}$ is independent of g_{mv}. For the Fukahori circuit g_{mv} can be adjusted via (6), $g_{mv} = I_T / [R I_o]$; in our example we used $I_o = 1000 I_T$ but other ratios could be used. Since a wide range of I_o / I_T is possible a wide range of K_o can be achieved. If for some reason other configurations are desired it may be worth looking for other VCCS circuits or incorporating gain in the CCCS. In any event the ability to separately adjust K_o and ω_c in active-R circuits should prove advantageous and is something realized by the circuits given here. And, because g_{mv} is variable with I_T / I_0 and the two current sources for I_T and I_0 can be conveniently adjusted by respective control voltages, any misalignments due to parameter variances can be easily compensated by adjustment via voltage control.

The integrator can replace the previously proposed bipolar one [6] while using the same synthesis technique to realize given transfer functions; with the circuits of the present paper more stable results occur since less predistortion is needed with the smaller ω_c.

References

[1] A. Soderstrand, "A simple synthesis technique for active R filter," in *Proc. IEEE 75 ISCAS*, pp. 261–265.

[2] O. A. Seriki, G. Indrajo, and R. W. Newcomb, "High frequency, extended CMOS active-R filters," in *Proc. Twenty-first Midwest Symp. CAS*, Aug. 1978, pp. 174–178.

[3] A. Soderstrand, "Design of active R filters using only resistors and operational amplifiers," *Int. J. Electronics*, vol. 40, no. 5, pp. 417–432, 1976.

[4] S. Srinivasan, "Synthesis of transfer functions using the operational amplifier pole," *Int. J. Electronics*, vol. 40, no. 1, pp. 5–13, 1976.

[5] J. R. Brand and R. Schaumann, "Active R filter: Review of theory and practice," *Proc. Inst. Elect. Eng., Part G, Electronic Circuits and Systems*, vol. 2, no. 4, pp. 89–101, July 1978.

[6] B. D. Berman and R. W. Newcomb, "Transistor–Resistor synthesis of voltage transfer functions," *IEEE Trans. Circuit Theory*, vol. CT-20, pp. 591–593, Sept. 1973.

[7] S. T. Liu and R. Newcomb, "The N-Darlington stage: An active R integrator," in *Proc. China 1985 Int. Conf. on Circuits and Systems*, Beijing, China, June 1985.

[8] K. Fukahori, "A bipolar voltage-controlled tunable filter," *IEEE J. Solid-State Circuits*, vol. SC-16, pp. 729–737, Dec. 1981.

[9] M. Herpy, *Analog Integrated Circuits*. New York: Wiley, 1980.

[10] A. Vladimirescu, A. R. Newton, and D. O. Pederson, "SPICE Version 2F.1 User's Guide," Dept. of Elect. Eng. and Computer Sci., Univ. of California, Berkeley, CA.

[11] M. S. Ghausi and K. R. Laker, *Modern Filter Design, Active RC and Switched Capacitor*. Englewood Cliffs, NJ: Prentice-Hall, 1981.